Mathematical Model for Effective Thermal Conductivity of Reinforced Concrete Including Round bar

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1. Introduction

After Fukushima, precise analysis of containment temperature and pressure is one of the important issues in nuclear safety. Lately, Noh et al. compared many effective thermal conductivity models and proposed modified Rayleigh model for evaluating effective thermal conductivity in containment wall of the standard Korean NPP OPR1000 [1]. However, previous models cannot exactly predict effective thermal conductivity of reinforced concrete composed of specific geometries [2].

The main purpose of this study is to propose effective thermal conductivity model of reinforced concrete including one round rebar as volume fraction of steel. Commonly, reinforced concrete consists of concrete and steel rebar. Mathematical model can be achieved by utilizing thermal network concept and Fourier's law for effective thermal conductivity [3]. Mathematical model is compared with results of numerical simulation by using ANSYS-CFX tool in this paper. For accurate thermal analysis of reinforced concrete, effective thermal conductivity model can be widely used in many fields such as building, civil engineering and nuclear.

2. Methods and Results

Modeling method is described in this section. Moreover, numerical simulations are conducted for evaluating effective thermal conductivity of reinforced concrete. Finally, the results of model and numerical simulation are also compared.

2.1 Modeling method

Effective thermal conductivity of reinforced concrete can be mathematically achieved by using thermal network concept and Fourier's law. Concrete block is assumed by a square L(m) on a side. Reinforced concrete is divided into concrete and steel as shown in Fig. 1.

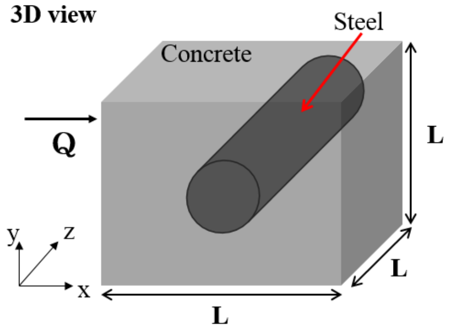

Fig. 1. Schematic of reinforced concrete containing one round rebar

Total thermal network can be described by summation of three thermal resistance for mathematical modeling of reinforced concrete in Fig. 2. Steel rebar is inserted at the direction of z axis in concrete. Heat is applied in the direction of x axis.

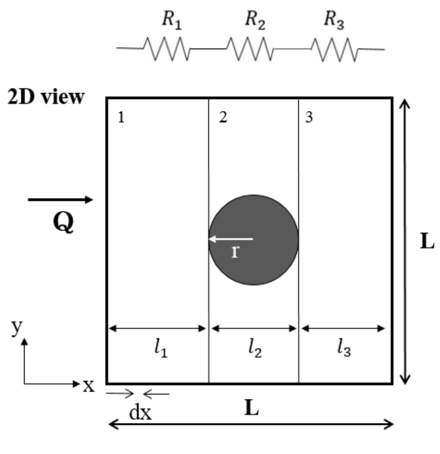

Fig. 2. 2-dimensional view of reinforced concrete containing one round rebar and its thermal network

Specific formula are described in following equations for obtaining effective thermal conductivity of reinforced concrete.

$$R_1 + R_3 = \frac{L-(l_1+l_3)}{k_c \cdot A} = \frac{L-2r}{k_c \cdot A} \quad (\because l_2 = 2r, k_1 = k_3 = k_c) \quad (1)$$

$$Q_{\text{total}} = Q_s + Q_c \;\; \text{at} \;\; R_2 \quad (2)$$

$$k_2 = \frac{Q_s + Q_c}{\frac{dT}{\mathrm{dx}} \cdot A} = \frac{k_s A_s}{A} + \frac{k_c A_c}{A} \quad \left(\because \mathrm{Q} = \mathrm{k} \cdot A\frac{dT}{\mathrm{dx}}\right) \quad (3)$$

Integral is performed about k_2 over 2r for obtaining effective thermal conductivity in part 2.

$$k_2 \cdot 2r = \frac{1}{A} \int_{2r} (k_s A_s + k_c A_c) dx \quad (4)$$

$$\therefore k_2 = \frac{1}{2rA} k_s(\pi r^2 L) + \frac{1}{2rA} k_c(2rL^2 - \pi r^2 L) \quad (5)$$

$$\mathrm{R}_2 = \frac{4r^2}{k_s(\pi r^2 L) + k_c(2rL^2 - \pi r^2 L)} \quad (6)$$

Thus, total thermal network is expressed as following equations.

$$R_{\mathrm{total}} = R_1 + R_2 + R_3 = \frac{L-2r}{k_c \cdot A} + \frac{4r^2}{k_s(\pi r^2 L) + k_c(2rL^2 - \pi r^2 L)} \quad (7)$$

$$k_{eff} = \frac{L}{R_{total} \cdot A} \quad (8)$$

Therefore, k_{eff} can be achieved such as (9). Concrete calculation is omitted.

$$\therefore k_{eff} = \frac{1}{\frac{1}{k_c} - \frac{1}{k_c}\left(\frac{4\phi_s}{\pi}\right)^{\frac{1}{2}} + \frac{4}{k_c\left(\frac{4\pi}{\phi_s}\right)^{\frac{1}{2}} + \pi(k_s - k_c)}} \left(\because \phi_s = \frac{\pi r^2}{L^2}\right) \quad (9)$$

2.2 Numerical simulation(CFD)

Numerical simulations for evaluating effective thermal conductivity of reinforced concrete containing one round rebar as volume fraction of steel. ANSYS CFX 16.2 based on Finite Volume Method(FVM) was used in steady-state condition. Thermal conductivity of concrete and steel was applied as 1.6W/m·K and 53W/m·K individually. The size of numerical domain is a square 0.1m on a side. Five cases are conducted as volume fraction of steel between about 3% and 24% in Table 1.

