Development of a Modified Kernel Regression Model for a Robust Signal Reconstruction

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1. Introduction

In order to achieve the desired operating configuration in any process plant, the system status needs to be accurately measured. Also, for a system to operate within the desired limit, it is essential to know the reliability of the plant measurements. Therefore, the demand for robust and resilient performance has led to the use of online-monitoring techniques to monitor the process parameters and signal validation. On-line monitoring and signal validation techniques are the two important terminologies in process and equipment monitoring. These techniques are automated methods of monitoring instrument performance while the plant is operating [1,2]. To implementing these techniques, several empirical models are used [1-4]. One of these models is nonparametric regression model, otherwise known as kernel regression (KR).

Unlike parametric models, KR is an algorithmic estimation procedure which assumes no significant parameters, and it needs no training process after its development when new observations are prepared; which is good for a system characteristic of changing due to ageing phenomenon. Although KR is used and performed excellently when applied to steady state or normal operating data, it has limitation in time-varying data that has several repetition of the same signal, especially if those signals are used to infer the other signals. In addition, many situations are related to variation and fluctuation, such as transient – start-up and shutdown mode. The major problem of KR in such a condition is that, the values of dependent variable in those points of the same value of the predictor variables assume value of the average of those dependent variable values. However, accurate estimation of the process signal can lead to the proper understanding of the equipment behaviours as well as enhance the online monitoring applications. Therefore, in this work, we proposed a modified KR model for robust signal reconstruction to resolve the setback of convectional KR.

2. Overview of Kernel Regression

In this section the convectional KR model is briefly introduced.

Given a matrix of memory data of predictor variables X and the response variable **y** as

$$\mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_n]^T$$

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,p} \end{bmatrix}$$

where p is the number of predictor variables, n is the number of memory vectors, and $x_{(i,j)}$ is the *i*th observation of the *j*th predictor variable.

Then, for any observed query vector,

$$\mathrm{x}_q = [x_1 \quad x_2 \quad \cdots \quad x_p]$$

the Euclidean distance, d, the kernel weight (Gaussian kernel), k and the estimated response variable, $\hat{\mathbf{y}}$ are calculated as follows:

$$\boldsymbol{d_i}(\boldsymbol{X_i}, \mathbf{x_q}) = \sqrt{(\boldsymbol{x_{i,1}} - \boldsymbol{x_1})^2 + (\boldsymbol{x_{i,2}} - \boldsymbol{x_2})^2 + \cdots + (\boldsymbol{x_{i,p}} - \boldsymbol{x_p})^2}$$

$$K_i(X_i, \mathrm{x}_q) = exp\left(\frac{-d_i^2}{2h^2}\right)$$

$$\hat{y}_i(\mathrm{x}_q) = \frac{\sum_{i=1}^{n} K(X_i, \mathrm{x}_q).\mathrm{y}_i}{\sum_{i=1}^{n} K(X_i, \mathrm{x}_q)}$$

where h is the bandwidth of the kernel.

The kernel regression can be modelled in three different ways [3]: inferential (IKR), auto-associative (AAKR) and hetero-associative (HAKR) models depending on the desired model for a particular application, as shown in Fig.1.

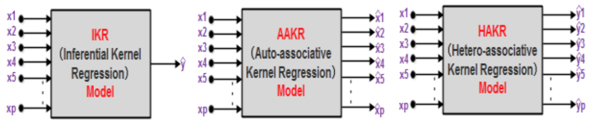

Fig. 1. Three different models of kernel regression

3. Methods and Results

In this section the approaches used to develop the modified KR are presented, and the results are discussed.

3.1 Development of Modified KR Model

The model flowchart is shown in Fig. 2. The modification is mainly on the improvement of measure of similarities used in KR prior to the distance calculation. We developed time dependent KR by taking the gradient/slope of the data into consideration. Generally, the historical data use to build the empirical models are collected at a particular time interval during operation. Let's assume that the data are evenly spaced with time interval of Δt, mathematically, to determine the slope or gradient of a non-linear function at any given point along the curve of the function, a straight line tangential to the curve is drawn at that point.

