

Accuracy Improvement of Axial Power Shape Reconstruction Using GMDH Algorithm

Seongpil Yum^a, Jaemin Kim^a, Ho Cheol Shin^b, Minyong Park^a, Jiwon Choe^a, Peng Zhang^a, and Deokjung Lee^{a*}

^aNuclear Engineering, UNIST, 50, UNIST-gil, Ulju-gun, Ulsan 44919

^bCore and Fuel Analysis Group, KHNP-CRI, 70, Yuseong-daero, Yuseong-gu, Daejeon 34101

*Corresponding author: deokjung@unist.ac.kr

1. Introduction

To monitor in-core power distributions, the reconstruction of axial power shape using in-core detector signals is essential. Core Operating Limit Supervisory System (COLSS) uses 5th order Fourier series expansion to reconstruct axial power shapes, which is known to have relatively large errors for saddle, top- and bottom-skewed shapes [1]. This paper applies Group Method of Data Handling (GMDH) algorithm [2-6] to improve the reconstruction accuracy. Reference three-dimensional power distributions are generated for Shinkori Unit 1 Cycle 3 by simulating Xe transients in BOC, MOC, and EOC, which include saddle, top- and bottom-skewed shapes. Using these power distributions, the axial power distributions were reconstructed by GMDH and the 5th order Fourier series method [7]. It was shown that GMDH algorithm reduced the average Root-Mean-Square-Error (RMSE) from 1.01% of Fourier series method, down to 0.17%.

2. Methods and Results

In this section, the 5th order Fourier series method and the GMDH algorithm will be presented with simple examples. Then, the accuracy of reconstructing power distribution of both methods will be evaluated.

2.1 5th order Fourier series method

The 5th order Fourier method uses five sine functions with Fourier weighting coefficients. The power distribution of core is represented as follows:

$$\phi(z) = \sum_{i=1}^5 a_i \sin\left(i\pi B \frac{z+\delta}{h}\right), \quad (1)$$

$$d_j = \int_j \phi(z) dz, \quad (2)$$

where

$\phi(z)$ = Neutron flux at axial location z ,

a_i = Fourier weighting coefficients,

h = Active core height,

δ = Extrapolated length

where neutron flux is assumed to be zero,

B = Axial buckling,

d_j = Detector signals at level j .

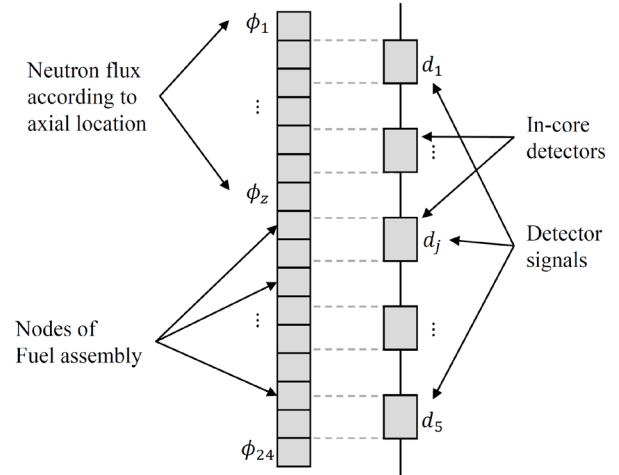


Fig. 1. Axially one-dimensional spatial detector system

Fig. 1 is a schematic drawing to show relative positions of in-core detectors according to axial location of nodes of fuel assembly.

The axial buckling, B , is defined as follows:

$$B \equiv \frac{H}{H + 2\delta}. \quad (3)$$

By substituting Eq. (1) into Eq. (2), detector signals are represented as follows:

$$d_j = \sum_{i=1}^5 \int_j a_i \sin\left(i\pi B \frac{z+\delta}{h}\right) dz. \quad (4)$$

