Development of Multi-Dimensional RELAP5 with Conservative Momentum Flux

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1. Introduction

The non-conservative form of the momentum equations are used in many codes [1,2,3,4]. But potential problems of using the non-conservative form of momentum equations have been investigated with CUPID code [8]. It tells us that using the non-conservative form in the non-porous or open body problem may not be good.

In this paper, two aspects concerning the multi-dimensional codes will be discussed. One of them is the properness of the type of the momentum equations. The other discussion will be the implementation of the conservative momentum flux term in RELAP5.

2. Momentum Equations in the Codes

2.1. Momentum equations in various forms

The multi-dimensional effects are simulated with the proper treatment of the momentum flux term in the momentum balance equations. The regular mass and momentum balance equations are written in conservative form as;

$$\frac{\partial \alpha_g \rho_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \boldsymbol{v}_g) = \Gamma_g \quad (1)$$

$$\frac{\partial \alpha_l \rho_l}{\partial t} + \nabla \cdot (\alpha_l \rho_l \boldsymbol{v}_l) = -\Gamma_g \quad (2)$$

$$\frac{\partial \alpha_g \rho_g \boldsymbol{v}_g}{\partial t} + \nabla \cdot (\alpha_g \rho_g \boldsymbol{v}_g \boldsymbol{v}_g) = -\alpha_g \nabla p - \alpha_g \rho_g F_g^w \boldsymbol{v}_g - \alpha_g \rho_g \alpha_l \rho_l F_i (\boldsymbol{v}_g - \boldsymbol{v}_l) + \Gamma_g \left((1-\theta)\boldsymbol{v}_g + \theta \boldsymbol{v}_l\right) + \alpha_g \rho_g \boldsymbol{g} \quad (3)$$

$$\frac{\partial \alpha_l \rho_l \boldsymbol{v}_l}{\partial t} + \nabla \cdot (\alpha_l \rho_l \boldsymbol{v}_l \boldsymbol{v}_l) = -\alpha_l \nabla p - \alpha_l \rho_l F_l^w \boldsymbol{v}_l - \alpha_g \rho_g \alpha_l \rho_l F_i (\boldsymbol{v}_l - \boldsymbol{v}_g) - \Gamma_g \left((1-\theta)\boldsymbol{v}_g + \theta \boldsymbol{v}_l\right) + \alpha_l \rho_l \boldsymbol{g} \quad (4)$$

$$\theta = \begin{Bmatrix} 1 \; if \; \Gamma_g \geq 0.0 \\ 0 \; if \; \Gamma_g < 0.0 \end{Bmatrix} \quad (5)$$

Non-conservative form of the momentum balance equation can be derived by expanding eqn.(3,4) and using mass conservation equation, eqn.(1,2);

$$\alpha_g \rho_g \frac{\partial \boldsymbol{v}_g}{\partial t} + \alpha_g \rho_g \boldsymbol{v}_g \nabla \cdot \boldsymbol{v}_g = -\alpha_g \nabla p - \alpha_g \rho_g F_g^w \boldsymbol{v}_g - \alpha_g \rho_g \alpha_l \rho_l F_i (\boldsymbol{v}_g - \boldsymbol{v}_l) + \Gamma_g \left((1-\theta)\boldsymbol{v}_g + \theta \boldsymbol{v}_l\right) - \Gamma_g \boldsymbol{v}_g + \alpha_g \rho_g \boldsymbol{g} \quad (6)$$

$$\alpha_l \rho_l \frac{\partial \boldsymbol{v}_l}{\partial t} + \alpha_l \rho_l \boldsymbol{v}_l \nabla \cdot \boldsymbol{v}_l = -\alpha_f \nabla p - \alpha_l \rho_l F_l^w \boldsymbol{v}_l - \alpha_g \rho_g \alpha_l \rho_l F_i (\boldsymbol{v}_l - \boldsymbol{v}_g) - \Gamma_g \left((1-\theta)\boldsymbol{v}_g + \theta \boldsymbol{v}_l\right) - \Gamma_g \boldsymbol{v}_l + \alpha_l \rho_l \boldsymbol{g} \quad (7)$$

