Feasibility of the APEC Method in Physics Analysis of a Burned PWR Core

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1. Introduction

In the conventional two-step procedure based on simplified equivalence theory (SET) [1], the node-wise equivalence constants consisting of the homogenized and group condensed cross sections (XSs) and assembly discontinuity factors (ADFs) are obtained from lattice calculations for a single fuel assembly (FA) with zero net current boundary condition. Since an infinite lattice has no neutron leakage and is unphysical, the B1 leakage correction method [2] taking into account the critical spectrum obtained from a critical buckling search has to be applied to the evaluation of the parameters. However, accuracy of the core nodal calculation based on the conventional method is quite limited when the node interface current is not close to zero and the neighborhood effect is rather strong in practical cores. In order to overcome this limitation, several approaches to functionalize the equivalence constants have been suggested by considering the FA boundary information in the actual core environment.

In a recent study by W. Kim and Y. Kim [3], the albedo-corrected parameterized equivalence constants (APEC) method was proposed by focusing on the two-group XS correction. In a new method, the flux- and volume-weighted constants (FWCs) are parameterized as a simple polynomial function of a node-wise current-to-flux ratio (CFR), a unique way to represent the spatial leakage of node. Then, the FWCs are corrected in-situ during nodal calculations by reflecting the group-wise actual leakage in the core. In the study, the effects of the APEC XS correction on nodal calculation were evaluated against a small pressurized water reactor (PWR) based fresh core problem. As a result, it is shown that the APEC method can be very effectively employed in adjusting the XSs and improving the accuracy of the conventional nodal analysis.

In this paper, effectiveness of XS correction by the APEC method is validated against a burned PWR core problem in terms of the multiplication factor, FA power and node-wise two-group XSs. The DeCART2D [4] code is used for various lattice calculations to determine the two-group XSs with different boundary conditions using a color-set model where the FA in question is surrounded by different FAs. In addition, an in-house nodal code employing the nodal expansion method (NEM) within the partial-current based coarse mesh finite difference (p-CMFD) formulation is used to demonstrate the impact of APEC XS update on the nodal calculation.

2. Methods and Results

2.1 APEC Method

The conventional FWCs can be quite different from the reference values which are obtained from whole core heterogeneous calculation. However, if one can generate the FWCs as a function of neutron leakage through FA interfaces, it is expected that the FWCs can be corrected by using the actual leakage information during the iterative core calculations. The resulting nodal equivalence for the homogenized FAs will then be improved, leading to a more accurate core analysis. With this kind of considerations of the actual interface conditions between FAs, the ad hoc critical spectrum correction may be eliminated in the conventional lattice calculations and the current two-step procedures can be applied to not only critical condition but also to any non-critical situations.

The leakage effects on the FWCs are considered by using the albedo information of the node surfaces. If a FA is symmetric, the FWCs have a strong relationship with the g th-group node-average CFR for node m, defined as below:

$$CFR_g^m = \frac{\sum_s J_g^{m,s}}{\sum_s \phi_g^{m,s}} \tag{1}$$

where the $J_g^{m,s}$ and $\phi_g^{m,s}$ are the group-wise net currents and surface fluxes at surface s of node m, respectively. It should be noted that the CFR is a normalized parameter representing a surface-integrated leakage of the FA.

In the standard and simplest APEC method, the XS changes, $\Delta\Sigma_{x,g}^m$, due to the non-zero leakage at a node interface are functionalized with the CFR as follows for node m, collision type x, and group g:

$$\Delta\Sigma_{x,1}^m = a_{x,1} CFR_1^m + b_{x,1} CFR_2^m + c_{x,1} \tag{2}$$

$$\Delta\Sigma_{x,2}^m = a_{x,2} CFR_2^m + b_{x,2} \left(CFR_2^m\right)^2 + c_{x,2} \tag{3}$$

