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# An Evaluation of Depletion Bias and Bias Uncertainty of the GBC cask with PLUS7 Fuels

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## 1. Introduction

The capability of on-site storage of used nuclear fuels (UNFs) generated in the domestic nuclear power plants is project to reach the limit from 2024. So it is necessary to utilize a dry storage cask (DSC) for interim storage of UNFs and hence the accurate nuclear criticality safety evaluation of the DSC is quite important. A nuclear criticality safety evaluation that applies burnup credit (BUC) to a DSC is performed mainly through a two-step process: (1) the determination of isotopic compositions within UNFs to be loaded into a DSC by a depletion analysis and (2) the determination of the $k_{eff}$ value with respect to the DSC by a criticality analysis [1]. In particular, the isotopic compositions by a depletion analysis should be estimated accurately because the concentrations of the nuclides contained in a UNF have a significant influence on the accuracies of depletion analysis and its subsequent criticality analysis. However, since no depletion computer code can calculate exactly nuclide compositions contained in a used nuclear fuel assembly (UNFA), it requires bias and bias uncertainty in terms of a reactivity difference, $\Delta k_{eff}$, by a depletion code for burnup credit criticality safety analyses. Thus, the objective of this work is to determine the bias and bias uncertainty in $k_{eff}$ resulted from biases and bias uncertainties in the calculated nuclide concentrations for the GBC-32 DSC system with 32 PLUS7 16X16 UNFAs. The SCALE 6.1/STARBUCS, SCALE 6.1/ TRITON code, and the IBM SPSS Statistics Version 23 program were used to evaluate the bias and bias uncertainty in $k_{eff}$.

## 2. Methods and Results

### *2.1 Methodology of Bias and Bias Uncertainty in $k_{eff}$*

In a depletion validation, the Monte Carlo uncertainty sampling method is used to represent the effects of nuclide concentration uncertainty on $k_{eff}$ values by sampling isotopic concentrations with uncertainty distributions developed from experimental data. The Monte Carlo uncertainty sampling method requires determination of biases and bias uncertainties in the calculated nuclide concentrations. The equations related to the calculation of bias and bias uncertainty in calculated nuclide concentrations were as follows [2]:

$$X_n^j = {M_n^j}\big/{C_n^j} \qquad (1)$$

$$\bar{X}_n = \sum_{j=1}^{N_n} {X_n^j}\big/{N_n} \qquad (2)$$

$$s_n = \sqrt{\sum_{j=1}^{N_n} {\left(X_n^j - \bar{X}_n\right)^2}\big/{(N_n - 1)}} \qquad (3)$$

$$\sigma_n = \begin{cases} s_n \cdot tf_2^n, & if\ N_n \geq 10 \\ s_n \cdot tf_1^n, & otherwise \end{cases} \qquad (4)$$

where

| | | |
|---|---|---|
| n | = | Burnup credit nuclide |
| j | = | Measured fuel sample |
| $X_n^j$ | = | Measured-to-Calculated concentration ratio |
| $M_n^j$ | = | Measured concentration |
| $C_n^j$ | = | Calculated concentration |
| $N_n$ | = | Number of evaluated fuel samples |
| $\bar{X}_n$ | = | Sample mean |
| $s_n$ | = | Sample standard deviation |
| $\sigma_n$ | = | Sample standard deviation adjusted for sample size |
| $tf_1^n$ | = | One-sided tolerance-limit factor |
| $tf_2^n$ | = | Two-sided tolerance-limit factor |

Isotopic bias and bias uncertainty values for PWR UNF are shown in Table I. The isotopic bias and bias uncertainty values were determined with Eqs. (2) and (4), respectively, where the nuclide concentration values for measured nuclides in fuel samples were calculated with SCALE 6.1 and the ENDF/B-VII nuclear data [2].