The phase intensive equations are written;

$$\frac{\partial \boldsymbol{v}_g}{\partial t} + \boldsymbol{v}_g \nabla \cdot \boldsymbol{v}_g = -\frac{1}{\rho_g} \nabla p - F_g^w \boldsymbol{v}_g - \alpha_l \rho_l F_i (\boldsymbol{v}_g - \boldsymbol{v}_l) + \frac{\Gamma_g}{\alpha_g \rho_g} \left((1-\theta)\boldsymbol{v}_g + \theta \boldsymbol{v}_l - \boldsymbol{v}_g\right) + \boldsymbol{g} \quad (8)$$

$$\frac{\partial \boldsymbol{v}_l}{\partial t} + \boldsymbol{v}_l \nabla \cdot \boldsymbol{v}_l = -\frac{1}{\rho_f} \nabla p - F_l^w \boldsymbol{v}_l - \alpha_g \rho_g F_i (\boldsymbol{v}_l - \boldsymbol{v}_g) - \frac{\Gamma_g}{\alpha_l \rho_l} \left((1-\theta)\boldsymbol{v}_g + \theta \boldsymbol{v}_l + \boldsymbol{v}_l\right) + \boldsymbol{g} \quad (9)$$

The immediate problem with these equations is that discretizing the eqn.(8,9) in finite volume method is not possible. But the finite difference approach can be adopted for discretizing those equations. To overcome this problem, Weller [7] used the modified non-conservative momentum equations as follows;

$$\begin{pmatrix} \boldsymbol{v}_g \cdot \nabla \boldsymbol{v}_g \equiv \nabla \cdot (\boldsymbol{v}_g \boldsymbol{v}_g) - \boldsymbol{v}_g (\nabla \cdot \boldsymbol{v}_g) \\ \boldsymbol{v}_l \cdot \nabla \boldsymbol{v}_l \equiv \nabla \cdot (\boldsymbol{v}_l \boldsymbol{v}_l) - \boldsymbol{v}_l (\nabla \cdot \boldsymbol{v}_l) \end{pmatrix} \quad (10)$$

It was realized that the estimated momentum fluxes with them are not correct because they are not reflecting mass flux effects correctly. To overcome this problem, the mass weighted modified non-conservative method [8] is used. In that, the following equality is used.

$$\begin{pmatrix} \alpha_g \rho_g \boldsymbol{v}_g \nabla \cdot \boldsymbol{v}_g \equiv \nabla \cdot (\alpha_g \rho_g \boldsymbol{v}_g \boldsymbol{v}_g) - \boldsymbol{v}_g \nabla \cdot (\alpha_g \rho_g \boldsymbol{v}_g) \\ \alpha_l \rho_l \boldsymbol{v}_l \nabla \cdot \boldsymbol{v}_l \equiv \nabla \cdot (\alpha_l \rho_l \boldsymbol{v}_l \boldsymbol{v}_l) - \boldsymbol{v}_l \nabla \cdot (\alpha_l \rho_l \boldsymbol{v}_l) \end{pmatrix} \quad (11)$$

Various forms of the momentum balance equations are used to implement the solution schemes for the individual codes. Table-1 shows such variations.

Table-1. Treatment of Momentum Equation in Codes

SPACE	RELAP5	RELAP5-3D	TRAC/ TRACE	CATHARE	COBRA-TF	CUPID
3-d.	1-d	3-d	3-d	3-d	3-d	3-d
non-cons.	non-cons.	non-cons.	non-cons.	mod. cons.	cons.	mod cons.
phase int.	phase int.	phase int.	phase int.	mass weight	regular	mass weight
rect cyl	network	rect cyl	rect cyl	rect cyl sph	rect	unst.
fvm	fdm	fdm	fdm	fvm	fvm	fvm

2.2. Discretization methods

Discretization of the multi-fluid governing equations can be performed through the finite volume method. Following discussions are made based on the Euler implicit finite volume discretization on the staggered mesh. For example, mass and momentum balance equations are discretized as follows;

$$\frac{\alpha_g^{n+1}\rho_g^{n+1} - \alpha_g^n \rho_g^n}{\Delta t} V + \sum_f \alpha_{g,f}^{n+1} \rho_{g,f}^{n+1} v_{g,f}^{n+1} A_f = {\Gamma_g}^{n+1} V \quad (12)$$

$$\frac{\alpha_l^{n+1}\rho_l^{n+1} - \alpha_l^n \rho_l^n}{\Delta t} V + \sum_f \alpha_{l,f}^{n+1} \rho_{l,f}^{n+1} v_{l,f}^{n+1} A_f = -{\Gamma_g}^{n+1} V \quad (13)$$