where $c_{x,g}$ is the constant term to consider the strong neighboring effect at a FA facing a baffle-reflector, otherwise is zero. The fast group XS change is a linear function of both fast and thermal group CFRs. This is because the energy spectrum change for the wide fast group is a coupled effect of fast and thermal groups. Especially, it is obvious that down-scattering XS change should depend on both fast and thermal group leakages. On the other hand, the thermal group XS change is a

quadratic polynomial function of the thermal group CFR only. Thus, the corrected XS, $\Sigma^m_{x,g}$, can be iteratively updated during nodal calculations as follows:

$$\Sigma^m_{x,g} = \Sigma^0_{x,g} + \Delta\Sigma^m_{x,g} \quad (4)$$

where $\Sigma^0_{x,g}$ is the FWCs obtained through single FA calculations, and $\Delta\Sigma^m_{x,g}$ is the XS change in Eqs. (2) and (3).

2.2 Benchmark Problem

In order to examine the effectiveness of the APEC XS correction against a burned core, a small PWR core problem was selected as a benchmark problem as shown in Fig. 1. Reference burnup is set to 20 MWD/kgU to avoid an effect according to depletion of the burnable absorber as well as to cover the cycle burnup on the current PWR design. To get reference solution at the specified burnup, the DeCART2D depletion calculation with an interval of 0.5 MWD/kgU was performed up to 25 MWD/kgU where FA power is 0.02895 MWt/FA/cm and temperature in each region is fixed as 600 K. At core-average burnup of 20 MWD/kgU, the reference k_{eff} is 1.039919 and Fig. 2 provides the FA-wise reference solutions such as burnup and power.

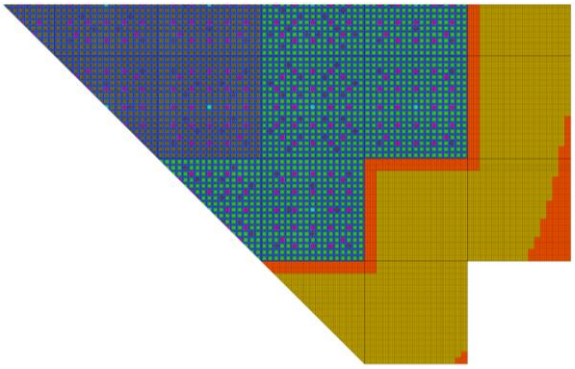


Fig. 1. Pin-wise 1/8 core configuration of the benchmark problem.

A2 22.6962 0.8528	A3 22.3164 0.8859	A2 22.2156 0.9838	B6 23.2728 1.2699	B2 17.5141 0.9729
	A2 22.4708 0.9154	A3 21.2688 0.9941	B5 21.8962 1.1990	B1 15.0069 0.8247
FA type FA burnup FA power		B5 22.5000 1.1742	B2 16.3830 0.8996	

Fig. 2. FA-wise reference solutions at core-average burnup of 20 MWD/kgU.

2.3 Lattice Calculations for APEC Method

As shown in Fig. 2, the problem consists of 6 types of FA. Thus, depletion calculations were performed for 6 single FA models. From these results, the FWCs and ADFs for each FA were obtained by linear interpolation between two burnup points nearest to its reference burnup.

In order to obtain the unknowns in the APEC XS correction functions, the change in XS data as function of different CFR values is required. However, arbitrarily albedo boundary conditions on a single FA cannot provide good results because they are likely to be a non-physical boundary condition. In order for the CFR at the boundary to be physically meaningful, it is necessary to change the CFR by arranging the actual environment around the FA so that physical CFRs are obtained, and a more accurate APEC functionalization is possible and can be applied to a wider range of problems.

To find physically acceptable CFR conditions in the inner core region, we suggest using the checkerboard color-set model shown in Fig. 3. For a certain FA, a non-zero CFR can be given by surrounding it with different FAs. Different surrounding FAs will give different CFRs at the boundary of the FA of interest. Then, the parameters in APEC function can be determined by fitting function to the data of change in XS and different CFR value. As the APEC XS correction functions have two unknowns for both fast and thermal groups, only two color-set calculations are necessary for each FA.

