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# The Conservative Momentum Flux and the Collocated Solution Algorithms For the Multi-Dimensional RELAP5

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## 1. Introduction

To overcome the potential problems of using the non-conservative form of momentum equations [1,2,3,4] that were identified by CUPID group [8], the conservative momentum flux have been successfully applied in the development of the RELAP5-MD (RELAP5-Multi-Dimensional) using the retarded temporal derivative terms [15]. Naturally it uses the orthogonal coordinates and staggered mesh structures as an extension of the original RELAP5 solving the contravariant components. As is well known, the structured mesh gives rise to the limitation to model the complex solution domains. Using the unstructured mesh is preferable to handle them. The prevailed method used in the CFD (Computational Fluid Dynamics) to implement the unstructured mesh is to solve the Cartesian velocity components using the collocated variable arrangements.

In this paper, two aspects concerning the RELAP5-MD will be discussed. One of them is the status of the validation of the code. The other discussion will be the implementation of the collocated solution algorithms using unstructured mesh in RELAP5.

## 2. New Momentum Equations in RELAP5-MD

### *2.1. The Conservative momentum equations*

In RELAP5-MD, following momentum equations are used in that, the temporal derivative terms are used to get more rigorous and consistent discretization.

$$\begin{aligned}\alpha_g\rho_g\frac{\partial \underline{u}_g}{\partial t}+\underline{u}_g\frac{\partial \alpha_g\rho_g}{\partial t}+\nabla\cdot(\alpha_g\rho_g\underline{u}_g\underline{u}_g)\\ =-\alpha_g\nabla p-\alpha_g\rho_g F_g^w\underline{u}_g-\alpha_g\rho_g\alpha_l\rho_l F_i(\underline{u}_g-\underline{u}_l)\\ +\Gamma_g\left((1-\theta)\underline{u}_g+\theta\underline{u}_l\right)+\alpha_g\rho_g\underline{g}\end{aligned} \quad (1)$$

$$\begin{aligned}\alpha_l\rho_l\frac{\partial \underline{u}_l}{\partial t}+\underline{u}_l\frac{\partial \alpha_l\rho_l}{\partial t}+\nabla\cdot(\alpha_l\rho_l\underline{u}_l\underline{u}_l)=\\ -\alpha_l\nabla p-\alpha_l\rho_l F_l^w\underline{u}_l-\alpha_g\rho_g\alpha_l\rho_l F_i(\underline{u}_l-\underline{u}_g)\\ -\Gamma_g\left((1-\theta)\underline{u}_g+\theta\underline{u}_l\right)+\alpha_l\rho_l\underline{g}\end{aligned} \quad (2)$$

### *2.2. The discretization of momentum equations for the staggered solution scheme*

The discretized and linearized forms on the momentum cell on the **face**, $f$ facing cells $c_0$ and $c_1$are as follows;

$$\begin{aligned}\alpha_{g,f}^n\rho_{g,f}^n\frac{u_{g,f}^{n+1}-u_{g,f}^n}{\Delta t}V_f+u_{g,f}^{n+1}\frac{\alpha_{g,f}^n\rho_{g,f}^n-\alpha_{g,f}^{n-1}\rho_{g,f}^{n-1}}{\Delta t}V_f\\ +\sum_{f'}\alpha_g^n\rho_g^n u_g^n u_{g,f'}^n A_{f'}=-\alpha_{g,f}^n\left(p_{c_1}^{n+1}-p_{c_0}^{n+1}\right)A_f\\ -\alpha_{g,f}^n\rho_{g,f}^n F_g^w u_{g,f}^{n+1}V_f-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^n\left(u_{g,f}^{n+1}-u_{l,f}^{n+1}\right)V_f\\ +\Gamma_g^n\left((1-\theta)u_{g,f}^{n+1}+\theta u_{l,f}^{n+1}\right)V_f+\alpha_{g,f}^n\rho_{g,f}^n gV_f\end{aligned} \quad (3)$$