If we let

$$H(i, j) = \int_j \sin\left(i\pi B \frac{z+\delta}{h}\right) dz, \quad (5)$$

then Eq. (4) becomes

$$d_j = \sum_{i=1}^5 H(i, j) a_i. \quad (6)$$

Eq. (6) can be rewritten in a matrix form as follows:

$$\mathbf{d} = \mathbf{H} \mathbf{a}. \quad (7)$$

In this calculation, matrix \mathbf{a} is the unknown. So it can be determined as below:

$$\mathbf{a} = \mathbf{H}^{-1} \mathbf{d}. \quad (8)$$

The axial power shape can be reconstructed by products of the Fourier series matrix \mathbf{S} and the Fourier series coefficient vector \mathbf{a} .

$$\mathbf{p} = \mathbf{S} \mathbf{a}, \quad (9)$$

where

\mathbf{p} = Vector of axial power distribution
in an assembly,

\mathbf{S} = Expanded matrix
by basic functions of Fourier series.

The matrix \mathbf{S} is expanded by basic functions of Fourier series which is as below:

$$\mathbf{S}(i, j) = \sin\left(i\pi B \frac{j + \delta}{h}\right) \quad (10)$$

Fig. 2 shows each basic function of Fourier series, which has different period.

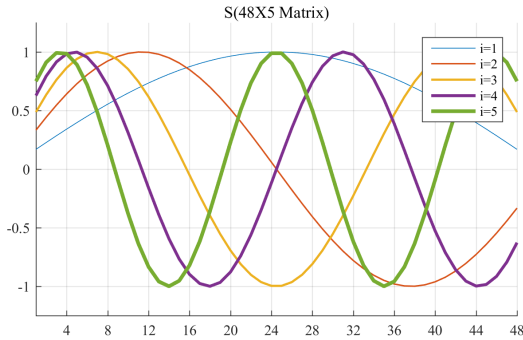


Fig. 2. Basis functions of matrix \mathbf{S}

2.2 GMDH algorithm

The GMDH algorithm is the process for constructing a high-order polynomial as follows:

$$y = a + \sum_{i=1}^m b_i x_i + \sum_{j=1}^m \sum_{i=1}^m c_{ij} x_i x_j + \sum_{k=1}^m \sum_{j=1}^m \sum_{i=1}^m d_{ijk} x_i x_j x_k + \dots \quad (11)$$

which relates m input variables, x_1, x_2, \dots, x_m to a single output variable y . This high-order polynomial is also consisted of so called Ivakhnenko polynomial which is as follows:

$$y = A + Bu + Cv + Du^2 + Ev^2 + Fuv. \quad (12)$$

Using the GMDH algorithm, the Ivakhnenko polynomials will be collected in each iteration, and eventually it is combined into a single equation representing the relationships between the input variables and the output variable. In the followings, the basic steps of the GMDH algorithm will be described.

Step 1: Divide data sets

To begin the GMDH algorithm, given data sets have to be divided into training and testing sets. The training set is used to train Ivakhnenko polynomials to find the polynomial which have optimal complexity. The testing set is used to evaluate the performance of the trained polynomials. In this paper, 75% of the total data is used for training and the remaining 25% is for testing.

Step 2: Construction next generation sets

The polynomials, that will produce next generation sets, are constructed as follows:

$$z = A + Bx_i + Cx_j + Dx_i^2 + Ex_j^2 + Fx_i x_j \quad (13)$$

where

$$(i, j) = [(1, 2), (1, 3), \dots, (m, m-1)],$$

and x_i and x_j indicate the columns of independent variables. This equation can be represented in a matrix form as follows:

$$\mathbf{X} \mathbf{a} = \mathbf{y} \quad (14)$$

where

$$\mathbf{X} = \begin{bmatrix} 1 & x_{i1} & x_{j1} & x_{i1}^2 & x_{j1}^2 & x_{i1} x_{j1} \\ 1 & x_{i2} & x_{j2} & x_{i2}^2 & x_{j2}^2 & x_{i2} x_{j2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{in_t} & x_{jn_t} & x_{in_t}^2 & x_{jn_t}^2 & x_{in_t} x_{jn_t} \end{bmatrix}, \quad \mathbf{a} = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n_t} \end{bmatrix}.$$

where n_t means the number of data for training, i.e., there is n_t of training data, and the rest of data is for testing. The coefficients matrix is the unknowns in this matrix calculation, and it can be found by applying the least square minimization. So this matrix calculation can be solved by Pseudo inverse matrix, and the process is as follows:

$$\mathbf{a} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}. \quad (15)$$

by using this coefficient found from Pseudo inverse calculation, the variable sets of next generations are produced by following way:

$$\mathbf{X} \mathbf{a} = \mathbf{z}, \quad (16)$$

where

$$\mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$$

In this matrix calculation, n indicates the total number of data. The coefficients of \mathbf{a} are produced from training set. And it's applied to whole data set including checking set to produce next generation variable set of \mathbf{z} .