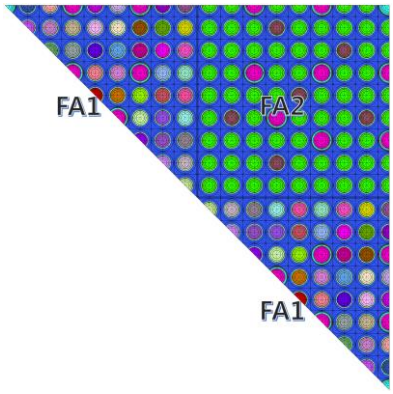
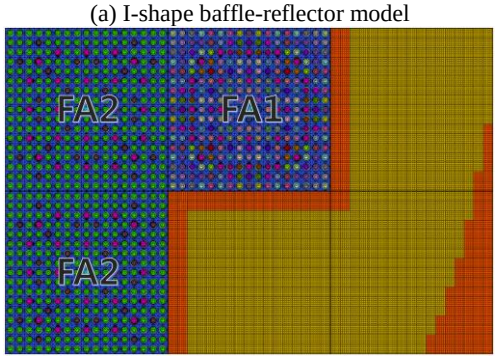


Fig. 3. Checkerboard color-set model for FAs loaded in the inner core region.

When a FA is facing a baffle-reflector region, it has a very different neutron energy spectrum and spatial power distribution from one in the inner core region or an infinite lattice. Therefore, additional parameter and color-set models are needed to functionalize XS change for such special FAs. As there are 3 parameters, 3 color-set calculations are necessary for fitting the XS change in the baffle-facing FA. Fig. 4 presents the new color-set models neighboring the baffle-reflector. To determine more accurate XS corrections, different model should be applied to depending on the shape of the baffle in spite of the same type of FA.

(a) I-shape baffle-reflector model



(b) L-shape baffle-reflector model

Fig. 4. Two color-set models for FAs facing baffle-reflector.

Lattice calculations for the APEC functionalization were performed for 15 color-set models. As shown in Table I, the color-set models at a particular burnup consist of 6 checkerboard models, 3 flat baffle models and 6 L-shape baffle models. At a specific burnup, 168 isotopic compositions for each fuel rod obtained from same burnup of single FA were inputted to the color-set model. To get more accurate solution, coefficients in Eqs. (2) and (3) should be linearly interpolated by using values obtained from two different burnups (1 and 2) as noted in Table I. Another burnup points covering a given reference burnup were chosen and additional 15 lattice calculations for same model with different burnup were carried out. In Table I, the number in parentheses stands for burnup, where it is assumed that color-set calculation is done at every 2.5 MWD/kgU.

Table I: List of color-set models for APEC method

No.	Burnup 1		Burnup 2		Model
	FA1[1)]	FA2[2)]	FA1[1)]	FA2[2)]	
1	A2(20.0)	A3(0.0)	A2(22.5)	A3(0.0)	Fig. 3
2	A2(20.0)	B5(20.0)	A2(22.5)	B5(22.5)	
3	A3(20.0)	A2(0.0)	A3(22.5)	A2(0.0)	
4	A3(20.0)	B6(20.0)	A3(22.5)	B6(22.5)	
5	B5(20.0)	B6(0.0)	B5(22.5)	B6(0.0)	
6	B6(20.0)	B5(0.0)	B6(22.5)	B5(0.0)	
7	B2(17.5)	A2(0.0)	B2(20.0)	A2(0.0)	Fig. 4 (a)
8	B2(17.5)	B2(17.5)	B2(20.0)	B2(20.0)	
9	B2(17.5)	B6(0.0)	B2(20.0)	B6(0.0)	
10	B1(15.0)	A2(0.0)	B1(17.5)	A2(0.0)	Fig. 4 (b)
11	B1(15.0)	B1(15.0)	B1(17.5)	B1(17.5)	
12	B1(15.0)	B6(0.0)	B1(17.5)	B6(0.0)	
13	B2(15.0)	A2(0.0)	B2(17.5)	A2(0.0)	
14	B2(15.0)	B2(15.0)	B2(17.5)	B2(17.5)	
15	B2(15.0)	B6(0.0)	B2(17.5)	B6(0.0)	

