Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 18-19, 2017

Application of Stochastic Model on Evaluation of Neutralization Probability for Physical Protection System Design

Min Ho Kang, Sung Soon Jang [a]
[a]*Physical Protection Devision, 1418 Yuseong-daero, Yuseong-gu, Daejeon 34101, Korea Institute of Nuclear Nonproliferation and Control*

1. Introduction

Since the 9·11 terrorist attack in 2001, the importance of physical protection measures to protect nuclear materials and nuclear facilities from emerging threats is increasing worldwide. Under this international situation, Korea established 'Design Basis Threat (DBT)' through 'Physical Protection Council' in December 2009. With the establishment of the DBT, the nuclear power companies need to construct a physical protection system that satisfies the established national DBT, and the physical protection regulatory agency should evaluate whether the installed physical protection system satisfies the DBT. It is important to establish the latest protection measures based on the result of risk assessment in which the risk of domestic nuclear facilities is evaluated based on DBT. Therefore, in this study, a methodology is proposed to improve the effectiveness and reliability of a physical protection system by using probabilistic methodology based on stochastic process to evaluate the risk of nuclear facilities.

2. Research Purpose and Method

The risk assessment method can be expressed as follows according to its general definition, which combines the probability of risk and the severity (damage) of a potential result.

$$R = P \times C$$

P is the probability of occurrence and *C* is the outcome (damage) of the attack. In this risk (*R*) formula, the probability (*P*) that an event occurs becomes the product of the attack probability (P_A) of the enemy against the facility and the probability ($P_{S|A}$) that an enemy will succeed if an attack occurs.

$$R = [P_A \times P_{S|A}] \times C$$

The probability that an attack will fail indicates how well the physical protection system defends. This is called physical protection system effectiveness (P_E). The equation is expressed as follows.

$$P_{S|A} = 1 - P_E$$

The effectiveness probability(P_E) is the product of probability of Interruption(P_I), which is the probability that reaction forces will arrive before the enemy completes the attack, and probability of neutralization(P_N), which is the probability that the reaction forces engage with the enemy to neutralize them.

$$P_E = P_I \times P_N$$

Taking the above equations into consideration, the risk is as follows.

$$R = [P_A \times (1 - P_I \times P_N)] \times C$$

In other words, when evaluating the effectiveness of the physical protection system of a nuclear power plant, it is important to not only detect and delay intruders with the detection system in the vulnerable entry path, but also to consider the engagement of reaction forces equipped with proper weapon systems to neutralize the enemy.

2.1 Need of probability process application

A probabilistic approach is needed to mathematically model the battle situation of two units, to predict the amount of damage of the two units, and to determine the probability of victory in a battle. In order to simulate the dynamic engagement situation over time, it is necessary to use a stochastic variable with an appropriate probability distribution to simulate conditions similar to the real world. Various methods such as Lanchester equation, Markov chain, and Monte Carlo method are used to estimate the neutralization probability. In this study, we present a methodology for deriving the neutrailization probability from a probabilistic Lanchester model using Markov chain model, which was developed by Taylor (1983) and furthered evolved by Ancker and Gafarian (1988), and Kingman (2002)[2]. Also, we present a new methodology that supplements the limitations of the Lanchester equation.

2.2 Probabilistic Lanchester Model

Lanchester presented the loss of two units in a simple mathematical model in combat situations. When B(t) and R(t) are defined as the troop capacities according to time, and *b* and *r* are defined as the killing powers of the Blue team and the Red team, the amount of military strength of each team according to time is expressed by the following equation[2]. Solving the differential equation of Eq.(1) gives Eq.(2).

$$\frac{dB(t)}{dt} = -rR(t), \quad \frac{dR(t)}{dt} = -bB(t) \tag{1}$$

$$\begin{aligned} B(t) &= \frac{1}{2}((B(0) - \sqrt{\frac{r}{b}}R(0))e^{\sqrt{br}t} \\ &\quad + (B(0) + \sqrt{\frac{r}{b}}R(0))e^{-\sqrt{br}t}) \\ R(t) &= \frac{1}{2}((R(0) - \sqrt{\frac{b}{r}}B(0))e^{\sqrt{br}t} \\ &\quad + (R(0) + \sqrt{\frac{b}{r}}B(0))e^{-\sqrt{br}t}) \end{aligned} \tag{2}$$