$$\frac{\alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1} - \alpha_{g,f}^n \rho_{g,f}^n v_{g,f}^n}{\Delta t} V_f + \sum_{f'} \alpha_g^{n+1}\rho_g^{n+1} v_g^{n+1} v_{g,f'}^{n+1} A_{f'} = -\alpha_{g,f}^{n+1}(p_{f''}^{n+1} - p_{f'''}^{n+1}) A_f - \alpha_{g,f}^{n+1}\rho_{g,f}^{n+1} F_g^{w(n+1)} v_{g,f}^{n+1} - \alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1} F_i^{n+1}(v_{g,f}^{n+1} - v_{l,f}^{n+1}) + \Gamma_g^{n+1}\left((1-\theta)v_{g,f}^{n+1} + \theta v_{l,f}^{n+1}\right) + \alpha_{g,f}^{n+1}\rho_{g,f}^{n+1} g \quad (14)$$

$$\frac{\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1} - \alpha_{l,f}^n \rho_{l,f}^n v_{l,f}^n}{\Delta t} V_f + \sum_{f'} \alpha_l^{n+1}\rho_l^{n+1} v_l^{n+1} v_{l,f'}^{n+1} A_{f'} = -\alpha_{l,f}^{n+1}(p_{f''}^{n+1} - p_{f'''}^{n+1}) A_f - \alpha_{l,f}^{n+1}\rho_{l,f}^{n+1} F_l^{w(n+1)} v_{l,f}^{n+1} - \alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1} F_i^{n+1}(v_{l,f}^{n+1} - v_{g,f}^{n+1}) - \Gamma_g^{n+1}\left((1-\theta)v_{g,f}^{n+1} + \theta v_{l,f}^{n+1}\right) + \alpha_{l,f}^{n+1}\rho_{l,f}^{n+1} g \quad (15)$$

Most of the generalized computer programs [1,2,3,8] of two-phase flow for industrial use adopt the semi-implicit scheme in their discretization procedure [11]. Further linearization is applied as long as the acoustic implicitness is met and the necessary implicitness for the source term is not hurt.

$$\frac{\alpha_g^{n+1}\rho_g^{n+1} - \alpha_g^n \rho_g^n}{\Delta t} V + \sum_f \alpha_{g,f}^{n} \rho_{g,f}^{n} v_{g,f}^{n+1} A_f = {\Gamma_g}^{n+1} V \quad (16)$$

$$\frac{\alpha_l^{n+1}\rho_l^{n+1} - \alpha_l^n \rho_l^n}{\Delta t} V + \sum_f \alpha_{l,f}^{n} \rho_{l,f}^{n} v_{l,f}^{n+1} A_f = -{\Gamma_g}^{n+1} V \quad (17)$$

$$\frac{\alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1} - \alpha_{g,f}^n \rho_{g,f}^n v_{g,f}^n}{\Delta t} V_f + \sum_{f'} \alpha_g^{n}\rho_g^{n} v_g^{n} v_{g,f'}^{n} A_{f'} = -\alpha_{g,f}^{n}(p_{f''}^{n+1} - p_{f'''}^{n+1}) A_f - \alpha_{g,f}^{n}\rho_{g,f}^{n} F_g^{w} v_{g,f}^{n+1} V_f - \alpha_{g,f}^{n}\rho_{g,f}^{n}\alpha_{l,f}^{n}\rho_{l,f}^{n} F_i^{n}(v_{g,f}^{n+1} - v_{l,f}^{n+1}) V_f$$

$$+\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{g,f}^n\rho_{g,f}^n gV_f \quad (18)$$

$$\frac{\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1}-\alpha_{l,f}^n\rho_{l,f}^n v_{l,f}^n}{\Delta t}V_f+\sum_{f'}\alpha_l^n\rho_l^n v_l^n v_{l,f'}^n A_{f'}$$
$$=-\alpha_{l,f}^n\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f-\alpha_{l,f}^n\rho_{l,f}^n F_l^w v_{l,f}^{n+1}V_f$$
$$-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^{n+1}\left(v_{l,f}^{n+1}-v_{g,f}^{n+1}\right)V_f$$
$$-\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{l,f}^n\rho_{l,f}^n gV_f \quad (19)$$

The same procedure can be applied to non-conservative form of momentum equations to get;

$$\alpha_{g,f}^n\rho_{g,f}^n\frac{v_{g,f}^{n+1}-v_{g,f}^n}{\Delta t}V_f+\sum_{f'}\alpha_g^n\rho_g^n v_g^n v_{g,f'}^n A_{f'}$$
$$-v_{g,f}^n\sum_{f'}\alpha_g^n\rho_g^n v_{g,f'}^n A_{f'}=-\alpha_{g,f}^n\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f$$
$$-\alpha_{g,f}^n\rho_{g,f}^n F_g^w v_{g,f}^{n+1}V_f-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^n\left(v_{g,f}^{n+1}-v_{l,f}^{n+1}\right)V_f$$
$$+\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f-\Gamma_g^n v_{g,f}^{n+1}V_f+\alpha_{g,f}^n\rho_{g,f}^n gV_f \quad (20)$$

