Transactions of the Korean Nuclear Society Autumn Meeting
Gyeongju, Korea, October 27-28, 2016

Albedo-corrected Parameterized Equivalence Constants for Cross-section Update in Nodal Calculation

Woosong Kim and Yonghee Kim*
Dept. of Nuclear and Quantum Eng., KAIST, 291 Daehak-ro, Yuseong-gu, Daejeon, Korea, 34141
**Corresponding author: yongheekim@kaist.ac.kr*

1. Introduction

Nodal equivalence theory is the cornerstone of modern reactor core analysis in which the heterogeneous core is simplified into a number of homogenized fuel assemblies. The main idea of the theory is to preserve the equivalency between an original heterogeneous assembly and a simplified homogenized assembly in terms of their reaction rates and node interface currents. Nowadays, simplified equivalence theory (SET) [1] is one of the most widely used techniques due to its computational efficiency. However, the SET efficiency on a single assembly homogenization is quite limited when the node interface current is not close to zero and the neighborhood effect is rather strong [2]. To overcome this limitation, several approaches to functionalize the equivalence constants have been suggested in order to achieve more accurate whole-core solution while maintaining the advantage of the conventional two-step procedure, such as boundary perturbation theory [3] and functional interface discontinuity factors [4].

In a recent study by W. Kim and Y. Kim [5], the albedo-corrected parameterized equivalence constants (APEC) method was proposed, in which fuel assembly two-group cross-sections are parameterized as a function of a node-average current-to-flux ratio (CFR), a unique way to represent the spatial leakage of node. In this study, the two-group homogenized cross-sections are functionalized based on the 2-D MOC lattice calculation and tested for a nodal analysis of small LWR problems.

2. Albedo-corrected Parameterized Equivalence Constant (APEC) method

Conventional flux-weighted constants (FWCs) [6] are based on the all reflective boundary condition. As such, they may be quite different from the reference values which are obtained from whole-core heterogeneous calculation. However, if one can generate the equivalence constants as a function of node interface condition at the lattice calculation and the cross-sections can be updated by using the actual leakage information during the iterative core calculation, it is expected that more accurate cross-sections will be obtained during the iteration and the resulting nodal equivalence for the homogenized fuel assemblies will then be improved, leading to a more accurate core analysis. With this kind of consideration of the actual interface condition between fuel assemblies, the tricky critical spectrum correction may be eliminated in the conventional lattice calculations.

2.1 Fuel assembly cross-section functionalization

The fuel assembly two-group cross-sections need to be functionalized for the APEC update during nodal calculation. Previous studies [5] showed that if a fuel assembly is symmetric, two-group homogenized cross-sections have a strong relationship with the node-average CFR at node *m*, defined as below:

$$CFR_g^m = \frac{\sum_s J_{g,m}^s}{\sum_s \phi_{g,m}^s}, \tag{1}$$

where the numerator is summation of g^{th} group surface outward net current of node *m* and the denominator is summation of g^{th} group surface flux of node *m*. Then, the cross-section changes due to the non-zero node interface condition are functionalized into 2nd order polynomial for the node-average CFR as follows:

$$\Delta\Sigma_{x,g}^m = a1_{x,g} CFR_g^m + a2_{x,g} \left(CFR_g^m\right)^2, \tag{2a}$$

$$\Sigma_{x,g}^m = \Sigma_{x,g}^0 + \Delta\Sigma_{x,g}^m, \tag{2b}$$

for node *m*, reaction *x*, and group *g*.

In order to determine the coefficients $a1_{x,g}$ and $a2_{x,g}$, a few color-set problem calculations are considered in addition to the conventional infinite lattice calculations, which are shown in Fig. 1. As the APEC cross-section correction function is of the second-order polynomial, only 2 color-set calculations are necessary for each fuel assembly. Total number of the required color-set calculations can also be reduced by using one color-set calculation results for both the two fuel assemblies used in the color-set problem.