These two equations always satisfies the relation of Eq.(3), which is called 'Lanchester's square law'.

$$rR^2 - bB^2 = \text{constant} \tag{3}$$

In other words, if we know *b* and *r* that indicate the killing powers of Blue team and Red team, and the capacities of their troops at the initial time, the loss of both troops after a certain time has elapsed can be predicted.

In a Lanchester's differential equation type of combat model, the disadvantage is that the damage of the troop is only a fixed constant value as shown in Eq.(3). To compensate for this, Taylor showed that the number of each group is an integer that is not negative, and it is assumed that no more than one casualty occurs at one transition. Also, the Lanchester model is extended to the probabilistic Lanchester model by using Markov chain, and this extended model is the most common form of the probabilistic Lanchester model[6].

When the capacity and the lethality of the Blue team are B and β, and those of the Red team are R and α, and when the troop capacities of both teams at time t were considered as troop status variables, state of markov chain is defined as (B_t, R_t), where B_t and R_t are natural numbers. The next state that can occur at any point in a state of markov chain is the case where one casualty occurs in Blue team or Red team, or both casualties do not occur. That is, the troop capacity state variables (B_t, R_t) have a markov property and can be expressed as a continuous time markov chain(CTMC) in which the above three state transitions occur. The probabilities for the three transitions are shown as follows.

$$\begin{aligned} &P\{(B_{t+dt}, R_{t+dt}) = (i-1, j) \mid (B_t, R_t) = (i, j)\} = \alpha R_t dt \\ &P\{(B_{t+dt}, R_{t+dt}) = (i, j-1) \mid (B_t, R_t) = (i, j)\} = \beta B_t dt \\ &P\{(B_{t+dt}, R_{t+dt}) = (i-1, j-1) \mid (B_t, R_t) = (i, j)\} = 1 - \alpha R_t dt - \beta B_t dt, \\ &\forall i = 1, \cdots, B, \forall j = 1, \cdots, R \end{aligned} \tag{4}$$

Let $P_{i_1 i_2 j_1 j_2}(t)$ be the probability that the state will transit from (j_1, j_2) to (i_1, i_2) for *t* time in the CTMC defined as below.

$$P_{i_1 i_2 j_1 j_2}(t) = P\{(B_{t+s}, R_{t+s}) = (i_1, i_2) \mid (B_s, R_s) = (j_1, j_2)\}, \ \text{for all s, t} \geq 0$$

Let us define variables, $q_{k_1 k_2 j_1 j_2}$ and $\nu_{j_1 j_2}$ in Eq.(5) and (6).

$$P_{k_1 k_2 j_1 j_2}(dt) = q_{k_1 k_2 j_1 j_2} dt, \ \ (k_1, k_2) \neq (j_1, j_2) \tag{6}$$

$$P_{j_1 j_2 j_1 j_2}(dt) = 1 - \nu_{j_1 j_2} dt \tag{7}$$

The variables $q_{k_1 k_2 j_1 j_2}$ and $\nu_{j_1 j_2}$ have the following values according to the probability in Eq.(4).

$$q_{k_1 k_2 j_1 j_2} = 0 \text{ except for } q_{i-1,\, jij} = \alpha j,\ q_{ij-1,\, ij} = \beta i \tag{8}$$

$$\nu_{ij} = \beta_i + \alpha_j \tag{9}$$

Let us define a new variable, $\mu_{i_1 i_2 j_1 j_2}$ as follows by using two variables defined as Eq.(8) and Eq.(9).

$$\mu_{i_1 i_2 j_1 j_2} = \begin{cases} q_{i_1 i_2 j_1 j_2}, & if \ (i_1, i_2) \neq (j_1, j_2) \\ -\nu_{i_1 i_2}, & if \ (i_1, i_2) = (j_1, j_2) \end{cases} \tag{10}$$