$$\alpha_{l,f}^n\rho_{l,f}^n\frac{v_{l,f}^{n+1}-v_{l,f}^n}{\Delta t}V_f+\sum_{f'}\alpha_l^n\rho_l^n v_l^n v_{l,f'}^n A_{f'}$$
$$-v_{l,f}^n\sum_{f'}\alpha_l^n\rho_l^n v_{l,f'}^n A_{f'}=-\alpha_{l,f}^n\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f$$
$$-\alpha_{l,f}^n\rho_{l,f}^n F_l^w v_{l,f}^{n+1}V_f-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^{n+1}\left(v_{l,f}^{n+1}-v_{g,f}^{n+1}\right)V_f$$
$$-\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f-\Gamma_g^n v_{l,f}^{n+1}V_f+\alpha_{l,f}^n\rho_{l,f}^n gV_f \quad (21)$$

The above discretized and linearized momentum equations are coupled simultaneous equations for velocities and rearranged as following form;

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}v_{g,f}^{n+1}\\ v_{l,f}^{n+1}\end{pmatrix}=\begin{pmatrix}r\\ s\end{pmatrix}\left(\delta p_{f''}^{n+1}-\delta p_{f'''}^{n+1}\right)+\begin{pmatrix}t\\ u\end{pmatrix} \quad (22)$$

where a, b, c, d, r, s, t and u are functions of the known properties. In Liles scheme, this equation is solved for intermediate velocities $v_{g,f}^*$ and $v_{l,f}^*$ and they are inserted into the mass and energy conservation equations to construct system pressures matrix [11]. RELAP5, CUPID and TRACE use the non-conservative momentum equations and follow the same procedure.

COBRA-TF, however, use fully conservative momentum equations like eqn.(16,17). It is very interesting to see that, unlike the case with the non-conservative momentum equations, some manipulation of the discretized momentum equations should be made before they can be solved for momentum fluxes, $\alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1}$ and $\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1}$. In COBRA-TF, the implicit velocities in eqn.(16,17) are changed to momentum fluxes as follows

$$\begin{pmatrix}v_{g,f}^{n+1}\rightarrow\dfrac{1}{\alpha_{g,f}^n\rho_{g,f}^n}\alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1}\\ v_{l,f}^{n+1}\rightarrow\dfrac{1}{\alpha_{l,f}^n\rho_{l,f}^n}\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1}\end{pmatrix} \quad (23)$$

Then, simultaneous equations are constructed for the momentum fluxes;

$$\begin{pmatrix}a' & b'\\ c' & d'\end{pmatrix}\begin{pmatrix}\alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1}\\ \alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1}\end{pmatrix}=\begin{pmatrix}r'\\ s'\end{pmatrix}\left(\delta p_{f''}^{n+1}-\delta p_{f'''}^{n+1}\right)+\begin{pmatrix}t'\\ u'\end{pmatrix} \quad (24)$$

where a', b', c', d', r', s' and u' are functions of the known properties. This equation is solved for intermediate mass fluxes, $\alpha_{g,f}^*\rho_{g,f}^*v_{g,f}^*$ and $\alpha_{l,f}^*\rho_{l,f}^*v_{l,f}^*$. Before they are inserted into the mass and energy equations, they are factored for intermediate velocities with the same relationships as eqn.(23);

$$\begin{pmatrix}\dfrac{1}{\alpha_{g,f}^n\rho_{g,f}^n}\alpha_{g,f}^*\rho_{g,f}^*v_{g,f}^*\rightarrow v_{g,f}^*\\ \dfrac{1}{\alpha_{l,f}^n\rho_{l,f}^n}\alpha_{l,f}^*\rho_{l,f}^*v_{l,f}^*\rightarrow v_{l,f}^*\end{pmatrix} \quad (25)$$