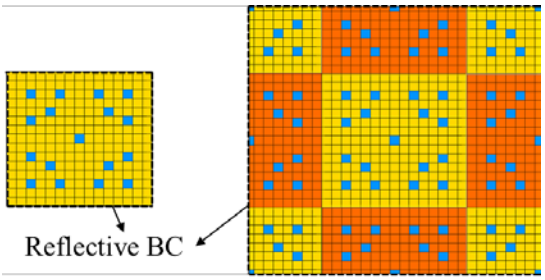

Fig. 1. Single lattice (left) and color-set problem (right)

2.2 APEC nodal calculation with p-CMFD acceleration

A nodal calculation code was developed to demonstrate the impact of APEC cross-section update

on the nodal analysis accuracy. While coarse-mesh finite difference (CMFD) method is very effective and popular for nodal calculations, the partial current-based CMFD (p-CMFD) method [7] was chosen for our APEC method implementation study. This is because in the p-CMFD acceleration method, there are two correction factors for each node interface to preserve both incoming and outgoing partial currents, while the CMFD method preserves only net current. Since partial currents are preserved in the p-CMFD method, both surface current and surface flux are subsequently preserved. Therefore, the CFR is preserved during p-CMFD calculation and the cross-section change by APEC update will also be preserved. This indicates that our APEC update does not need any additional iteration loop and is triggered just after the p-CMFD partial current correction factor updates. The APEC method implemented p-CMFD nodal code flowchart is shown in Fig. 2.

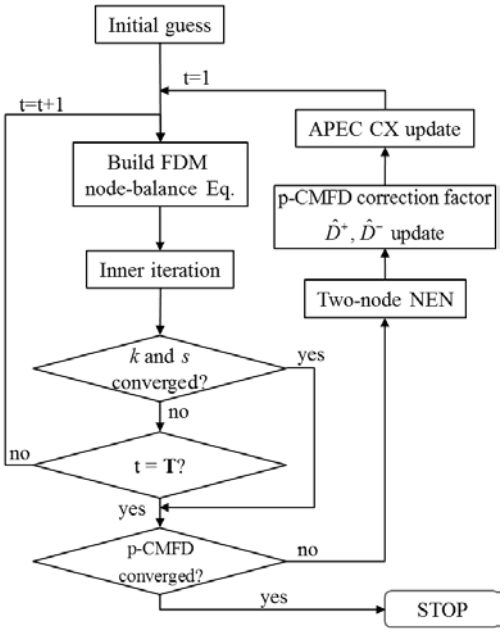

Fig. 2. Flowchart of the APEC implementation on a two-node NEM p-CMFD nodal calculation

3. Results and discussions

The effect of APEC cross-section update was tested using a two-step procedure. A two-node NEM p-CMFD code was used for assembly-homogenized core nodal calculations and a 2-D MOC based lattice code, DeCART2D [8] was used for the lattice calculation and color-set, and the reference whole-core transport calculation. For a test problem, a small benchmark problem was designed by modifying the hot zero power (HZP) SMART 2-D core problem from the DeCART2D sample input [8]. While the original SMART has 6 fuel assembly types and 11 fuel assemblies in 1/8 core, our modified small core has only 2 fuel assembly types and 5 fuel assemblies in 1/8 core. The 1/8 core configuration of those two cores is shown in Fig. 3 and some design parameters of each fuel assembly type are shown in Table I.

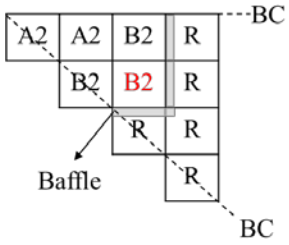

Fig. 3. 1/8 core configuration small core problem

Table. I. Fuel assembly data in small core problem

FA type	U-235 enrichment (wt%)	Fuel pin #	Guide tube #	Gd pin #
A2	2.82	256	25	8
B2	4.88	256	25	8

3.1 Cross-section functionalization results

Besides two infinite lattice calculations for (A2) and (B2) fuel assemblies in the test problem, 3 color-set calculations were performed to determine the APEC cross-section function coefficients. For the (A2) fuel assembly cross-section functionalization, (A2, A3) and (A2, B2) paired color-set problems were used. For the (B2) fuel assembly, (A2, B2) and (B2, BX) paired color-set problems were used, where (BX) fuel assembly is similar to (B2) but has 68 Gd pins, in order to simulate very strong neutron leakage of (B2) fuel assembly at peripheral region.