1) FA of interest
2) Surrounding FA to get different albedo information

2.4 Numerical Results

The two-group XSs of FAs in nodal calculations were used in the following three different ways: 1) reference XS from the whole core transport calculation, 2) FWC from a single FA calculation without the criticality correction, and 3) XS corrected by the basic APEC method. In addition, 8 checkerboard models are added to achieve further improved accuracy in XS correction and the results using these will be denoted as APEC2. Moreover, an improved APEC function, which is called APEC-SI, is also employed by accounting for the neutron spectrum change in a fuel assembly in terms of a spectral index (SI) defined as the fast-to-thermal flux ratio. For all cases, the ADFs are used along with 6 position-dependent reflector XSs and surface-dependent discontinuity factors (DFs) came from the reference DeCART2D calculation. It should be mentioned that the ADFs are determined in the single FA analysis without the B1 criticality calculation and they are not updated in the current APEC algorithm.

Table II: Comparison of k_{eff} and errors in FA power

	k_{eff}	Δρ (pcm)	Error in FA power (%)	
			Max.	RMS
1) Ref. XS	1.041641	159.0	1.91	1.19
2) FWC	1.045037	470.9	4.94	2.59
3) APEC	1.044392	411.9	3.44	1.66
4) APEC2	1.043867	363.6	3.19	1.59
5) APEC2-SI	1.043744	352.4	2.95	1.47
6) FWC-B1	1.045553	518.2	2.88	1.72

1.09	1.29	1.27	0.77	-0.90
-4.94	-3.92	-1.87	2.04	2.83
-3.44	-2.44	-0.77	1.81	1.62
-3.19	-2.27	-0.78	1.82	1.44
-2.95	-2.07	-0.72	1.70	1.41
-2.88	-1.82	0.43	2.73	0.70
	1.37	0.85	0.17	-1.83
	-3.17	-1.72	1.70	1.42
	-1.74	-0.99	1.04	0.33
	-1.65	-1.00	1.28	0.12
	-1.49	-0.97	1.19	0.10
	-0.97	0.26	1.70	-1.70
Ref. XS		0.33	-1.91	
FWC		0.68	0.06	
APEC		0.76	-0.35	
APEC2		0.84	-0.56	
APEC2-SI		0.76	-0.54	
FWC-B1		0.81	-2.35	

Fig. 5. Comparison of relative error (%) in FA-wise power.

Table II summarizes results of the nodal calculations mentioned above. For comparison, the results (noted as FWC-B1) using conventional FWC from a single FA calculation considering the critical spectrum are added

in this table. One can note that the simple APEC XS correction leads to noticeable improvement in the accuracy of nodal calculation compared to the FWC when the used number of color-set models are sufficient. Especially, maximum error and root mean square (RMS) error in FA power distribution are significantly decreased. The FA-wise powers are also compared for the above 6 approaches. The relative errors in the FA power for the diffusion nodal analyses are given in Fig. 5. It is observed that the FWC-based nodal solutions show a relatively large discrepancy from the reference solution in terms of the neutron multiplication factor and power profile, which is ascribed to the simple application of the macroscopic XS without any special modeling of zone-dependent Xe concentration. On the other hand, the APEC2-SI gives slightly more improved results than basic APEC scheme and shows as accurate as conventional FWC-B1.

Figure 6 shows the relative error in the FA-wise FWC XSs and Fig. 7 shows improvement of the two-group XSs by adding the color-set models. The APEC XS correction is more effective when the FA is facing a baffle-reflector region. However, it is also noted that the APEC XS correction is rather marginal in several fuel assemblies.