Table 1: Test matrix for numerical simulations

Diameter of round bar(mm) (KS R 3504)	Volume fraction (%)
19	2.8
32	8.0
41	13.2
51	20.4
55	23.8

Numerical domains are shown in Fig. 3. Constant heat flux is 1000W/m^2 in the left side of concrete block. Moreover, constant convective heat transfer coefficient and room temperature were also applied in the outside air of right side concrete for the boundary conditions.

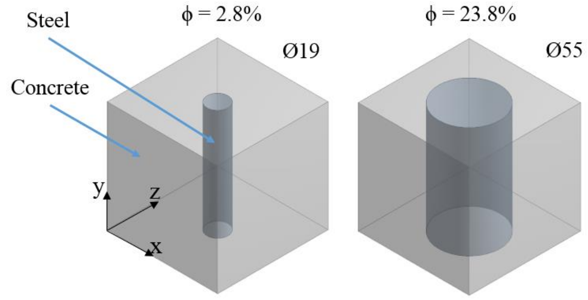

Fig. 3. Numerical domain in reinforced concrete for volume fractions ϕ=0.028 and ϕ=0.238

Temperature distributions are shown in Fig. 4. Steel rebar played an important role in temperature distribution. Distortion of temperature distribution becomes larger as increasing volume fraction of steel.

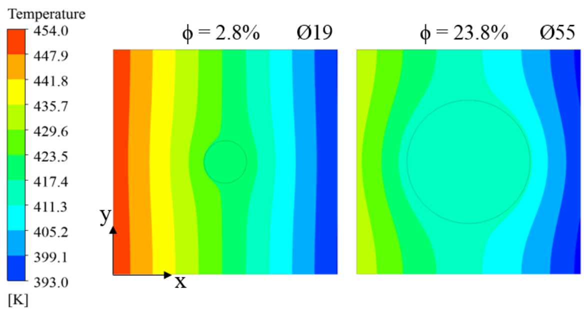

Fig. 4. Temperature distributions in reinforced concrete for volume fractions ϕ=0.028 and ϕ=0.238

2.3 Results

Mathematical model for evaluating effective thermal conductivity of reinforced concrete containing one round rebar was compared with results of numerical simulation in Figure. 5.

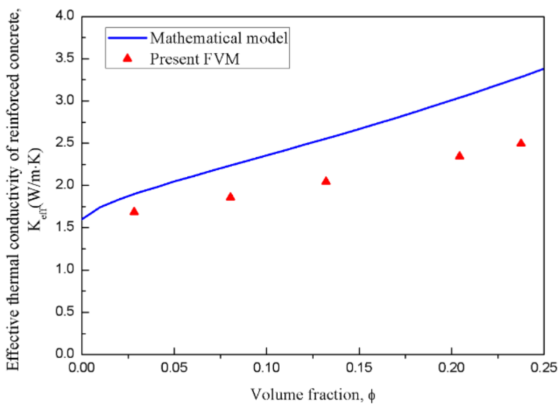

Fig. 5. Effective thermal conductivity of reinforced concrete versus volume fraction of steel rebar

Effective thermal conductivity of reinforced concrete generally increases as growing volume fraction of steel in results of numerical simulation. The result of mathematical model slightly overestimates CFD results over all. Moreover, differences between model and CFD results become larger as increasing volume fraction of steel as shown in Figure. 5.

3. Conclusions

The effective thermal conductivity model of reinforced concrete is proposed. Moreover, the effective thermal conductivity of reinforced concrete is numerically investigated by numerical simulation (ANSYS CFX 16.2). Mathematical model was compared with CFD results. Model value generally overestimates CFD results. Furthermore, differences between model and CFD results get larger as increasing volume fraction of steel. Therefore, thermal network has to be more complexed for predicting accurate effective thermal conductivity in further research.

Nomenclature

k_{eff}: Effective thermal conductivity in reinforced concrete (W/m·K)
k_c: Concrete thermal conductivity (W/m·K)
k_s: Steel thermal conductivity (W/m·K)
ϕ_s: Volume fraction of steel
R_{total}: Total thermal resistance (K/W)
R_i: i-th thermal resistance (K/W)
Q_{total}: Total heat quantity (W)
Q_c: Heat quantity to concrete (W)
Q_s: Heat quantity to steel (W)
A_c: Area of concrete in mixed part (m^2)
A_s: Area of steel in mixed part (m^2)
r: Radius of round rebar (m)
l_i: Length of i-th part (m)
L: One side length of concrete wall (m)

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