Thus, by taking an arbitrary point x' to be immediately after the point x_1 and before the point x_2 which occurs half-way of Δt, then we have,

$$x' = x_1 + m(\frac{1}{2}\Delta t)$$

where m is the first derivative or slope/gradient of the function at point x_1.

In this case, we need to determine the slope of the function at each data point. To calculate the slope at any given point, we applied the finite difference derivative approximations which are derived from the Taylor series expansion,

$$x(t+\Delta t) = x(t) + \Delta t\frac{dx(t)}{dt} + \frac{(\Delta t)^2}{2!}\frac{d^2x(t)}{dt^2} + \frac{(\Delta t)^3}{3!}\frac{d^3x(t)}{dt^3} + \cdots\cdots$$

For nxp matrix X of the given data, the forward difference, backward difference, and central difference derivative approximation based on three points can be evaluated respectively as follows:

$$m_{i,j} = \frac{dx}{dt} = \frac{-x_{i+2,j} + 4x_{i+1,j} - 3x_{i,j}}{2\Delta t}$$

$$m_{i,j} = \frac{dx}{dt} = \frac{3x_{i,j} - 4x_{i-1,j} + x_{i-2,j}}{2\Delta t}$$

$$m_{i,j} = \frac{dx}{dt} = \frac{x_{i+1,j} - x_{i-1,j}}{2\Delta t}$$

These yield the slope/gradient matrix *M* as

$$M = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,p} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n,1} & m_{n,2} & \cdots & m_{n,p} \end{bmatrix}$$

Fig. 1. Flowchart of the developed model.

Hence, the transformed/adjusted X matrix can then be expressed as

$$X' = \begin{bmatrix} x_{1,1} & \cdots & x_{1,p} \\ \vdots & \ddots & \vdots \\ x_{n,1} & \cdots & x_{n,p} \end{bmatrix} + \begin{bmatrix} m_{1,1} & \cdots & m_{1,p} \\ \vdots & \ddots & \vdots \\ m_{n,1} & \cdots & m_{n,p} \end{bmatrix} * \frac{1}{2}\Delta t$$

$$X' = \begin{bmatrix} x'_{1,1} & x'_{1,2} & \cdots & x'_{1,p} \\ \vdots & \vdots & \ddots & \vdots \\ x'_{n,1} & x'_{n,2} & \cdots & x'_{n,p} \end{bmatrix}$$

where

$$x'_{i,j} = x_{i,j} + m_{i,j}\left(\frac{1}{2}\Delta t\right)$$

Then, the Euclidian distance for any given query vector can be calculated as

$$d_i(X_i, x_q) = \sqrt{\frac{(x'_{i,1} - x'_1)^2}{\sigma_1^2} + \frac{(x'_{i,2} - x'_2)^2}{\sigma_2^2} + \cdots + \frac{(x'_{i,p} - x'_p)^2}{\sigma_p^2}}$$

Where σ_p^2 is the variance of the transformed predictor variable p

This distance is then used to calculate the kernel weight using Gaussian kernel given in section 2, and estimate the dependent variables.

There are two path modes in the flowchart presented in Fig.1: the training path mode and the execution path mode. The proposed model at first, takes the training path by taking the historical data collected during plant operation from which the model is trained based on the developed algorithms. When a satisfactory training model is achieved, the model stored the trained data which can then be used for online monitoring to predict the signal values for any online query data vector at a particular time. It is worth noting that, as indicated in the flowchart, the model needs to get the initial three successive query vectors to enable it evaluate the gradient of those three vectors using finite difference derivative approximation that depends on three data points for evaluations. At this initial condition, the model evaluates the gradient of the three vectors using forward, central, and backward differences for the first, second, and third vectors respectively. Subsequently, the model continues to stores the two immediate previous vectors and uses it to evaluate the gradient of any subsequent available query vector using central difference approximation.

3.2 Results

In order to test our developed model, two difference kinds of data are used. Firstly, we used the lab experimental data generated from heat conduction experiment to evaluate the performance of the model and compare the results with the convectional KR. Secondly, we used the real time simulation data from compact nuclear simulator (CNS) provided by KAERI.