Assuming *U* has a matrix size of BR × BR with an element $\mu_{i_1 i_2 j_1 j_2}$ and *P(t)* is a transition probability matrix with an element $P_{i_1 i_2 j_1 j_2}(t)$, the following equation is satisfied.

$$P'(t) = UP(t) \tag{11}$$

Solving Eq.(11) gives the following.

$$P(t) = \exp(Ut) \tag{12}$$

And then,

$$\exp(Ut) = \sum_{n=0}^{\infty} \frac{(Ut)^n}{n!} \tag{13}$$

Using Eq.(12) and Eq.(13), the probability that the state transitions over time *t* can be calculated, and if the initial troop states of the Blue team and the Red team are given, the probability distributions of the troop capacities of Blue team and Red team at any time can be calculated by this method of calculation.

2.3 Shooting time interval model

A previous study showed that the method of reducing the average shooting time interval is relatively more effective than the method of raising the hit probability to reduce the engagement time and increase the probability of victory in a normal battle situation. However, as shown in Table 1, in the war game model operated by the Korean Army, instead of using the probability distribution on firing time interval, one shot time is calculated by defining the shooting procedure for each weapon system and applying the average time for each shooting procedure. Here, the average shooting time means a constant at which the same time is always input[3].

This means that it is necessary to present a more realistic method of deriving the shooting time since defining the unit time of a specific event as the average interfiring time in a simulation may give results different from reality.

Table 1: Comparison Inter-firing time interval and variables in DNS and AWAM model[3]

	German ground forces C4ISR Effect Analysis Model(DNS)	Ground weapon effect analysis model (AWAM)
Scale	Army division level	Army battalion, Regimental level
Inter-firing time interval	Calculate shooting time by constant averaging of aiming time and shooting preparation time	Define a constant by constant average times of aiming, reloading, and holding
PH (Prob. of Hit)	Classified according to fire, burial, target type, distance, degree of protection	Classified according to posture depending on distance, exposure, and movement
PK (Prob. of Kill)	Conditional probability that depends on PH	Classified according to target position depending on distance, exposure

If Lanchester differential equations use simple averaging over the firing time interval, a previous study using probability distribution suggested that if shooting time interval is limited to the case of exponential distribution and Erlang-2 distribution, analytical solution can be obtained. Moreover, regardless of analytical solution, analytical proximal solution or simulation, it is suggested that it is more desirable to use the hit interval time model, because it is possible to reduce analysis time and effort in the stochastic modeling of process of transition from hit to hit, rather than dealing with the process of transition from shoot to shoot.

However, since there is a limitation in obtaining information about the hit interval time probability distribution and only the data about the shooting time interval or the hit probability is available in many cases, the hitting interval time probability distribution, that is, the probability density function or the cumulative probability distribution function, should be derived from the firing interval time probability distribution characteristics. By overcoming the limits of the Lanchester square law and by exploiting the firing time probability distribution and not the constant firing time interval, modeling and results that are more similar to the actual battlefield situation could be derived[1].

In order to obtain the probability density function, we first define variables and symbols as follows. N is the number of fires until hit and p is the hit probability, $F_n(t)$ is the n-convolution of cumulative probability function of shooting time interval distribution. If the probabilistic density function and the cumulative distribution function of the random variable of hitting interval are defined as $h(t)$ and $H(t)$ as described above, for now the cumulative distribution function is as follows.

$$\begin{aligned} H(t) &= P(T \le t) \\ &= \sum_{n=1}^{\infty} P(T \le t \mid N = n) P(N = n) \\ &= \sum_{n=1}^{\infty} F_n(t) p q^{n-1} \end{aligned} \qquad (14)$$

The probability density function, $h(t)$ can be obtained by differentiating cumulative density function Eq.(14).

$$h(t) = \sum_{n=1}^{\infty} f_n(t) p q^{n-1} \qquad (15)$$

The hit time interval probability density function, $h(t)$ can be expressed by considering both the first hit, the second hit, or the case of hit after several failed hits.