Table I: Isotopic bias and bias uncertainty for PWR UNF [2]

| Burnup | 15 < Burnup ≤ 40 GWd/MTU | | |
|---|---|---|---|
| Nuclide | Number of samples | Isotopic bias | Isotopic bias uncertainty |
| U-234 | 43 | 0.9119 | 0.1749 |
| U-235 | 69 | 0.9907 | 0.0416 |
| U-238 | 69 | 1.0017 | 0.0042 |
| Pu-238 | 65 | 1.1500 | 0.0923 |
| Pu-239 | 69 | 0.9587 | 0.0375 |
| Pu-240 | 69 | 0.9801 | 0.0317 |
| Pu-241 | 69 | 1.0108 | 0.0514 |
| Pu-242 | 69 | 1.0647 | 0.0783 |
| Am-241 | 27 | 0.9312 | 0.2077 |

The equations related to the calculation of nuclide concentration values for use in $k_{eff}$ calculations were given by [2]

$$c_{n,b}^k = \begin{cases} c_{n,b} \cdot (\bar{X}_n^b + \sigma_n^b \cdot R_n^k|_{normal}), if\ N_n \geq 10 \\ c_{n,b} \cdot (\bar{X}_n^b + \sigma_n^b \cdot R_n^k|_{uniform}), otherwise \end{cases} \quad (5)$$

$$\bar{k}_{eff} = \sum_{i=1}^{N_C} k_{eff}^i / N_C \quad (6)$$

$$\sigma_{k_{eff}} = \sqrt{\sum_{i=1}^{N_C} (k_{eff}^i - \bar{k}_{eff})^2 / (N_C - 1)} \quad (7)$$

$$\beta_i = k_{eff-REF} - \bar{k}_{eff} \quad (8)$$

$$\Delta k_i = \sigma_{k_{eff}} \times tf_1^{N_C} \quad (9)$$

where

| | | |
|---|---|---|
| k or i | = | Index of a criticality calculation |
| b | = | Burnup of a PWR UNF |
| $c_{n,b}^k$ | = | Concentration adjusted for isotopic bias and bias uncertainty |
| $c_{n,b}$ | = | Calculated concentration by code |
| $\bar{X}_n^b$ | = | Isotopic bias |
| $\sigma_n^b$ | = | Isotopic bias uncertainty |
| $R_n^k\|_{normal}$ | = | Random number sampled from the standard normal distribution |
| $R_n^k\|_{uniform}$ | = | Random number sampled from the uniform distribution, -1 ~ +1 |
| $N_C$ | = | Number of calculated $k_{eff}$ values |
| $k_{eff}^i$ | = | $k_{eff}$ value for criticality calculation i |
| $\bar{k}_{eff}$ | = | Sample mean of the $k_{eff}$ values |
| $\sigma_{k_{eff}}$ | = | Sample standard deviation of the $k_{eff}$ values |
| $\beta_i$ | = | Bias in $k_{eff}$ resulting from Table I |
| $k_{eff-REF}$ | = | $k_{eff}$ for calculated concentrations without adjustments |
| $\Delta k_i$ | = | Bias uncertainty in $k_{eff}$ resulting from Table I |
| $tf_1^{N_C}$ | = | One-sided tolerance-limit factor for the normal distribution |

Thus, the bias and bias uncertainty in $k_{eff}$ resulted from biases and bias uncertainties in the calculated nuclide concentrations are determined as follows [2].

$$\beta_i + \Delta k_i = \begin{cases} (\bar{k}_{eff} - k_{eff-REF}) + \sigma_{k_{eff}} \times tf_1^{N_C}, if\ \bar{k}_{eff} > k_{eff-REF} \\ \sigma_{k_{eff}} \times tf_1^{N_C}, otherwise \end{cases} \quad (10)$$

The Monte Carlo uncertainty sampling method is computationally intensive because a significant number of fuel composition simulations are necessary to ensure that the underlying probability distributions are adequately sampled and that the Monte Carlo estimates of $\bar{k}_{eff}$ and $\sigma_{k_{eff}}$ have reached convergence. Convergence is considered to be achieved when $\bar{k}_{eff}$ and $\sigma_{k_{eff}}$ values change insignificantly (e.g., within ±0.0005) with additional simulation [2].

## *2.2 Main Input Settings*

### 2.2.1 Design Data of domestic NFA

The NFA applied to this paper was the PLUS7 16X16 assembly which has been used in the Hanbit nuclear power plant. It is referred from Ref. 1 for the detailed configuration of the NFA and the burned location in the reactor core [1]. The radial and axial burnup distributions of all nuclear fuel rods were assumed to be uniform. In addition, the nuclides considered for the application of BUC were only the nine major actinides recommended in Ref. 2: U-234, U-235, U-238, Pu-238, Pu-239, Pu-240, Pu-241, Pu-242, and Am-241 [2].

