Two-Fluid Equations with a Temporally- and Spatially-Varying Fluid Porosity

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1. Introduction

As a result of a large-break loss-of-coolant accident, a rise in the internal pressure in the fuel rod pins due to overheating may cause a permanent deformation of the cladding. The coolability for the ballooned cladding geometry has received attention as a design-extended condition in a nuclear power plant. Most existing studies, however, assumed a pre-deformed geometry of the cladding [1]. The effect of the cladding deformation in time was not investigated.

The purpose of this study is to derive the general two-fluid equations for a porous media which can reflect a geometry deformation. The fluid porosity can vary spatially and temporally. The validity of the derived equations was investigated by a SPACE code simulation in terms of mass error.

2. General Two-Fluid Equations

Consider the averaged phases of gas, continuous liquid, droplet, and solid phases in a total averaging volume fixed in time (Fig. 1).

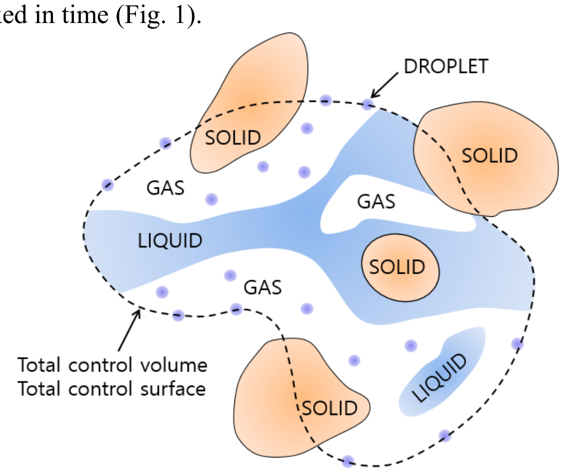

Fig. 1. Gas, continuous liquid, droplet, and solid phases in the total averaging volume.

The total volume is the sum of phasic volumes.

$$V = V_s + V_g + V_l + V_d \tag{1}$$

The subscripts *s*, *g*, *l*, and *d* signify the solid, gas, continuous liquid, and droplet fields, respectively. The fluid porosity (ε) and the volume fraction (α_k) for phase *k* are defined as follows:

$$\varepsilon \equiv \frac{V - V_s}{V} , \tag{2}$$

$$\alpha_k \equiv \frac{V_k}{V - V_s} = \frac{V_k}{\varepsilon V} . \tag{3}$$

Applying the Liebnitz rule, the divergence theorem, and the averaging theorem, one obtains the conservation equations for mass, momentum, and internal energy:

$$\frac{\partial}{\partial t}(\varepsilon \alpha_k \rho_k) + \nabla \cdot (\varepsilon \alpha_k \rho_k \mathbf{u}_k) = \varepsilon \Gamma_{ik} , \tag{4}$$

$$\varepsilon \alpha_k \rho_k \left(\frac{\partial \mathbf{u}_k}{\partial t} + \mathbf{u}_k \cdot \nabla \mathbf{u}_k \right) = -\varepsilon \alpha_k \nabla p_k + \varepsilon \alpha_k \nabla \cdot \boldsymbol{\tau}_k + \nabla \cdot (\varepsilon \alpha_k \boldsymbol{\tau}_k^{\mathrm{Re}}) + \varepsilon \Gamma_{ki} (\mathbf{u}_{ik} - \mathbf{u}_k) + \varepsilon \mathbf{f}_{ik} + \varepsilon \mathbf{f}_{sk} + \varepsilon \alpha_k \rho_k \mathbf{g} \tag{5}$$

$$\begin{aligned} &\frac{\partial}{\partial t}(\varepsilon \alpha_k \rho_k e_k) + \nabla \cdot (\varepsilon \alpha_k \rho_k e_k \mathbf{u}_k) \\ &= -\nabla \cdot [\varepsilon \alpha_k (\mathbf{q}_k + \mathbf{q}_k^{\mathrm{Re},e})] + \varepsilon Q_{ik} + \varepsilon Q_{sk} \\ &\quad - p_k \nabla \cdot (\varepsilon \alpha_k \mathbf{u}_k) + \varepsilon \Gamma_{ik} h_{ik} - p_k \frac{\partial}{\partial t}(\varepsilon \alpha_k) + \varepsilon \alpha_k \Phi_k \end{aligned} \tag{6}$$

The momentum equation is expressed in the non-conservative form. It is assumed in this study that the fluid porosity is explicitly given as a function of time, being independent of fluid motion, like a fuel cladding ballooning.

3. One-Dimensional Equations

The one-dimensional equation can be obtained by simplifying Eqs. (4)~(6). The mass equation for one-dimensional flow is

$$\frac{\partial}{\partial t}(\varepsilon \alpha_k \rho_k) + \frac{\partial}{\partial x}(\varepsilon \alpha_k \rho_k u_k) = \varepsilon \Gamma_{ik} . \tag{7}$$

For a turbulent flow, the velocity profile is nearly flat across the pipe; thus, $\boldsymbol{\tau}_k^{\mathrm{Re}}$ is small. The term $\nabla \cdot \boldsymbol{\tau}_k$ originates from the surface integral of viscous stresses at the inlet and outlet. Because the flow is perpendicular to the inlet and outlet, $\nabla \cdot \boldsymbol{\tau}_k$ is negligibly small. With those assumptions, one can reduce Eq. (5) in the form

$$\begin{aligned} &\alpha_k \rho_k \left(\frac{\partial u_k}{\partial t} + u_k \frac{\partial u_k}{\partial x} \right) \\ &= -\alpha_k \frac{\partial p_k}{\partial x} + \Gamma_{ki}(u_{ik} - u_k) + f_{ik} + f_{sk} + \alpha_k \rho_k g_x \end{aligned} \tag{8}$$

It is interesting to note that the fluid porosity disappears in the above equation. The non-conservative form of the momentum equation greatly reduces the complexity of the momentum equation.