$$\begin{aligned}\alpha_{l,f}^n\rho_{l,f}^n\frac{u_{l,f}^{n+1}-u_{l,f}^n}{\Delta t}V_f+u_{l,f}^{n+1}\frac{\alpha_{l,f}^n\rho_{l,f}^n-\alpha_{l,f}^{n-1}\rho_{l,f}^{n-1}}{\Delta t}V_f\\ +\sum_{f'}\alpha_l^n\rho_l^n u_l^n u_{l,f'}^n A_{f'}=-\alpha_{l,f}^n\left(p_{c_1}^{n+1}-p_{c_0}^{n+1}\right)A_f\\ -\alpha_{l,f}^n\rho_{l,f}^n F_l^w u_{l,f}^{n+1}V_f-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^{n+1}\left(u_{l,f}^{n+1}-u_{g,f}^{n+1}\right)V_f\\ -\Gamma_g^n\left((1-\theta)u_{g,f}^{n+1}+\theta u_{l,f}^{n+1}\right)V_f+\alpha_{l,f}^n\rho_{l,f}^n gV_f\end{aligned} \quad (4)$$

where, $V_f$ and $A_f$ are volume and area of the face cell respectively. The second terms of eqn.(3,4) are regarded as source terms. Mass derivative terms in them are constructed with the $n-1$ step values.

It is interesting to note that the terms $\Gamma_g^n v_{g,f}^{n+1}V_f$ and $\Gamma_f^n v_{l,f}^{n+1}V_f$ used in the discretization of the non-conservative form of the momentum equations are evaluated in old time step. They are equivalent to the following equations;

$$\Gamma_g^n V_f=\frac{\alpha_{g,f}^n\rho_{g,f}^n-\alpha_{g,f}^{n-1}\rho_{g,f}^{n-1}}{\Delta t}V_f+\sum_{f'}\alpha_{g,f'}^n\rho_{g,f'}^n v_{g,f'}^n A_{f'} \quad (5)$$

$$\Gamma_f^n V_f=\frac{\alpha_{l,f}^n\rho_{l,f}^n-\alpha_{l,f}^{n-1}\rho_{l,f}^{n-1}}{\Delta t}V_f+\sum_{f'}\alpha_{l,f'}^n\rho_{l,f'}^n v_{l,f'}^n A_{f'} \quad (6)$$

It means that it is a retarded correction, like the mass derivative term in eqn.(3,4).

### *2.3. The semi-implicit solution procedure*

The above discretized and linearized momentum equations are coupled simultaneous equations for velocities and rearranged as following form;

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}u_{g,f}^{n+1}\\ u_{l,f}^{n+1}\end{pmatrix}=\begin{pmatrix}r\\ s\end{pmatrix}\left(\delta p_{f''}^{n+1}-\delta p_{f'''}^{n+1}\right)+\begin{pmatrix}t\\ u\end{pmatrix} \quad (7)$$

where $a, b, c, d, r, s, t$ and $u$ are functions of the known properties.

In Liles' semi-implicit scheme [11], this equation is solved for intermediate velocities $u_{g,f}^*$ and $u_{l,f}^*$ and they are inserted into the mass and energy conservation equations to construct system pressures matrix. Solving the pressure matrix back substitution is made. Iteration is performed until the residual is reduced to the permitted level. RELAP5, CUPID and TRACE use the non-conservative momentum equations and follow the same procedure. But they use one-step procedure.

### *2.4. Inserting the conservative momentum flux terms*

There are three corrections to be made to change the original RELAP5 to 3-dimensional code RELAP5-MD with fully conservative momentum flux terms with temporally non-conservative momentum balance equations.

a. Remove 1-dimensiobnal momentum flux terms.
b. Insert 3-dimensiobnal momentum flux terms
c. Remove the term, $\Gamma_k^n v_f^{n+1}$.
d. Add the mass derivative term;

The verification of the implementation is checked through the simple 2-dimensional conceptual flow test simulation [14]. Once the verification of the modified code is confirmed, assessments of the typical experiments are performed. And it is applied to the analysis of the large break LOCA for APR-1400 to check the applicability.