### 2.2.2 Arrangement of UNFAs in DSC

The type of a DSC which accommodate the PLUS7 16X16 UNFAs was the Generic 32 PWR-assembly Burnup Credit (GBC-32) cask. As shown in Fig.1, the DSC is capable of accommodation up to 32 UNFAs and it is referred from Ref. 1 for the detailed configuration of the DSC and the arrangement of the UNFAs in the DSC. In general, a nuclear criticality safety analysis has been evaluated commonly under the condition of the storage of 32 UNFAs with the same initial enrichment, final burnup, axial burnup distribution, and cooling time [1]. The cooling times which means the period of wet storage in a used nuclear pool, of all UNFAs were set to 5 years.

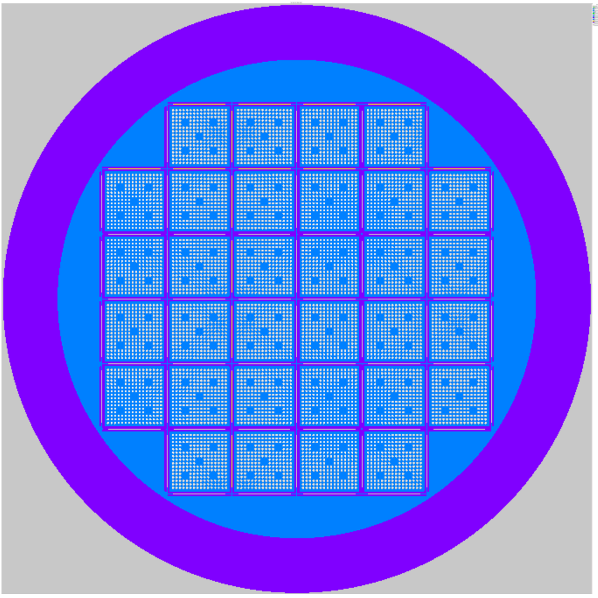

Fig. 1. Arrangement of 32 UNFAs in the DSC.

## *2.3 Generation of Nuclear Cross Section Libraries*

Isotopic compositions by a depletion analysis should be estimated accurately. One way for an accurate estimation of isotopic compositions is to apply accurate cross section libraries to a depletion analysis. Thus, the new one-group cross section libraries of the ORIGEN code were generated with respect to the PLUS7 16X16 NFA using the SCALE 6.1/TRITON code. Fig. 2 shows the representative configuration of the PLUS7 16X16 NFA.

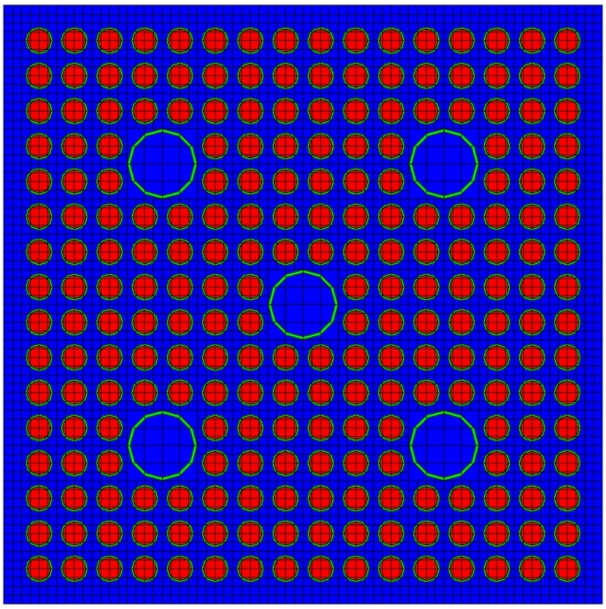

Fig. 2. Configuration of the PLUS7 16X16 NFA.

14 cross section libraries were newly generated in dependence of 14 initial enrichments ranged from 0.5 to 6.0 wt. % U-235 and 41 specific burnups ranged from 0 to 69,200 MWD/MTU. The SCALE 6.1/STARBUCS code for a NCSE with BUC was performed using these new one-group cross section libraries.