Step 3: Check the compatibility of new sets

The RMSE calculated by checking sets can be used for screening out data sets which are not effective at predicting. And this step, the testing sets are considered only, and the calculation form is as follows:

$$RMSE_t^j = \sum_{i=n_t+1}^n (y_i - z_{ij})^2 \quad (17)$$

where

j = Order of column

i = Order of row

n_t = Number of training set

$n - n_t$ = Number of checking set

t = Iteration number

The variable columns which have larger value of the RMSE than the predetermined criteria, those columns will be removed in that generation. This process is called as a self-organizing, and it is similar to the natural selection process. The worse variable sets are not selected and the only good data can survive. In this step, if the minimum value of RMSE among the new generation's columns decreases, the calculation process goes back to step 2, and repeats the same process. But if it increases, then it means that this new generation is fitting to the training set more accurately, but the prediction accuracy for the testing sets is less accurate, *i.e.*, it's getting worse at predicting variables of the checking sets. This is called as an overfitting, and if this phenomenon occurs, then this iteration is the end of GMDH algorithm, and the column sets which has the least RMSE will be the final Ivakhnenko polynomials .

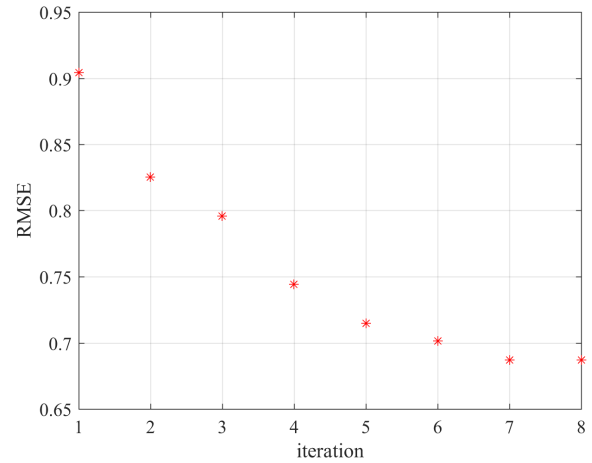


Fig. 3 RMSE as stopping criterion

2.3 Results

To test the validity of the GMDH based axial power reconstruction method, the data sets of Shinkori unit 1 cycle 3 were used to construct the axial power shapes. The final form of polynomials of axial power distribution at node z is represented by $P(z)$. $P(1)$ and $P(10)$ are the examples of power distribution at 1st node and 10th node in this research.

$$P(1) = (-0.0033) \left((-0.019) + (d_1 0.282 + d_1 d_2 0.020) 1.047 + (d_3 1.793 + d_4 (-1.468))^2 (-0.0258) \right) 1.341 + \left((d_2 0.485 + d_2^2 0.028 + d_3 (-0.253)) 1.182 + (d_3 1.793 + d_4 (-1.468)) (-0.188) \right) (-0.339)$$

$$P(10) = \left((d_1 (-0.430) + d_1^2 0.0112 + d_2 0.861) 0.514 + (d_1 0.144 + d_1 d_3 (-0.00296) + d_3 0.372) 0.485 \right) 0.875 + \left((d_3 1.109 + d_3^2 (-0.0121) + d_4 (-0.573)) 0.923 + (d_3 1.109 + d_3^2 (-0.0121) + d_4 (-0.573))^2 (-0.187) + (d_2 0.568 + d_2^2 (-0.0669) + d_2^2 0.0474)^2 0.257 \right) 0.124$$

and there are more polynomials for each position of 24-nodes.