The two-group homogenized cross-section differences between infinite lattice and color-set problems are calculated and they are fitted into the 2nd order polynomial functions of node-average CFR, as described in Eq. (2a). The results are plotted with the difference between the reference cross-section from the whole-core transport calculation and the infinite lattice cross-section for comparison. Figs. 4~7 show cross-section change results of (B2) fuel assembly for (4) fast group absorption cross-section, (5) thermal group fission yield and absorption cross-section, (6) down-scattering cross-section, and (7) up-scattering cross-section.

While thermal group cross-section changes agree very nicely with the reference (Figs. 5 and 7), fast group cross-section changes are not well represented by Eq. (2a) in most cases (Fig. 4). However, fast group cross-section change and its importance are relatively small compared to the thermal group. Therefore, the fast group cross-section change was ignored in the current APEC method, except for the down-scattering cross-section, which is important and its 2nd-order fitting is fairly good.

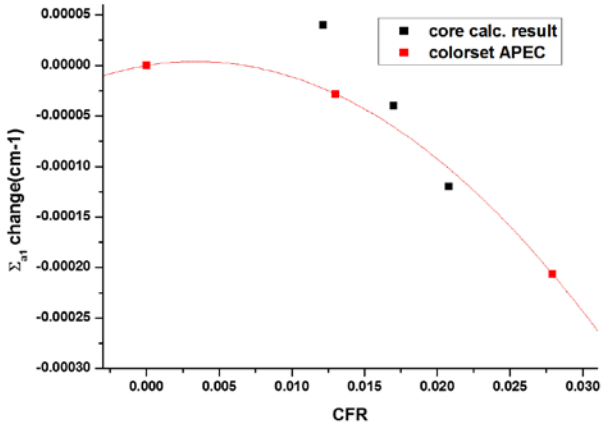

Fig. 4. $\Sigma_{a,1}$ change by CFR (B2)

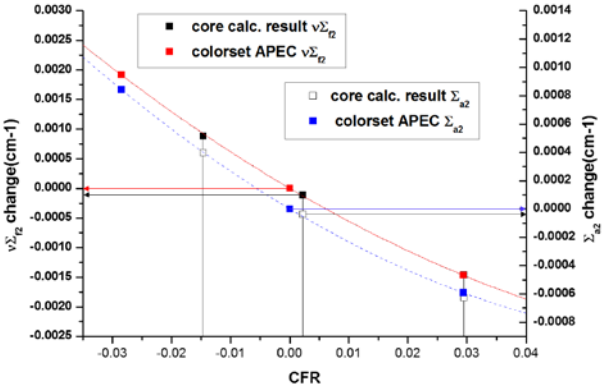

Fig. 5. $\nu\Sigma_{f,2}$ and $\Sigma_{a,2}$ change by CFR (B2)

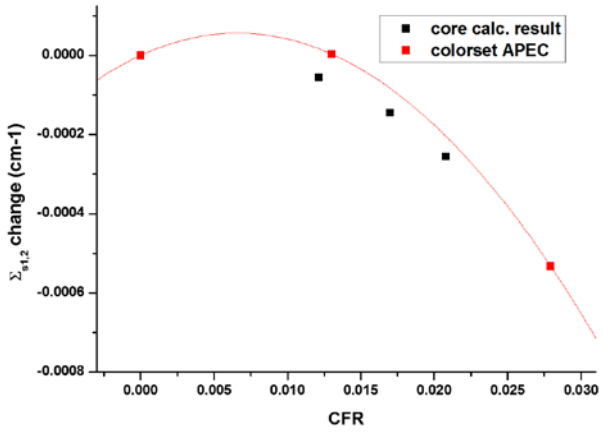

Fig. 6. $\Sigma_{s1\rightarrow2}$ change by CFR (B2)

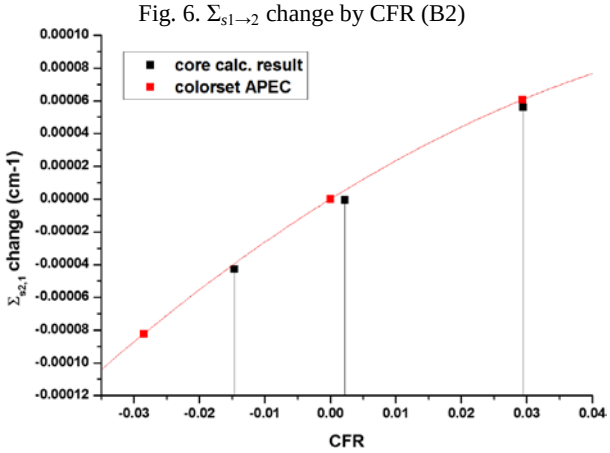

Fig. 7. $\Sigma_{s2\rightarrow1}$ change by CFR (B2)