## *2.4 Search for Initial Enrichment Values*

The burnup-dependent nuclide concentrations for the GBC-32 DSC were determined so that the $k_{eff\text{-}REF}$ value was 0.94. The $k_{eff\text{-}REF}$ value was determined by applying an assumed allowance for biases and uncertainties of 0.01 to the recommended $k_{eff}$ value of 0.95 for general cask criticality safety analyses [2]. Thus, the appropriate initial enrichment values for which the $k_{eff\text{-}REF}$ value of the DSC system was made as 0.94 were searched as a function of specific burnup using the SCALE 6.1/STARBUCS code. The loading curve for the GBC-32 DSC with 32 PLUS7 16X16 UNFAs was able to be developed using these initial enrichment values and final specific burnups. Fig. 3 shows the loading curve for the GBC-32 DSC with 32 PLUS7 16X16 UNFAs, where the region above the red line is $k_{eff\text{-}REF} < 0.94$, i.e. the region is acceptable to store 32 PLUS7 16X16 UNFAs in the DSC, whereas the region below the red line is $k_{eff\text{-}REF} > 0.94$, i.e. the region is unacceptable to store.

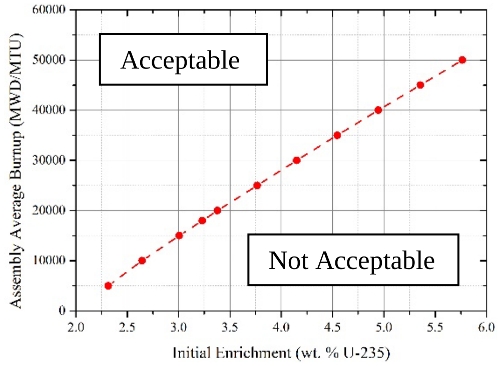


Fig. 3. Loading curve for the GBC-32 DSC.

## *2.5 Generation of Random Numbers*

In the Monte Carlo uncertainty sampling procedure, a normal distribution model is used to determine isotopic bias and bias uncertainty values if more than 10 measured concentration values are available for a nuclide. Random numbers drawn from the standard normal distribution (i.e., the normal distribution with the distribution mean of zero and standard deviation of unity) are used as shown in Eq. (5) to simulate nuclide concentration variations within the range of uncertainty [2]. Random numbers, $R_n^k|_{normal}$, were generated using the IBM SPSS Statistics Version 23 program. Fig. 4 shows 900 random numbers which have the normal distribution with the distribution mean of about 0.0104 and standard deviation of about 0.9600.

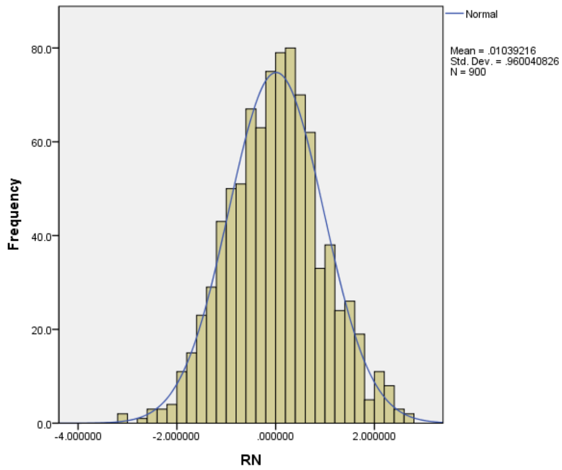


Fig. 4. 900 random numbers with a normal distribution.

## *2.5 Results and Evaluations*

In previous sections, the fundamental parameters to model the GBC-32 DSC with the PLUS7 16X16 UNFAs, the isotopic bias and bias uncertainty values for PWR UNF compositions (in Table I), the initial enrichments for the $k_{eff\text{-}REF}$ value of the DSC, and the random numbers with the normal distribution were obtained so far. Using these data, 100 $k_{eff}^{i}$ values for the final specific burnup of 30,000 MWD/MTU were computed by means of the SCALE 6.1/STARBUCS code and the 238-group ENDF/B-VII cross-section library. It was checked if the calculated $k_{eff}^{i}$ values passed the Shapiro-Wilk normality test at the 0.05 level using the IBM SPSS Statistics Version 23 program. Fig. 5 shows the $k_{eff}$ value for each index of a criticality calculation, $k_{eff}^{i}$, the average $k_{eff}$ value ranged from the first index to the target index, $\bar{k}_{eff}$, and the $k_{eff}$ value for the calculated nuclide concentrations with no adjustments, $k_{eff\text{-}REF}$.