For comparing error distribution of power reconstructions for each case, for each assembly, the RMSE of each assembly is calculated as follows:

$$RMSE_j = \frac{1}{24} \sum_{z=1}^{24} (y_j(z) - P_j(z))^2 \quad (18)$$

where

j = Number of cases,

z = Axial location in assembly,

$P(z)$ = Power reconstruction at location z ,

$y(z)$ = Reference power distribution at location z .

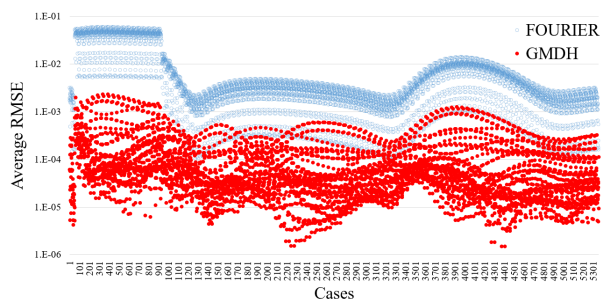
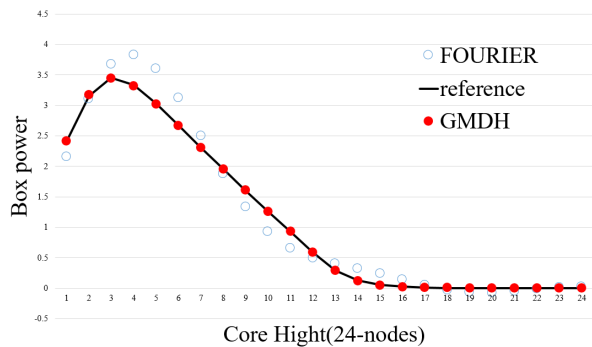
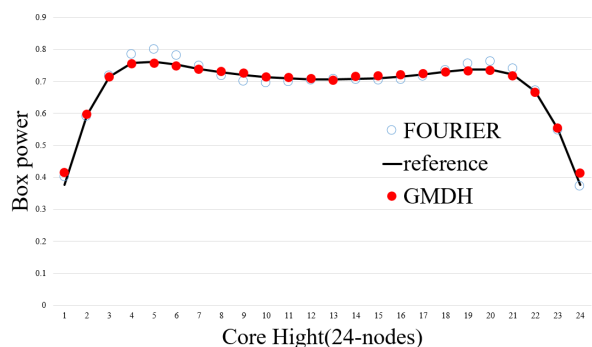


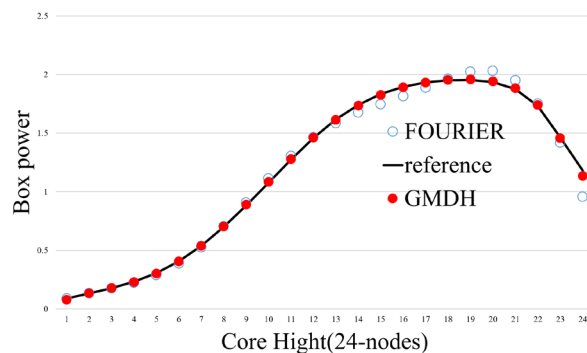
Fig. 4 RMSE for Fourier series method and GMDH



(a)



(b)



(c)

Fig. 5 Power reconstructions according to (a)bottom-skewed shape, (b)saddle shape, and (c)top-skewed shape.

As in Fig. 4, the RMSE of each case of GMDH algorithm has smaller error than the 5th order Fourier series method. Fig. 5 shows three cases from the data sets, and the GMDH shows good accuracy at fitting curve compared with the Fourier series method. Especially in Bottom-skewed, and Top-skewed shapes, Fourier series method shows relatively large error in a convex and concave region of power distribution whereas the GMDH algorithm still shows good accuracy.

3. Conclusions

This paper presents the accuracy improvement of axial power reconstruction using GMDH algorithm with the first order polynomial basis function compared to the COLSS Fourier series expansion algorithm. In the future study, higher order basis functions, such as trigonometric, exponential, or high-order polynomials will be applied in GMDH algorithm to further reduce the error in top/bottom skewed power shapes.

Acknowledgement

This work was partially supported by KETEP, which is funded by the Korean government Ministry of Trade, Industry and Energy. (No. 20131610101850)

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