This process is simply equivalent to solving the equations;

$$\alpha_{g,f}^n\rho_{g,f}^n\frac{v_{g,f}^{n+1}-v_{g,f}^n}{\Delta t}V_f+\sum_{f'}\alpha_g^n\rho_g^n v_g^n v_{g,f'}^n A_{f'}$$
$$=-\alpha_{g,f}^n\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f-\alpha_{g,f}^n\rho_{g,f}^n F_g^w v_{g,f}^{n+1}V_f$$
$$-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^n\left(v_{g,f}^{n+1}-v_{l,f}^{n+1}\right)V_f$$
$$+\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{g,f}^n\rho_{g,f}^n gV_f \quad (26)$$

$$\alpha_{l,f}^n\rho_{l,f}^n\frac{v_{l,f}^{n+1}-v_{l,f}^n}{\Delta t}V_f+\sum_{f'}\alpha_l^n\rho_l^n v_l^n v_{l,f'}^n A_{f'}$$
$$=-\alpha_{l,f}^n\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f-\alpha_{l,f}^n\rho_{l,f}^n F_l^w v_{l,f}^{n+1}V_f$$
$$-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^{n+1}\left(v_{l,f}^{n+1}-v_{g,f}^{n+1}\right)V_f$$
$$-\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{l,f}^n\rho_{l,f}^n gV_f \quad (27)$$

In other words, it is equivalent to the fact that the following assumption is made in the temporal derivative terms;

$$\begin{pmatrix}\dfrac{\partial\alpha_g\rho_g}{\partial t}=0\\ \dfrac{\partial\alpha_l\rho_l}{\partial t}=0\end{pmatrix} \quad (28)$$

This setting might be understood as a kind of linearization. But it is a kind of the inconsistent discretization [12]. Therefore, COBRA-TF and its derivatives such as COBRA/TRAC [5] and WCOBRA/ TRAC [13] may need to be investigated carefully.

Instead of the above inconsistent linearization, one can get more rigorous and consistent discretization if the non-conservative time derivative term is used;

$$\alpha_g\rho_g\frac{\partial\boldsymbol{v}_g}{\partial t}+\boldsymbol{v}_g\frac{\partial\alpha_g\rho_g}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\boldsymbol{v}_g\boldsymbol{v}_g\right)$$
$$=-\alpha_g\nabla p-\alpha_g\rho_g F_g^w\boldsymbol{v}_g-\alpha_g\rho_g\alpha_l\rho_l F_i\left(\boldsymbol{v}_g-\boldsymbol{v}_l\right)$$
$$+\Gamma_g\left((1-\theta)\boldsymbol{v}_g+\theta\boldsymbol{v}_l\right)+\alpha_g\rho_g\boldsymbol{g} \quad (29)$$

$$\alpha_l\rho_l\frac{\partial\boldsymbol{v}_l}{\partial t}+\boldsymbol{v}_l\frac{\partial\alpha_l\rho_l}{\partial t}+\nabla\cdot\left(\alpha_l\rho_l\boldsymbol{v}_l\boldsymbol{v}_l\right)=$$
$$-\alpha_l\nabla p-\alpha_l\rho_l F_l^w\boldsymbol{v}_l-\alpha_g\rho_g\alpha_l\rho_l F_i\left(\boldsymbol{v}_l-\boldsymbol{v}_g\right)$$
$$-\Gamma_g\left((1-\theta)\boldsymbol{v}_g+\theta\boldsymbol{v}_l\right)+\alpha_l\rho_l\boldsymbol{g} \quad (30)$$

Their discretized and linearized forms are as follows;

$$\alpha_{g,f}^n\rho_{g,f}^n\frac{v_{g,f}^{n+1}-v_{g,f}^n}{\Delta t}V_f+v_{g,f}^{n+1}\frac{\alpha_{g,f}^n\rho_{g,f}^n-\alpha_{g,f}^{n-1}\rho_{g,f}^{n-1}}{\Delta t}V_f$$
$$+\sum_{f'}\alpha_g^n\rho_g^n v_g^n v_{g,f'}^n A_{f'}=-\alpha_{g,f}^n\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f$$
$$-\alpha_{g,f}^n\rho_{g,f}^n F_g^w v_{g,f}^{n+1}V_f-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^n\left(v_{g,f}^{n+1}-v_{l,f}^{n+1}\right)V_f$$
$$+\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{g,f}^n\rho_{g,f}^n gV_f \quad (31)$$

$$\alpha_{l,f}^n\rho_{l,f}^n\frac{v_{l,f}^{n+1}-v_{l,f}^n}{\Delta t}V_f+v_{l,f}^{n+1}\frac{\alpha_{l,f}^n\rho_{l,f}^n-\alpha_{l,f}^{n-1}\rho_{l,f}^{n-1}v_{l,f}^{n-1}}{\Delta t}V_f$$
$$+\sum_{f'}\alpha_l^n\rho_l^n v_l^n v_{l,f'}^n A_{f'}=-\alpha_{l,f}^n\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f$$
$$-\alpha_{l,f}^n\rho_{l,f}^n F_l^w v_{l,f}^{n+1}V_f-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^{n+1}\left(v_{l,f}^{n+1}-v_{g,f}^{n+1}\right)V_f$$
$$-\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{l,f}^n\rho_{l,f}^n gV_f \quad (32)$$

The second terms of eqn.(31,32) are regarded as source terms. Mass derivative terms in them are constructed with the $n-1$ step values.