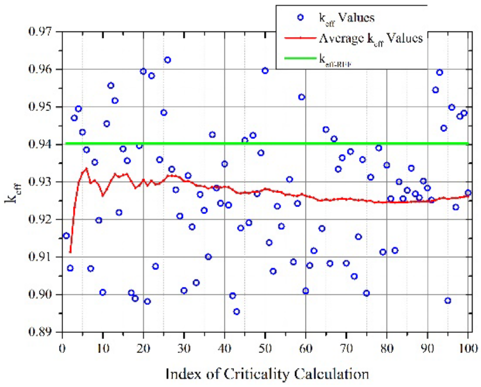


Fig. 5. $k_{eff}$ estimates by the Monte Carlo simulations.

As a result, the bias in $k_{eff}$, $\beta_i$, was 0.014108 by Eq. (8) through the result data in Fig. 5. Since the one-sided tolerance-limit factor for the normal distribution corresponding to $N_C$=100, at a 95% probability, 95% confidence level, $tf_1^{100}$, was 1.927 in Ref. 3 [3], the bias uncertainty in $k_{eff}$, $\Delta k_i$, was 0.035151 by Eq. (9) through the result data in Fig. 5. Therefore, since the average $k_{eff}$ value ranged from the first index to the last index, $\bar{k}_{eff}$, was smaller than the $k_{eff}$ value for the calculated nuclide concentrations with no adjustments, $k_{eff\text{-}REF}$ as shown in Fig. 5, the $k_{eff}$ bias and $k_{eff}$ bias uncertainty value, $\beta_i + \Delta k_i$, for this nuclear criticality safety analysis was 0.035151 by Eq. (10).

### 3. Conclusions

In this work, the bias and bias uncertainty in $k_{eff}$ resulting from biases and bias uncertainties in the calculated nuclide concentrations were determined for the GBC-32 DSC system with 32 PLUS7 16X16 UNFAs. First, the new one-group cross section libraries of the ORIGEN code were generated with respect to the PLUS7 16X16 NFA using the SCALE 6.1/TRITON code. Second, the appropriate initial enrichment values for which the $k_{eff\text{-}REF}$ value of the DSC system was to be 0.94 were searched as a function of specific burnup using the SCALE 6.1/STARBUCS code. Third, 900 random numbers with the normal distribution were generated using the IBM SPSS Statistics Version 23 program. At last, 100 $k_{eff}^{i}$ values for the final specific burnup of 30,000 MWD/MTU were computed by means of the SCALE 6.1/STARBUCS code. From the results calculated in these conditions, the following conclusions are drawn.

1. The average $k_{eff}$ value ranged from the first index to the last index, $\bar{k}_{eff}$, was smaller than the $k_{eff}$ value for the calculated nuclide concentrations with no adjustments, $k_{eff\text{-}REF}$ for all index of a criticality calculation.
2. The $k_{eff}$ bias and $k_{eff}$ bias uncertainty value, $\beta_i + \Delta k_i$, for this nuclear criticality safety analysis was 0.035151, which was the high value. Because the index of a criticality calculation was not enough and the sample standard deviation of the $k_{eff}$ values from the Monte Carlo simulations, $\sigma_{k_{eff}}$, was high.
3. It is expected to be able to decrease the $k_{eff}$ bias and $k_{eff}$ bias uncertainty value if the index of a criticality calculation increases. Because the sample standard deviation of the $k_{eff}$ values from the Monte Carlo simulations and the one-sided tolerance-limit factor, $tf_1^{Nc}$, decease as the index of a criticality calculation increases.

In the future, this work will be extended for consideration of large number of UNFA burnups and the increased number of random samplings for more general conclusion.

### REFERENCES

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[3] R. E. Odeh and D. B. Owen, Tables for Normal Tolerance Limits, Sampling Plans, and Screening, Marcel Dekker, Inc., 1980.