Two-fluid equations for a nuclear system with arbitrary motions

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1. Introduction

The two-fluid equations have played a vital role in safety analysis of nuclear power plants. The equations are obtained by time- or spatial-averaging of mass, momentum, and energy conservation equations over fixed and stationary control volumes, which are appropriate for a nuclear system fixed on Earth surface.

Recently, ocean nuclear systems have received a great attention in Korea [1]. Ocean nuclear systems include a seabed-type plant, a floating-type plant, and a nuclear-propulsion ship. We asked ourselves, "What governing equations should be used for ocean nuclear systems?" Since ocean nuclear systems are apt to move arbitrarily, the two-fluid model must be formulated in the non-inertial frame of reference that is undergoing acceleration with respect to an inertial frame.

Two-phase flow systems with arbitrary motions are barely reported. Kim et al. (1996) added the centripetal and Euler acceleration forces to the homogeneous equilibrium momentum equation embedded in the RETRAN code. However, they did not look into the mass and energy equations.

The purpose of this study is to derive general two-fluid equations in the non-inertial frame of reference, which can be used for safety analysis of ocean nuclear systems.

2. Single-phase flow

First, we review the single-phase conservation equations in a non-inertial frame. The non-inertial (arbitrarily moving) frame $\mathrm{i'j'k'}$ is located by position vector $\mathbf{R}$ relative to the absolute frame ijk, as shown in Fig. 1. The non-inertial frame rotates with an angular velocity Ω. A particle is instantaneously located with respect to the moving frame by position vector $\mathbf{r}'$. Then, the position of the particle is given by $\mathbf{R}+\mathbf{r}'$ in the absolute frame.

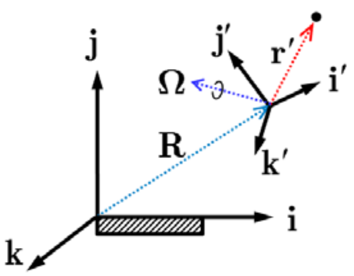

Fig. 1. Frame transformations

In the absolute frame, the mass, momentum, and internal energy equations are

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{u})=0\,, \tag{1}$$

$$\rho\frac{D\mathbf{u}}{Dt}=-\nabla p+\nabla\cdot\left[\mu(\nabla\mathbf{u}+(\nabla\mathbf{u})^T)+\lambda(\nabla\cdot\mathbf{u})\mathbf{I}\right]+\rho\mathbf{g}\,, \tag{2}$$

$$\rho\frac{De}{Dt}=-\nabla\cdot\mathbf{q}-p\nabla\cdot\mathbf{u}+\boldsymbol{\tau}:\nabla\mathbf{u}\,. \tag{3}$$

According to frame transformation, spatial gradient of scalars and vectors is invariant. In addition, Lagrangian time derivative of scalars is invariant. As a result, the scalar equations such as mass, internal energy, and enthalpy are formally unchanged. In other words, all variables in Eqs. (1) ~ (3) can be simply replaced by variables measured in the moving frame.

$$\frac{\partial \rho'}{\partial t}+\nabla\cdot(\rho'\mathbf{u}')=0\,, \tag{4}$$

$$\rho'\frac{D'e'}{Dt}=-\nabla\cdot\mathbf{q}'-p'\nabla\cdot\mathbf{u}'+\boldsymbol{\tau}':\nabla\mathbf{u}'\,, \tag{5}$$

The prime symbols denote fluid properties measured in the moving frame.

In contrast, the time derivative of vectors has the following relation:

$$\frac{d}{dt}=\frac{d'}{dt}+\boldsymbol{\Omega}\times\,, \tag{6}$$

where d/dt and d'/dt are the time derivatives in the absolute and moving frames, respectively. Accordingly, the momentum equation is written as [2]

$$\rho'\left(\frac{\partial\mathbf{u}'}{\partial t}+\mathbf{u}'\cdot\nabla\mathbf{u}'\right)=-\nabla p'+\nabla\cdot\boldsymbol{\tau}'+\rho'\mathbf{g}'+\mathbf{f}_{\text{fictitious}}\,, \tag{7}$$

$$\mathbf{f}_{\text{fictitious}}=-\rho[\ddot{\mathbf{R}}+\dot{\boldsymbol{\Omega}}\times\mathbf{r}+2\boldsymbol{\Omega}\times\mathbf{u}-\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})]\,. \tag{8}$$

Four additional terms account for the acceleration of the origin of the moving frame, the Coriolis force, the Euler force, and the centripetal force, respectively. These forces are called fictitious forces. The Coriolis force is a function of the rotational velocity of a two-phase flow system and the fluid velocity.

3. Two-phase flow

The two-fluid equations are obtained by time- or spatial-averaging of mass, momentum, and energy conservation equations. Since the scalar equations are formally unchanged, the two-fluid equation forms for mass, internal energy, and enthalpy are the same as existing ones.

On the other hand, the fictitious forces must be included in the two-fluid momentum equation. For spatial averaging, the following force terms must be added to the existing momentum equation.

$$\begin{aligned}\mathbf{F}_{\text{fictitious}} &= \frac{1}{V}\int_V f_{\text{fictitious}} dV \\ &= -\alpha_k \overline{\overline{\rho}}_k [\ddot{\mathbf{R}} + \dot{\mathbf{\Omega}} \times \hat{\mathbf{r}}_k + 2\mathbf{\Omega} \times \hat{\mathbf{u}}_k + \mathbf{\Omega} \times (\mathbf{\Omega} \times \hat{\mathbf{r}}_k)]\end{aligned} \quad (9)$$

$\ddot{\mathbf{R}}$ accounts for three translation motions of a ship. The last three terms are related to rotational motions of a ship. We will discuss the difference between the time averaged momentum equation and the spatial averaged equation.

As for the interfacial jump conditions, if the effect of surface tension is neglected, the interfacial jump conditions is formally unchanged.

3. Conclusions

The two-fluid equation forms for scalar properties such as mass, internal energy, and enthalpy equation in the moving frame are the same as those in the absolute frame. On the other hand, the fictitious effect must be included in the momentum equation. In the near future, we will investigate the relative importance of the Coriolis force to other force terms.

REFERENCES

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