The effect of conduction is small for vapor-water fluid. In addition, for a turbulent flow, the velocity and temperature are nearly uniform across the pipe. With those assumptions, one can reduce Eq. (6) in the form

$$\begin{aligned}&\frac{\partial}{\partial t}(\varepsilon\alpha_k\rho_k e_k)+\frac{\partial}{\partial x}(\varepsilon\alpha_k\rho_k e_k u_k)\\&=-p_k\frac{\partial}{\partial t}(\varepsilon\alpha_k)-p_k\nabla\cdot(\varepsilon\alpha_k u_k)\,.\\&+\varepsilon(Q_{ik}+Q_{sk}+\Gamma_{ik}h_{ik}+\alpha_k\Phi_k)\end{aligned}\qquad(9)$$

4. Validation

The mass and energy matrix equations of SPACE were modified according to Eqs. (7) and (9). The momentum equation was not modified since the porosity is absent in the non-conservative form of the momentum equation, as shown in Eq. (8). The FLECHT-SEASET heating rods were assumed to be deformed at the middle elevation. The simulation conditions were the same as those for 31054 test, except for time-dependent porosity. The fluid porosity was set to decreases as shown in Fig. 2.

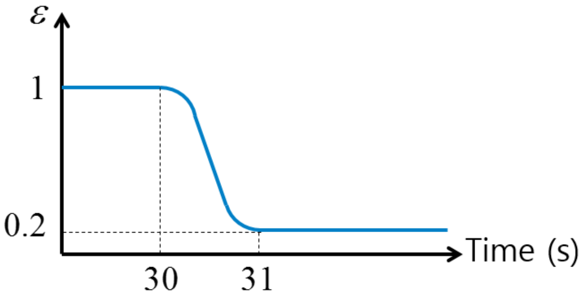

Fig. 2. Fluid porosity variation with time

For $30 < t < 31$ s, the fluid porosity is given as

$$\varepsilon = 1-2.4(t-30)^2+1.6(t-30)^3\,. \qquad (10)$$

Figure 3 shows the variation of the mass error with time. As shown, the mass error is very small. This result tell us that the derived equations are correct in terms of mass conservation.

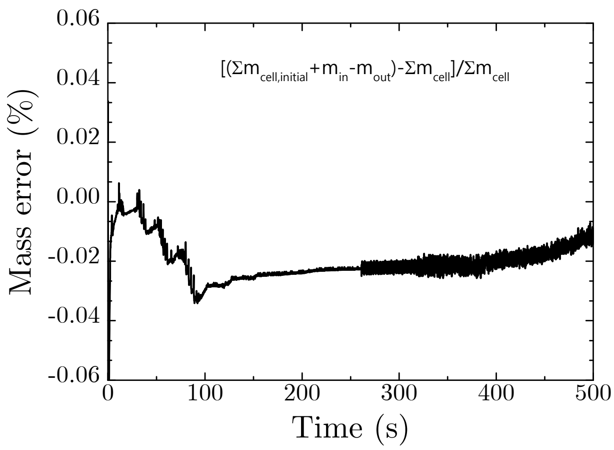

Fig. 3. Mass error variation with time

We performed another simulation with the mass and energy matrix equations in which the term $\Delta\varepsilon/\Delta t$ was intentionally excluded. Figure 4 shows the result. The mass error increases rapidly at 30 s. This result indirectly means that the one-dimensional equations we obtained are correct in terms of mass conservation.

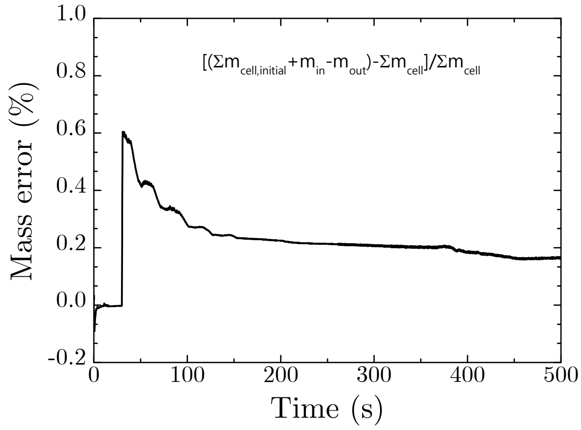

Fig. 4. Mass error variation when the term $\Delta\varepsilon/\Delta t$ is intentionally excluded.

4. Conclusions

We derived the general two-fluid equations considering the fluid porosity which may vary spatially and temporally. The simulation results showed that the derived equations are correct in terms of mass conservation. These equations will be used for SPACE code. The fluid porosity will be determined by a fuel code.

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