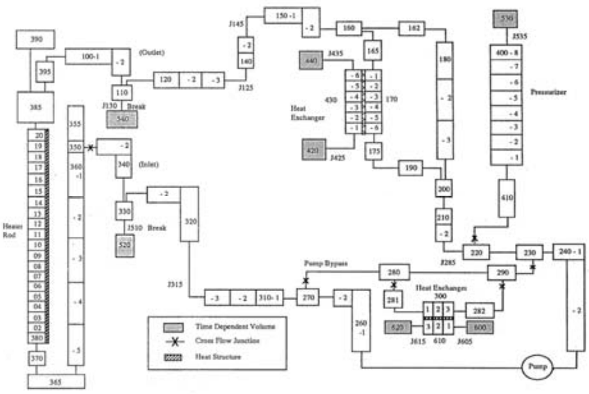



Fig.1. THTF Nodalization

## 3. Validation Tests and Applicability Tests of the RELAP5-MD

### *3.1. Simulation of the THTF test 105*

The effects of the conservative momentum flux during the blowdown period is tested against the THTF experiment, test 105. The nodalization of the test facility is shown in Fig.1. Only the test section, pipe 380 is replaced by the conservative momentum flux case. As one can see in Fig.2, there is no big difference between the original case (blue) and the modified case (red). The retarded correction term effect is also checked (grey) but there is almost no effect.

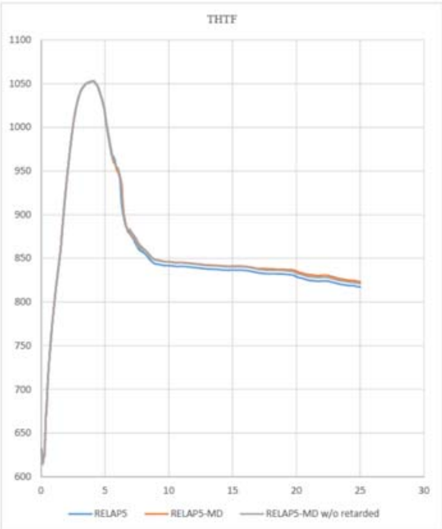



Fig.2. THTF Cladding Temperature

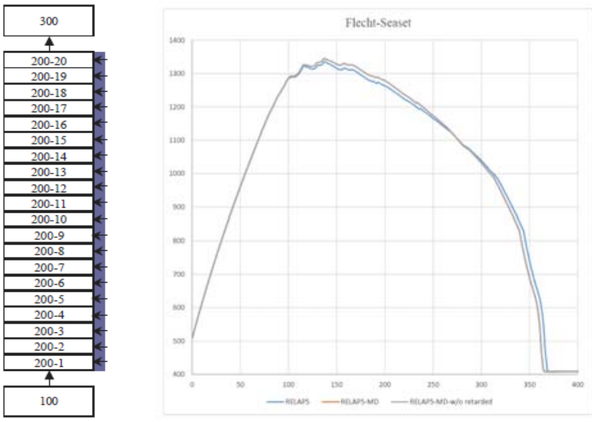



Fig.3. FLECHT-SEASET test nodalization and Results

### *3.2. FLECHT-SEASET test Simulation*

The effects of the conservative momentum flux during the reflood period is tested against the FLECHT-SEASET experiment, test-31504. The nodalization of the test facility is shown in Fig.3. As one can see in Fig.3, there is no big difference between the original case (blue) and the modified case (red). The retarded correction term effect is also checked (grey) but there is almost no effect. Since the red and grey is overlapped red one is not shown. So the retarded correction term is practically no effect during reflood period. Since this term is reflecting the compressibility of the fluid, the small compressibility during low pressure period makes the effect negligibly small.

### *3.3. Two-dimensional Downcomer Model for APR-1400 LOCA Analysis*.

The typical downcomer model for the APR-1400 is shown in Fig.4. Usually 6 pipes are used for modelling the downcomer. The connections to the cold/hot legs are done through the exit faces of the downcomer pipes. It means that cross-flow junction option is not used.