$$h(t) = pf(t) + pqf^{(2)}(t) + pq^2 f^{(3)}(t) + \cdots \qquad (16)$$

Laplace transformation on both left and right sides in Eq.(16) is as follows.

$$\begin{aligned} L_k(s) &= pL_f(s) + pqL_f^2(s) + pq^2 L_f^3(s) + \cdots \\ &= pL_f(s)[1 + qL_f(s) + q^2 L_f^2(s) + \cdots] \\ &= pL_f(s)[1 - qL_f(s)] \end{aligned} \qquad (17)$$

The $h(t)$ can be obtained by taking a Laplace inversion on both sides in Eq.(17). That is, if there is information on the probability density function of shooting time interval *f(t)*, the probability distribution function of hit time interval can be obtained.

In most cases, it is difficult to obtain an explicit solution manually due to the complexity of the function when taking Laplace inversions. Hence, the numerical analysis approach to obtain this solution is to be left as a research project in the future

3. Conclusions

In the evaluation of the effectiveness of a physical protection system, the neutralization probability of the infiltrating enemy by the reaction force should be identified as an important factor, but it is difficult to obtain information on an accurate calculation method through combat simulation. The most used methodology for finding neutralization probability is markov chain model. Based on that neutralization probability is proportional to the number of shooters of each troop if weapon systems and degree of training are identical between each troop, the probability can be obtained. In this paper, the different method was proposed that the troop capacity over time can be predicted by using the Lanchester equation based on the stochastic process theory which predicts troop capacity by using differential equations. The above two models have something in common with variable, which indicate the killing powers of each weapon system. However little information is known about predicting the value. Lastly, since Lanchester equation has the limitation that the average shooting time as a constant value is used in simulation for predicting the probability, the shooting interval time as a probability distribution was developed to improve the reliability of the probability. However, the limitation that it can only simulate the engagement between an infiltrating enemy and a reaction force with a single weapon system, an engagement of two groups with mixed weapon systems is difficult to simulate still exists. The comparison of the three models is shown in the following Table 2.

Table 2: Comparison of methodology for finding of neutralization probability

	Markov Chain	Lanchester's equation	Shooting time interval
Main Merit	Easy understanding in transition of status	More clear insights of performance measure	Exploiting shooting time probability distribution
Solution Form	Acquiring of explicit results	Acquiring of matrix geometric solutions	Limiting on finding analytical solution
Weapon System	Single	Single/ Mixed	Single

Since invaders are highly likely to be equipped with a variety of weapons systems to sabotage nuclear facilities, future works on probabilistic analysis of the attributes of a mixed weapon system and a study of the troop transfer during engagements will enable simulation of more realistic battlefield situations.

Acknowledgement

This work was sponsored by funding from the Korea Foundation of Nuclear Safety, Republic of Korea.

REFERENCES

[1] Y.K. Hong, Finding Interkilling Time Probability Distribution in Stochastic Combat, Military Operation Research Society of Korea, Vol.28(2), p. 56-69, 2002.
[2] D.H. Park, D.H. Kim, H.G. Moon, H.Y. Shin, Gaussian Approximation of Stochastic Lanchester Model for Heterogeneous Forces, Journal of the Korean Institute of Industrial Engineering, Vol.42(2), p. 86-95, 2016.
[3] Y.H. Jeong, Y.K. Hong, A Stochastic Combat Simulation Model with Heterogeneous Weapon System, The Korea Society for Simulation, Vol.18(1), p. 53-62, 2009.
[4] Y.K. Hong, Probability Distribution of Battle Duration in Stochastic Combats, Military Operation Research Society of Korea, Vol.29(2), p. 1-12, 2003.
[5] J.F.C. Kingman, Stochastic aspects of Lanchester's theory of warfare, Journal of Applied Probability, Vol.39(3), p. 455-465, 2002.
[6] J.G. Taylor, Lanchester Models of Warfare, Military Applications Section, Operations Research Society of America, Virginia, 1983.