Effects of Experimental Conditions on Estimation Uncertainty of Weibull Distribution: Applications for Crack Initiation Testing

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1. Introduction

It is well known that stress corrosion cracking (SCC) is one of the main material-related issues in operating nuclear reactors [1,2]. To predict the initiation time of SCC, the Weibull distribution is widely used as a statistical model representing SCC reliability [3,4].

The typical experimental procedure of an SCC initiation test involves an interval-censored cracking test with several specimens. From the result of the test, the experimenters can estimate the parameters of Weibull distribution by maximum likelihood estimation (MLE) or median rank regression (MRR) [5,6].

However, in order to obtain the sufficient accuracy of the Weibull estimators, it is hard for experimenters to determine the proper number of test specimens and censoring intervals. Therefore, in this work, the effects of some experimental conditions on estimation uncertainties of Weibull distribution were studied through the Monte Carlo simulation.

2. Approach

The cumulative distribution function (CDF) of the two parameter Weibull distribution is frequently used as a cracking probability function and given by:

$$F(t;\beta,\eta) = 1 - exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right];\ t \geq 0;\ \ \beta,\eta > 0, \quad (1)$$

where t is time, β is the shape parameter and η is the scale parameter of the Weibull distribution. For MRR method, the median rank is computed by Benard's approximation [5] and a nonlinear curve fitting solver in a least squared sense is employed for the regression procedure. For MLE method, the likelihood function for the interval and right censored data [7] is used. Numerical approach was adopted to find out maximum likelihood point which related to the ML estimators.

The estimated Weibull distributions by MRR and MLE are different, even though both estimators were derived from the same experiment result [6]. Then, it is in curious that which estimator is more precise as compared to the true behavior of the cracking probability. Unfortunately, there is no MLE theory yet available to set the estimation confidence for interval censored data [5]. Therefore, the Monte Carlo simulation is used to evaluate uncertainties of each estimation methods quantitatively by conducting the simulated cracking tests. Considered experimental condition factors of this simulation study are 1) True Weibull parameters ($\beta_{true}, \eta_{true}$), 2) The number of specimen, 3) End cracking fraction (ECF) and 4) Censoring interval.

In our earlier study [6], we calculated the estimation uncertainties of Weibull distribution as a function of experimental conditions and estimation methods (see Fig. 1).

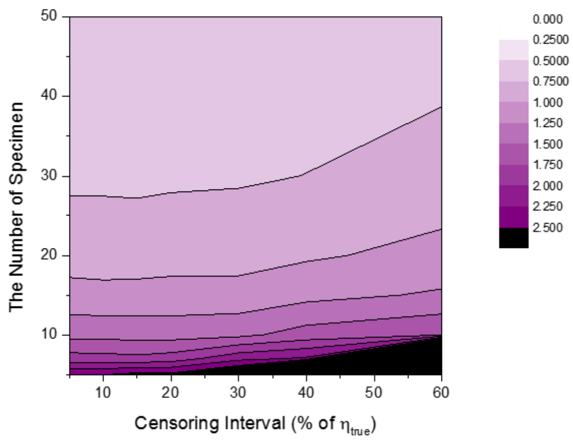

Fig. 1. Distributions of $SLCI_{90\%}(\hat{\beta}_{MLE})$ at $\beta_{true} = 2$ (test duration: 120% of η_{true}) [6].

For SCC initiation of nuclear materials, β is larger than unity (i.e., hazard function is increased with time) [3, 8]. Therefore, as a rule of thumb, it is reasonable to narrow the censoring interval with time. We set the censoring interval decreased depending on the cracking fraction of the specimens to reflect this time-dependent censoring interval (TDCI) effect. Time-independent censoring interval (TICI) case was also studied as a control group. Table 1 shows the range of this simulation study.

Table I: Range of the simulation

The number of specimen [ea.]	5 ~ 50
Starting censoring interval [% of η_{true}]	5 ~ 50
End cracking fraction (ECF)	0.6, 0.8, 1.0
β_{true}	2, 3, 4

Figure 2 shows the comparison of two simulation experiment examples. It is well represented that censoring interval is decreased after the cracking in the TDCI case (see Fig. 2b). "Weibull_True" is the assumed true probability behavior of SCC initiation, "Weibull_MLE" and "Weibull_MRR" are the estimated Weibull distributions by MLE or MRR methods [5,6], respectively.