It is interesting to note that the terms $\Gamma_g^n v_{g,f}^{n+1}V_f$ and $\Gamma_f^n v_{l,f}^{n+1}V_f$ in the eqn.(20,21), are evaluated in old time step. They are equivalent to the following equations;

$$\Gamma_g^n V_f=\frac{\alpha_{g,f}^n\rho_{g,f}^n-\alpha_{g,f}^{n-1}\rho_{g,f}^{n-1}}{\Delta t}V_f+\sum_{f'}\alpha_{g,f'}^n\rho_{g,f'}^n v_{g,f'}^n A_{f'} \quad (33)$$

$$\Gamma_f^n V_f=\frac{\alpha_{l,f}^n\rho_{l,f}^n-\alpha_{l,f}^{n-1}\rho_{l,f}^{n-1}}{\Delta t}V_f+\sum_{f'}\alpha_{l,f'}^n\rho_{l,f'}^n v_{l,f'}^n A_{f'} \quad (34)$$

It means that it is a retarded correction like the mass derivative term in eqn.(31,32).

Lastly, it can be noted that using the phase intensive form, eqn.(8,9) instead of the non-conservative form, eqn.(6,7) makes, basically, no difference because solving the simultaneous momentum equations for both cases are equivalent except the scale factors, $\alpha_{k,f}^n\rho_{k,f}^n$.

2.3. Development of multi-Dimensional RELAP5 by inserting the conservative momentum flux terms

Since RELAP5 is basically developed through the one-dimensional non-conservative finite difference approach, at a first glance, it seems to be very difficult to implement the momentum flux in conservative form. But a little careful investigation is enough to recognize that the implemented algorithms in RELAP5 are directly applicable to the conservative form. Instead of the spatially non-conservative equations, eqn.(6,7), equivalently, one can solve the temporally non-conservative equations, eqn.(29,30).

There are three corrections to be made to change RELAP5 to 3-dimensional code with fully conservative momentum flux terms with temporally non-conservative momentum balance equations.

1. Remove 1-dimensiobnal momentum flux terms.

2. Insert 3-dimensiobnal momentum flux terms
3. Remove the term, $\Gamma_k^n v_f^{n+1}$.
4. Add the mass derivative term;

The validity of the implementation is checked through the simple 2-dimensional conceptual flow test simulation [14]. Once the validity of the modified code is confirmed, it is applied to the analysis of the large break LOCA for APR-1400.