3. Conclusions

Feasibility of XS correction by the APEC method has been investigated with a burned PWR core problem in terms of the multiplication factor, node-wise power and two-group XSs. The APEC XS corrections lead to noticeable improvement in the accuracy of nodal results compared to the conventional FWC. However, it was also found that more accurate burnup-dependent APEC functions should be considered for a substantial improvement of the nodal analysis in a highly burned core. It is expected that APEC XS correction will be a lot more effective if microscopic Xe XS is considered in the tabulation of burnup-dependent macroscopic XSs of FAs.

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0.06	0.06	0.15	-0.23	0.25
0.01	0.01	-0.04	0.05	-0.05
-0.29	-0.27	-0.31	0.99	0.34
0.00	0.05	-0.61	-0.15	-0.20
-0.18	-0.10	-0.43	0.62	1.07
-0.22	-0.09	-0.81	0.29	0.28
-0.37	-0.39	-0.10	1.33	1.65
0.08	0.14	-1.33	0.46	-1.57
	0.09	0.16	-0.18	0.85
	0.01	-0.08	0.05	-0.14
	-0.30	-0.23	1.00	-0.45
	-0.10	-0.89	-0.09	0.25
	-0.18	-0.54	0.77	1.23
	-0.24	-1.14	0.35	0.28
	-0.37	0.23	1.44	1.22
	-0.09	-2.23	0.40	-2.46
D1		-0.20	0.97	
D2		0.09	-0.14	
Σa1		0.98	-0.60	
Σa2		0.20	-0.01	
νΣf1		0.84	1.03	
νΣf2		0.62	0.29	
Σ12		1.13	0.92	
Σ21		1.38	-2.63	

Fig. 6. Relative error (%) in FA-wise FWC XSs.

-0.13	-0.10	0.00	0.37	-0.17
0.01	0.00	0.04	0.10	0.03
0.04	0.05	-0.12	0.03	-0.21
0.02	0.08	-0.77	0.01	-0.46
-0.30	-0.19	-0.58	0.79	0.10
-0.20	-0.07	-0.98	0.48	0.00
0.20	0.15	0.12	-0.27	0.43
-0.01	0.00	-0.29	0.07	-0.17
	-0.08	0.06	1.29	-0.19
	0.01	0.11	0.10	0.13
	0.01	-0.20	-0.38	-0.33
	-0.09	-1.17	0.09	-0.19
	-0.30	-0.71	2.49	0.35
	-0.24	-1.43	0.57	-0.23
	0.14	0.09	-1.71	0.48
	-0.10	-0.20	-0.19	0.60
D1		0.73	-0.14	
D2		0.02	0.13	
Σa1		-0.09	-0.31	
Σa2		0.55	-0.53	
νΣf1		1.63	0.20	
νΣf2		1.02	-0.30	
Σ12		-0.87	0.47	
Σ21		-0.15	0.16	

(a) APEC

-0.12	-0.10	0.00	0.24	-0.17
0.01	0.00	0.07	-0.01	0.04
0.03	0.04	-0.12	0.14	-0.21
0.02	0.07	-0.76	0.00	-0.46
-0.28	-0.18	-0.58	0.57	0.09
-0.21	-0.07	-0.96	0.46	0.00
0.15	0.11	0.11	0.04	0.43
-0.02	0.00	-0.21	-0.20	-0.18
	-0.07	0.04	0.36	-0.19
	0.01	0.15	-0.02	0.14
	-0.01	-0.19	0.02	-0.34
	-0.09	-1.14	0.06	-0.21
	-0.27	-0.75	0.68	0.35
	-0.24	-1.39	0.52	-0.25
	0.10	0.11	-0.09	0.48
	-0.10	-0.13	-0.28	0.45
D1		0.31	-0.14	
D2		-0.07	0.15	
Σa1		0.12	-0.31	
Σa2		0.55	-0.54	
νΣf1		0.82	0.20	
νΣf2		0.99	-0.30	
Σ12		-0.07	0.48	
Σ21		-0.25	0.05	

(b) APEC2

Fig. 7. Relative error (%) in FA-wise APEC XSs.