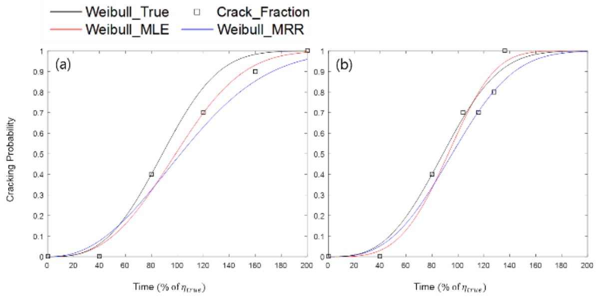

Fig. 2. Two examples of simulation experiment at (a) TICI case and (b) TDCI case (number of specimen: 10 ea., starting censoring interval: 40 % of η_{true}, ECF: 1.0, β_{true}: 3.0).

Like examples in Fig. 2, 20,000 times of simulated cracking tests (and also estimations) were carried out for each case in the simulation range. From the results of the simulations, 5th, 50th and 95th percentiles of Weibull estimators were driven from each case. These estimators were converted to the standardized error. The standardized error of Weibull estimators was defined as follows:

$$SE(\hat{\beta}) = \frac{\hat{\beta}-\beta_{true}}{\beta_{true}}\,;\; SE(\hat{\eta}) = \frac{\hat{\eta}-\eta_{true}}{\eta_{true}}, \qquad (2)$$

where, $\hat{\beta}$ and $\hat{\eta}$ are the estimated Weibull parameters by MRR or MLE.

3. Results and Discussion

By using a Monte Carlo simulation, uncertainties of Weibull estimators were quantified in various conditions of experimental cases. And, as already anticipated, the simulation result shows that the TDCI case returns small estimation uncertainties as compared to the those of TICI case when "starting censoring interval" is equal. However, TDCI case contains much higher mean censoring times (MCT). Figure 3 shows this increasing MCT effect well. To compare fairly between the TDCI and TICI cases, we set the same MCT line as uncertainty evaluation criterion. In this study, the value of MCT criterion is selected as 10 or 20.

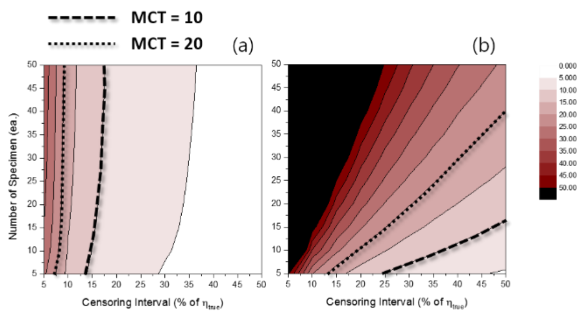

Fig. 3. Distributions of MCT at (a) TICI case and (b) TDCI case (ECF: 1.0, β_{true}: 3.0).

Interestingly, there is almost no difference for estimation uncertainty between the case of TICI and TDCI when the same MCT criterion is adopted. Figure 4 shows some examples showing this little uncertainty difference for β_{MLE}.

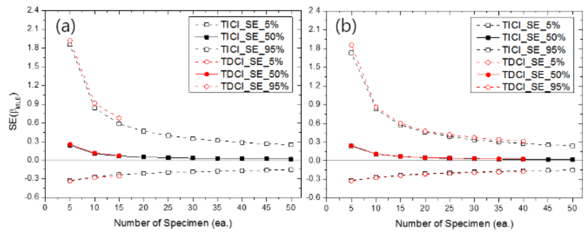

Fig. 4. Effect of specimens number on SE($\hat{\beta}_{MLE}$) when (a) MCT = 10 and (b) MCT = 20 case (ECF: 1.0, β_{true}: 3.0).

4. Conclusions

The main goal of this work is to suggest quantitative estimation uncertainties for experimenters who want to develop probabilistic SCC initiation model by a cracking test. Widely used MRR and MLE are considered as estimation methods of Weibull distribution. By using a Monte Carlo simulation, uncertainties of MRR and ML estimators were quantified in various experimental cases. And we compared the uncertainties between the TDCI and TICI cases. The same MCT line is adopted as an uncertainty evaluation criterion. For this criterion, there is almost no difference between the TDCI and TICI cases.

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