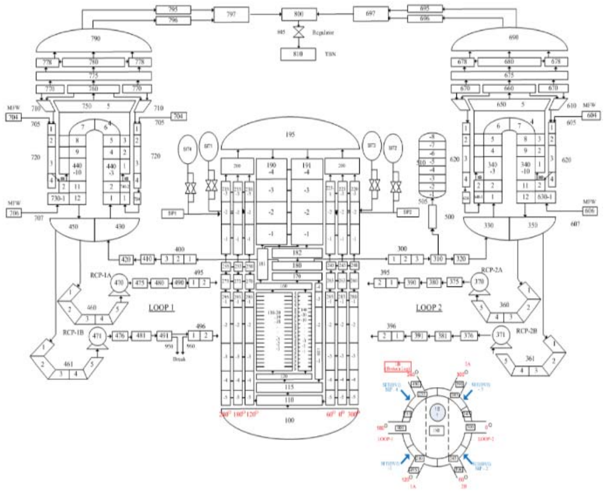



Fig.4. APR-1400 Nodalization

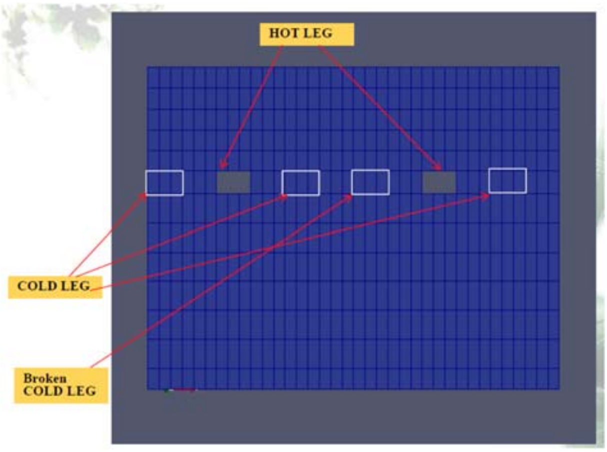



Fig.5. Downcomer Nodalization for RELAP5-MD

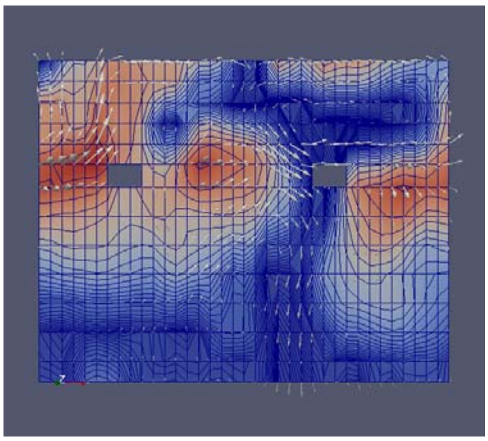


Fig.6. Downcomer Flow Pattern at End-of-Blowdown (31sec.)

For this study, 36 pipes are used for modelling the downcomer to have finer resolution of the flow field. The newly developed RELAP5-MD uses the full cross-flow connections to implement the cross flow convection terms. New nodalization for the downcomer is shown in

Fig.5. Only the downcomer pipes are solved with the conservative momentum flux. The purpose of this study is the first try to assess the applicability of the conservative momentum flux implementation with the real plant calculation. Therefore, any detailed study of the calculation results has not been performed yet. But brief look at the flow distribution will be introduced.

Flow pattern in the downcomer during refill period may be interesting to see. Fig.6. is the flow pattern and the vapor fraction distribution in the downcomer for the RELAP5-MD.

In the upper downcomer, flow merges to only one point on top of the broken cold leg. At end-of-blowdown time (31sec.) liquid flow penetrates to the lower plenum below the broken cold leg. It is reasonable because liquid is collected near the broken cold leg before the penetration. Also, the liquid velocity vector plot seems to be reasonable.

## 4. The Collocated Solution Algorithms for Unstructured Mesh

### *4.1. The discretization of momentum equations for the collocated solution scheme*

The solution procedure of the collocated semi-implicit method is already used by CUPID [8]. The discretized and linearized forms on the momentum cell on the **cell**, $c$ are almost same form if face cell parameters should be replaced by cell parameters. The gradient of pressure should be evaluated by the face pressure, $p_f$, which is defined by;

$$p_f = \frac{\left\{\xi . \dfrac{p_{c0}}{\rho_{c0}} + (1-\xi) . \dfrac{p_{c1}}{\rho_{c1}}\right\}}{\left\{\dfrac{\xi}{\rho_{c0}} + \dfrac{1-\xi}{\rho_{c1}}\right\}} \tag{8}$$

$$\xi = \frac{|d\underline{r}_1|}{|d\underline{r}_{01}|}, 1-\xi = \frac{|d\underline{r}_0|}{|d\underline{r}_{01}|} \tag{9}$$

$d\underline{r}_0$ and $d\underline{r}_1$ are the displacement vectors from face center to the individual cell centers. $d\underline{r}_{01}$ is the displacement vector from the cell center $c_0$ and the cell center $c_1$.