3.2 2-D small core problem

Two small core problems were solved by a two-node NEM p-CMFD code. The core configurations of those two test problems are the same as in Fig. 3, but one includes a standard baffle-reflector and the other has water reflector without baffle, in order to remove errors from baffle-water homogenization in the analysis. Core configurations of two test problems are shown in Fig. 8. Nodal calculations were performed using 1) two-group parameters from conventional SET with infinite lattice cross-sections or 2) APEC cross-section update. Simple ADFs were used for both cases. The error criteria for k_{eff} and source were both 10^{-10} and 4x4 nodes per assembly were used.

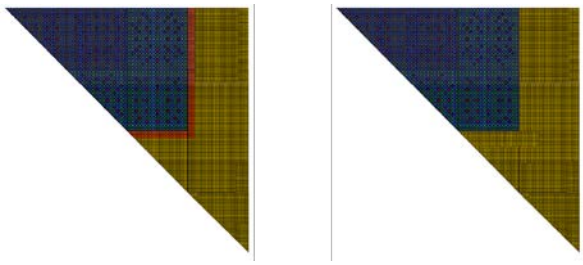

Fig. 8. Core configuration with standard baffle-reflector (left) and without baffle (right)

As a result of nodal calculation, APEC cross-section change showed noticeable improvement on the accuracy of k_{eff} that Δρ decreased by 62.4% (Table II). The assembly normalized power distribution was also improved by the APEC method that maximum error and RMS error were decreased by 23.5% and 19.4% (Fig. 9). Until the solution converges, conventional SET method took 352 outer iterations and APEC cross-section update took 351 outer iterations. Regarding that APEC cross-section updates are just substitution of node-average CFR to Eq. (2) at the end of p-CMFD correction factor update, the computing cost for APEC cross-section update is negligible.

Table II. standard baffle-reflector problem k_{eff} results

	k_{eff}	Δρ (pcm)
Reference (DeCART2D)	1.17417	-
Conventional SET	1.17678	189.5
APEC*	1.17515	71.3

* Applied to only thermal group cross-sections and $\Sigma_{s1\rightarrow2}$

				Max.	RMS
Reference	1.2246	1.1159	0.9339	-	-
SET	-1.83%	-1.10%	1.19%	1.83%	1.24%
APEC	-1.40%	-0.93%	1.10%	1.40%	1.00%
		1.2359	0.7199		
		0.72%	1.11%		
		0.38%	0.90%		

Fig. 9. Assembly normalized power distribution and error

In order to see the convergence behavior, thermal group $\nu\Sigma_f$ was plotted against outer iteration number, at the red colored fuel assembly (B2) in Fig. 3. By using APEC cross-section update, the cross-section values

approach from infinite lattice cross-section to the reference value (Fig. 10). As a result, at the same fuel assembly, thermal group cross-section error is reduced by more than 90% by APEC update (Table III). However, fast group cross-sections are not corrected and down-scattering cross-section correction does not fit as nicely as shown in Fig. 6.

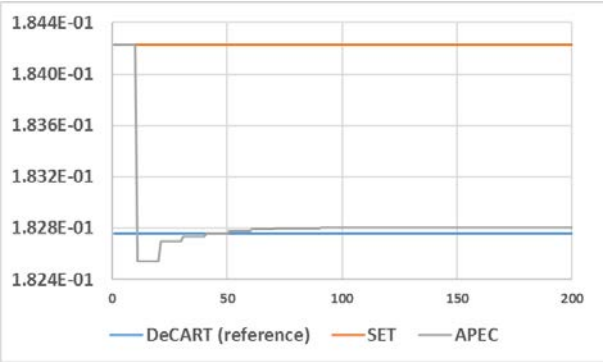

Fig. 10. $\nu\Sigma_{f2}$ by outer iteration

Table III. Two-group homogenized cross-section error at the peripheral fuel assembly (B2)