This data is a cooling and heating data collected during heating up from cool-down mode, which represents a normal transient data. Although, this data may not be transient enough, however, we used it to demonstrate the performance of the developed model. The two kinds of the data used are respectively shown in Fig. 2 and Fig. 3. In both Fig. 2 and Fig. 3, x_1 and x_2 are independent variables that are used to estimate/predict the value of Y. The results are shown in Figs. 4, 5, 6, and 7. The mean square error (MSE) are calculated and used to evaluate the performance of the models as,

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_{iest} - y_i)^2$$

where y_{iest} is the estimated value and y_i is the actual value. N is the number of data points/observations.

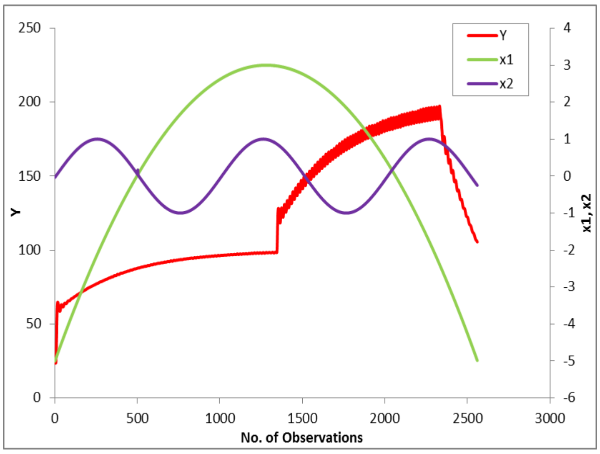

Fig. 2. Lab Experimental data.

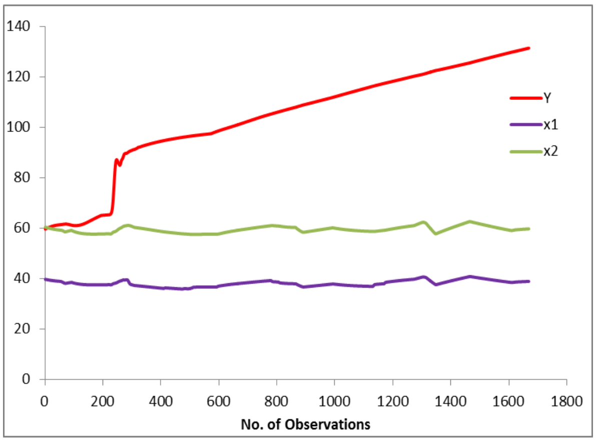

Fig. 3. CNS data collected during heating up cold-down mode.

From the results of the lab experimental data shown in Fig. 4 and Fig. 5 respectively for modified KR and Convectional KR, the MSE of the convectional KR is about 62 times larger than that of modified KR. In another word, the accuracy of a modified KR prediction is 62 times better than that of convectional KR. We discovered that the convectional KR has several deviations due to the fact that, it does not take the fluctuations or changes or slopes of the data series into consideration. Whereas, the modified KR though has shown a greater performance, also have few points of deviation. This might be due to the differences between the gradients of the data points which are not taking into consideration presently in this study, for computing the weight of similarities between the data points. We discovered that the impact is largely the contribution from the dependent variable x_1 shown in Fig. 2. However further studies is ongoing to resolve this issues so as to make the model more general.

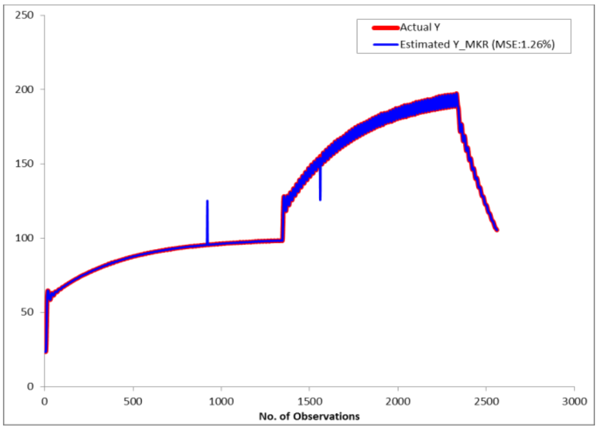

Fig. 4. Estimated dependent variable, Y by a modified KR model (Lab experimental data).