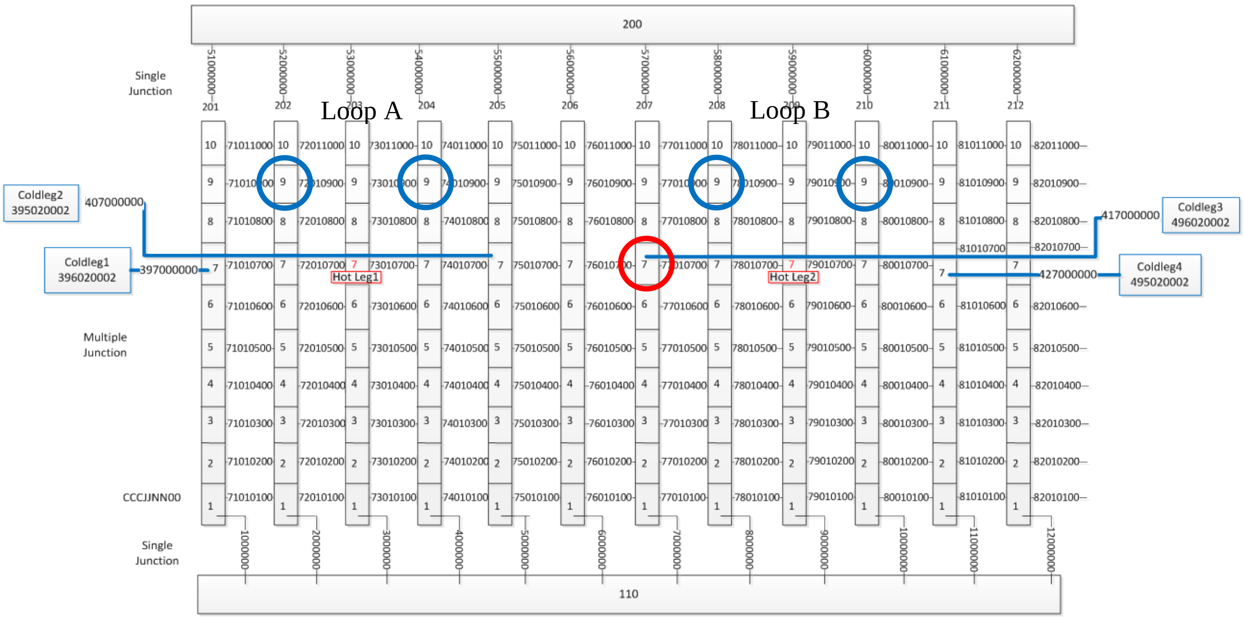

Fig.1. Downcomer Nodalization with 12 Pipes (blue circle: SIT injection point, red circle: Break Point)

3. Application of RELAP5-Multi-D to the 2-dimensional Downcomer Model for APR-1400 LOCA Analysis.

The typical downcomer model for the APR-1400 is shown in Fig.2. Usually 6 pipes are used for modelling the downcomer. The connections to the cold/hot legs are done through the exit faces of the downcomer pipes. It means that cross-flow junction option is not used.

For this study, 12 pipes are used for modelling the downcomer to have finer resolution of the flow field. Cross flow junction options are fully utilized for connecting 12 pipes. The newly developed RELAP-5-Multi-D uses the full cross-flow connections to implement the cross flow convection terms. New nodalization for the downcomer is shown in Fig.1. The purpose of this study is the first try to assess the applicability of the conservative momentum flux implementation with the real plant calculation. Therefore, any detailed study of the calculation results has not been performed yet. But brief look at the comparison calculations between cross flow model and the full 2-dimensiunal model will be introduced.

3.1. Comparison of the Peak Cladding Temperatures (PCT)

As shown in Fig.3, the peak cladding temperatures between the two cases are performed. During the blowdown, the cladding temperature shows the similar trend. The reason for this trend can be understood by the fact that the flow during the blowdown period is mainly determined by the pressure gradient from the core to the break point. The flow field distribution in the downcomer may not give any serious effect on the pressure gradient which is mainly determined by the mass-energy blowdown rate at the break.

During the refill period, Fig.3 shows some difference in PCTs. Multi-D case sows a little lower temperature at around 30 seconds. But it is not significant at all. The collapsed water level at downcomer (Fig.5) and at core (Fig.4) shows similar behavior. This similar behavior may be mainly due to the fact that the Direct Vessel Injection (DVI) flow from the Safety Injection Tank (SIT) with fluidic device injects sufficient flow to fill the downcomer almost up to the level of cold leg. Therefore, two-phase flow dynamics at the downcomer does not affect to determine the collapsed water level. It is contrasted to the conventional PWRs without DVIs where the dynamics affects the collapsed water level appreciably.

3.2. Comparison of the Flow Pattern during Refill Period

Flow pattern in the downcomer during refill period may be interesting to see any difference between the cross-flow model and the Multi-D model. Fig.6. is the flow pattern and the vapor fraction distribution in the downcomer for cross-flow model case. Fig.7. is the equivalent picture for the Multi-D model case.

In the upper downcomer, flow shows the tendency to merge to the center point of the SIT injection points for the respective loops, i.e., pipe 203 and pipe 209 for the cross-flow case as can be seen in Fig.1. But, for Multi-D case, flow merges to only one point on top of the break location, pipe 207. Further study has to be made to confirm that this behavior is physically explainable.