### *4.2. Development of the procedure for the collocated solver*

The discretized cell momentum equations are solved for the cell pressure gradient to yield, $\underline{\gamma}_{kc0}^n$ and $\beta_{kc0}$ for phase $k$of cell $c0$;

$$\underline{u}_{kc0}^{n+1} = \underline{\gamma}_{kc0}^n + \beta_{kc0}\, \nabla p_{c0}^{n+1} \tag{10}$$

Explicit velocity can be evaluated if the old time cell pressure gradient is used;

$$\underline{u}_{kc0}^{*} = \underline{\gamma}_{kc0}^n + \beta_{kc0}\, \nabla p_{c0}^{n} \tag{11}$$

Implicit cell velocity can be expressed by the incremental pressure $p'_{c0} = p_{c0}^{n+1} - p_{c0}^n$, as follows;

$$\underline{u}_{kc0}^{n+1} = u_{kc0}^{*} + \beta_{kc0}\, \nabla p'_{c0} \tag{12}$$

Face velocities are necessary to evaluate the convective terms. And they might be linearly interpolated from its neighboring cell center values. However, this can lead to the well-known checker-boarding oscillation of a pressure field. The Rhie-Chow scheme is widely used as a remedy. If the following face value function is defined for any parameters, $\phi$;
where;

$$\phi_f \equiv F(\phi_{c_0}, \phi_{c_1}) = \xi\phi_{c_0} + (1-\xi)\phi_{c_1} \tag{13}$$

Then,

$$\underline{u}_{k,f}^{*} = F(\underline{u}_{kc_0}^{*}, \underline{u}_{kc_1}^{*}) = \xi\underline{u}_{kc_0}^{*} + (1-\xi)\underline{u}_{kc_1}^{*} \tag{14}$$

With the following approximation;

$$F(\beta_{kc_0}\nabla p'_{c_0}, \beta_{kc_1}\nabla p'_{c_1}) \approx F(\beta_{kc_0}, \beta_{kc_1}) F(\nabla p'_{c_0}, \nabla p'_{c_1}) \tag{15}$$

Face velocity is expressed as;

$$\underline{u}_{k,f}^{n+1} = F(\underline{u}_{kc_0}^{n+1}, \underline{u}_{kc_1}^{n+1}) = \underline{u}_{k,f}^{*} + F(\beta_{kc_0}, \beta_{kc_1}) F(\nabla p'_{c_0}, \nabla p'_{c_1}) \tag{16}$$

Define;

$$\nabla p'_f \equiv \frac{(p'_{c_1} - p'_{c_0})\underline{n}_f}{|d\underline{r}_{01}|}\, N_d \tag{17}$$

And, if the face velocity is evauated by;

$$\underline{u}_{k,f}^{n+1} \equiv \underline{u}_{k,f}^{*} + F(\beta_{kc_0}, \beta_{kc_1}) \nabla p'_f \tag{18}$$

The above approach guarauntees the implicit form of Rhie-Chow velocity interpolation;

$$\underline{u}_{kf}^{n+1} = F(\underline{u}_{kc_0}^{n+1}, \underline{u}_{kc_1}^{n+1}) + F(\beta_{kc_0}, \beta_{kc_1}) \left\{ \begin{array}{c} \dfrac{(p'_{c_1} - p'_{c_0})\underline{n}_f}{|d\underline{r}_{01}|} \\ -F(\nabla p'_{c_0}, \nabla p'_{c_1}) \end{array} \right\} \tag{19}$$

If the normal vector $n_f$ for the face $f$is applied to eqn.(18), one can get the similar face normal component, which is equivalent to the eqn.(7). Once the face velcocity parameters are evaluated, the rest of the solution procedure is the same as the original RELAP5 procedure untill the last step to estimate the new time cell velocity.