	SET error (%)	APEC error (%)
CFR_1	7.645	4.969
CFR_2	-9.395	-3.990
D_1	0.961	0.961
D_2	-0.318	-0.044
Σ_{a1}	-0.387	-0.387
Σ_{a2}	0.572	0.049
$\nu\Sigma_{f1}$	0.481	0.481
$\nu\Sigma_{f2}$	0.800	0.022
$\Sigma_{s2\to1}$	-2.480	0.123
$\Sigma_{s1\to2}$	0.379	0.430

The same analysis was done for the small core problem without baffle to see impacts of the simple baffle-reflector homogenization. Results of the calculations are shown in Table IV and Figure 11. It is clear that the APEC cross-section update improves the nodal accuracy a lot: eigenvalue error is reduced by a factor of 5 (78.0% reduction in error) and the maximum and RMS errors in nodal powers are also reduced by 31.43% and 29.2%, respectively. These results indicate that the position-dependent baffle-reflector cross sections are quite different from those obtained with the simple baffle-reflector homogenization, as shown in Table V. In addition, this reveals that the actual flux discontinuity factors on the core boundary should be quite baffle-dependent.

Table IV. k_{eff} results for the no-baffle problem

	k_{eff}	Δρ (pcm)
Reference (DeCART2D)	1.17223	-
Conventional SET	1.17437	155.6
APEC*	1.17270	34.2

* Applied to only thermal group cross-sections and $\Sigma_{s1\to2}$

				Max.	RMS
Reference	1.2352	1.1189	0.9301	-	-
SET	-1.40%	-0.80%	0.87%	1.40%	0.96%
APEC	-0.96%	-0.62%	0.70%	0.96%	0.68%
		1.2312	0.7178		
		1.01%	0.47%		
		0.65%	0.31%		

Fig. 11. Assembly normalized power distribution and error

Table V. Errors in baffle-reflector cross sections

Baffle type	L-shape	flat
D_1	0.56%	0.56%
Σ_{a1}	-1.63%	-1.50%
$\Sigma_{s1\to2}$	-14.27%	-1.94%
D_2	0.26%	0.20%
Σ_{a2}	-0.42%	-2.21%
$\Sigma_{s2\to1}$	6.82%	5.20%

4. Conclusions

In the two-group homogenization of PWR fuel assembly, both thermal-group and down-scattering cross sections can be well functionalized as a function of an albedo information on the boundary. The APEC method can be very effectively utilized in adjusting the thermal-group cross sections and improving the accuracy of the conventional nodal analysis for PWR cores. For accurate parameterization of the cross section, a color-set configuration should be also analyzed, and the additional computational cost is rather marginal. Currently, extension of the APEC method to the fast-group cross sections and baffle-reflector homogenization is ongoing.

REFERENCES

[1] K. S. Smith, Assembly Homogenization Techniques for Light Water Reactor Analysis, Progress in Nuclear Energy, 17, 303 (1986).
[2] K. T. Clarno and M. L. Adams, Capturing the Effect of Unlike Neighbors in Single-Assembly Calculations, Nucl. Sci. Eng., Vol. 149, p. 182 (2005).
[3] Rahnema F., Nichita E.M., Leakage corrected spatial (assembly) homogenization technique, Annals of Nuclear Energy 24 (6), 477-488 (1997).
[4] J. J. Herrero et al., Neighborhood-corrected interface discontinuity factors for multi-group pin-by-pin diffusion calculations for LWR, Annals of Nuclear Energy, 46, 106-115 (2012)
[5] W. Kim and Y. Kim, Feasibility of Albedo-corrected Parameterized Equivalence Constants for Nodal Equivalence Theory, ANS MC2015, Nashville, TN, April 19-23 (2015)
[6] A. F. Henry, Nuclear-Reactor Analysis, The MIT Press, Cambridge, Massachusetts (1975).
[7] N. Z. Cho, The Partial Current-Based CMFD (p-CMFD) Method Revisited, Transactions of the Korean Nuclear Society Autumn Meeting Gyeongju, Korea, October 25-26 (2012)
[8] J. Y. Cho, DeCART2D v1.0 User's Manual, KAERI/TR-5116/2013