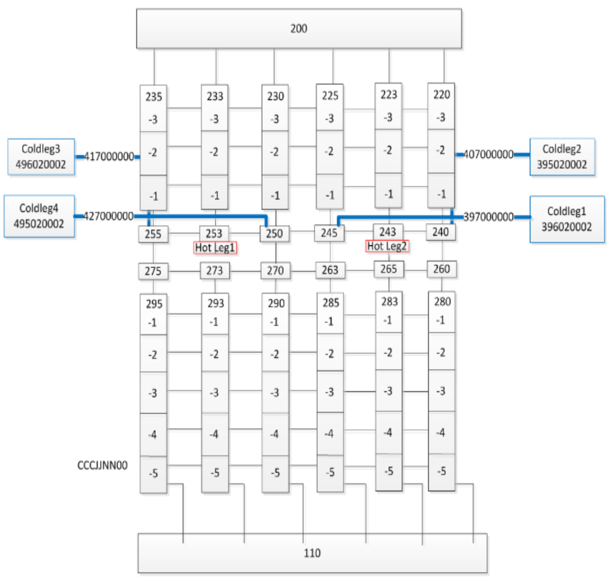

Fig.2. Typical 6 pipes Downcomer Nodalization for APR-1400

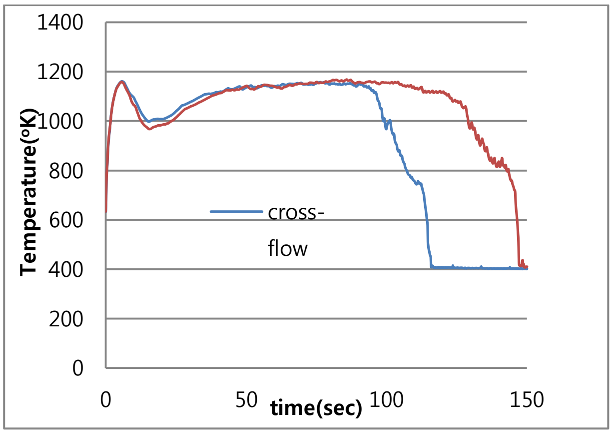

Fig.3. Peak Clad Temperature

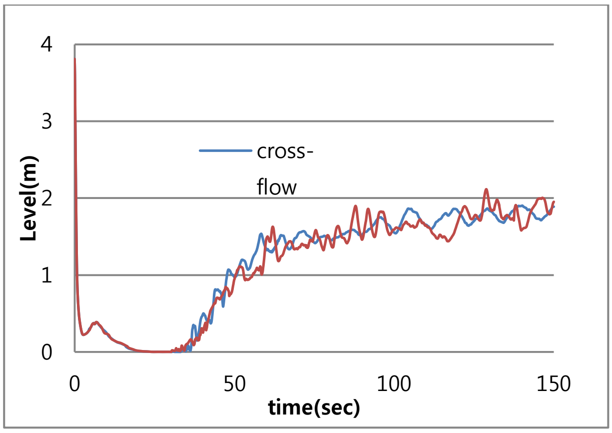

Fig.4. Core Collapsed Water Level

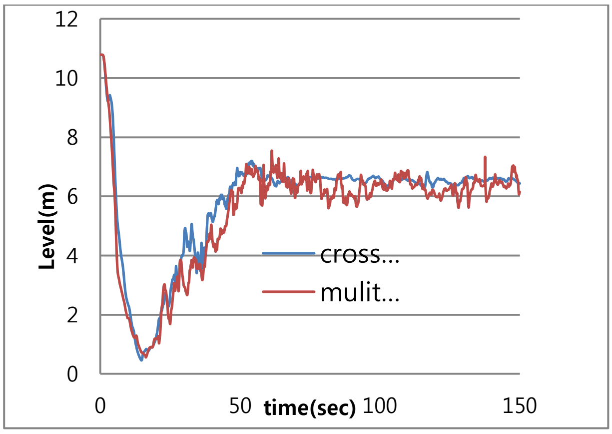

Fig.5. Downcomer Collapsed Water Level

4. Discussions and Perspectives

From the present study and former [14], it is shown that the RELAP5 Multi-D with conservative convective terms is applicable to LOCA analysis. And the implementation of the conservative convective terms in RELAP5 seems to be successful. Further efforts have to be made on making it more robust.

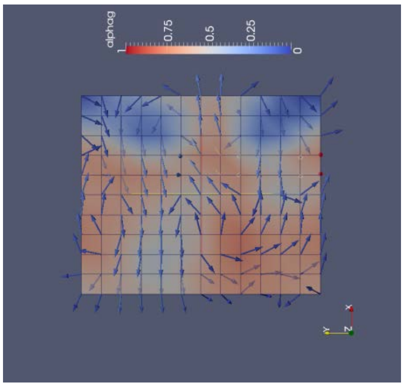

Fig.6. Vapor Fraction and Liquid Velocity by Cross-Flow Model

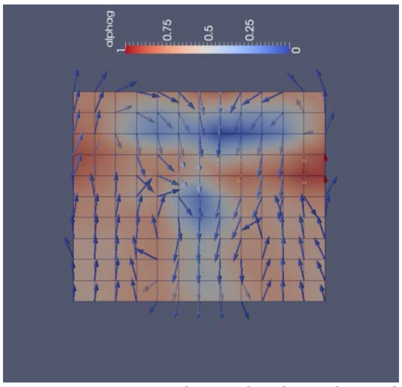

Fig.7. Vapor Fraction and Liquid Velocity by Multi-D

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