Cell velocity is evaluated by the following interpolation for minimizing the numerical diffusion;

$$\underline{u}_{k,c_0}^{n+1} = \underline{u}_{k,c_0}^{*} + \frac{1}{V_{c_0}} \sum_f \frac{(1-\xi)}{N_d} |d\underline{r}_{01}| |\underline{S}_f| (\underline{u}_{k,f}^{n+1} - \underline{u}_{k,f}^{*}) \tag{20}$$

### *4.3. Implementing the collocated solver*

One of the big tasks to implement the collocated solver is that the interfacial drag coefficient is evaluated base on the junction flow regime map defined at the face in the original RELAP5. Therefore, the interfacial drag coefficient evaluation procedure at cell center should be constructed. Since the wall drag is originally evaluated by face averaging of the cell evaluated drag values. Therefore, new solver can use the original cell values.

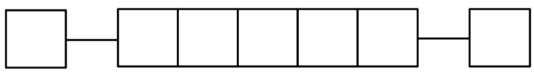


Fig.7. One-dimensional test Cells and Faces

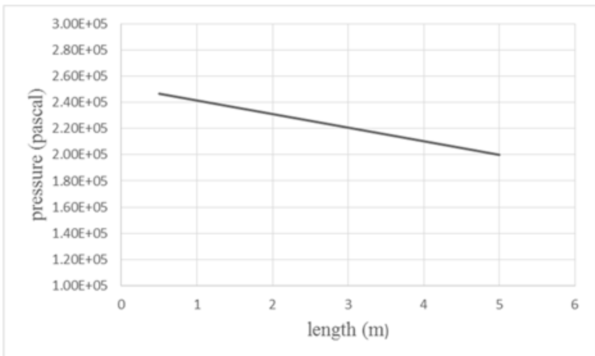



Fig.8. Pressure drop for Injection Test

## 5. Verification Tests for the Collocated Solution Scheme

### *5.1. One-dimensional tests*

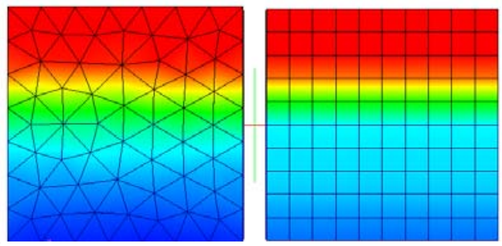
Five node pipe (Fig.7) is used to verify the implementation of the collocated solver. Area of the pipe is 1.0m x 1.0m and the height is also 1.0m. Inlet and outlet conditions are simulated by time dependent volume and time dependent and/or single junctions. Various combinations of the pipe flow were tested through various combination of the boundary conditions and initial conditions. Two-phase condition was tested as well. Vertical and horizontal pipe also tested. Through all these tests it was concluded that the implementation of the collocated solution scheme were successful.

As an example, the pressure drop along the pipe is shown in Fig.8 for the single phase injection problem. Flow injection (10.0m/sec single phase liquid with 300ºK) from the left hand side time dependent volume and time dependent junction develops the pressure drop along the pipe. The right hand side boundary is constructed with a single junction and a time dependent volume with constant pressure of 2bar (Fig.7).

Important aspect of the simulation is that the pressure drop does not show any saw-tooth type fluctuation. It means that the Rhie-Chow scheme is implemented correctly.

### *5.2. Two-dimensional tests*

2-dimensional settle down problem were investigated to see the effect of the unstructured mesh. As shown in Fig.9, 10.0m x 10.0m x 1.0m cube is modeled. 10 x 10 nodes were tested for the hexagonal mesh test (right hand side). The unstructured mesh with triangular column are also shown (left hand side). At the beginning time of the simulation, all cells are filled with 50% gas volume fraction. Phase separation by gravity is completed about 50 seconds later. Smooth settle down of two-phase reveals that the collocated solver is implemented correctly.



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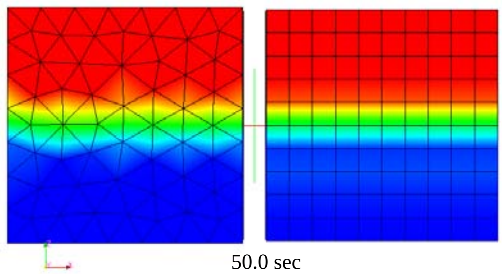

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Fig.9. Two-dimensional Settle-Down Problem

## 6. Discussions and Perspectives

Through all these verification simulation tests it was concluded that the collocated solution scheme is implemented correctly. The unstructured mesh handling capability should be developed and implemented in RELAP5-MD for the future.

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