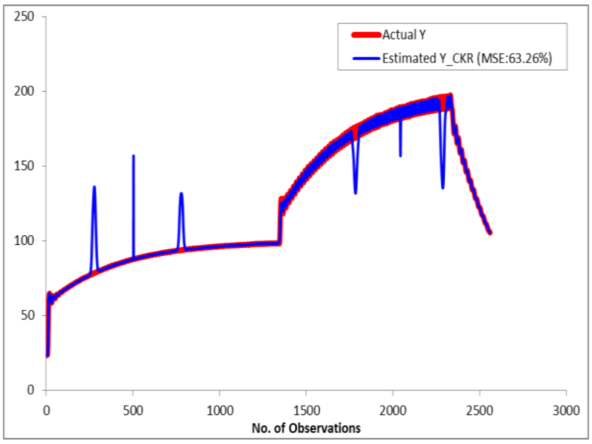

Fig. 5. Estimated dependent variable, Y by a convectional KR model (Lab experimental data).

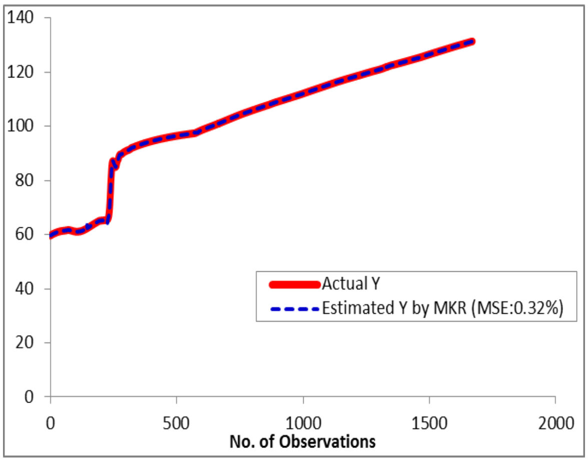

Fig. 6. Estimated signal by modified KR model (CNS data).

The results of the prediction from CNS data are respectively shown in Fig. 6 and Fig. 7 for modified KR and convectional KR. Also, as shown in the results, the modified KR still performed better than the convectional KR. In this case, the convectional KR MSE is much lower than its MSE in experimental data. This is because the dependent variables are not too transients compare to experimental data, and the fluctuations are not extremely frequent as that of the experimental data.

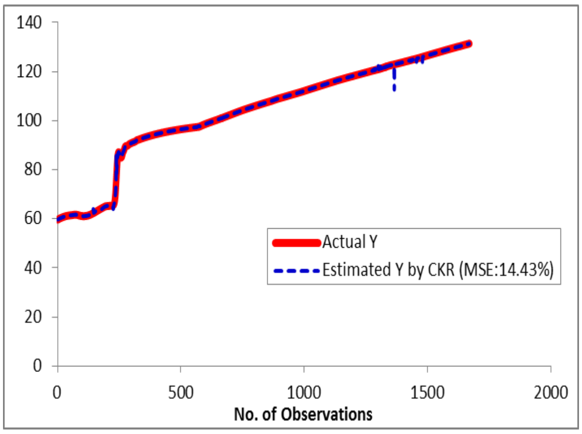

Fig. 7. Estimated signal by Convectional KR model (CNS data).

4. Conclusions

The convectional KR has limitation in correctly estimating the dependent variable when time-varying data with repeated values are used to estimate the dependent variable especially in signal validation and monitoring. Therefore, we presented here in this work a modified KR that can resolve this issue which can also be feasible in time domain. Data are first transformed prior to the Euclidian distance evaluation considering their slopes/changes with respect to time. The performance of the developed model is evaluated and compared with that of conventional KR using both the lab experimental data and the real time data from CNS provided by KAERI. The result shows that the proposed developed model, having demonstrated high performance accuracy than that of conventional KR, is capable of resolving the identified limitation with convectional KR. We also discovered that there is still need to further improve our model to make it more generalized as well for more robustness than the current performance. Therefore, the further study is on-going to resolve these issues and to also considering the different domains (time and frequency domains) as well as different level of derivatives.

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