

Review of Condensation Mass Flux in the Presence of Non-condensable Gas

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1. Introduction

Film-wise condensation on the wall at the presence of non-condensable gas (NCG) such as air is of great importance in advanced passive nuclear power plant, in particular, in passive containment cooling system. Several condensation models have been suggested by several authors, and among them Collier's model is most sophisticate[1]. His approach is based on the thermodynamic property profile in the boundary layer, which can be said 'diffusion layer' because the property variation is driven by gas species diffusion. Conceptual profile is given in Fig. 1.

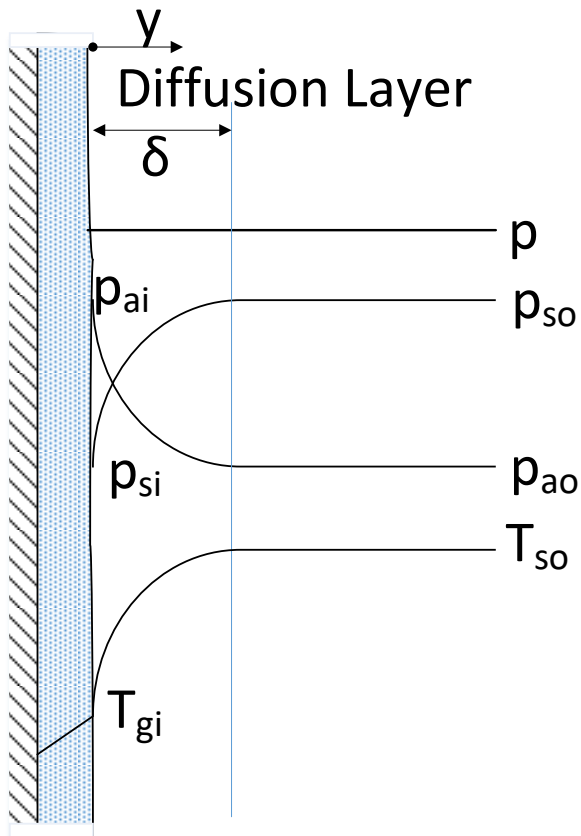


Fig. 1. Conceptual thermodynamic property profile near the liquid film.

Such a concept has been used in several literatures [2,3,4,5]. However, some of the earlier studies are pointed out to have some mistakes, and this is believed caused by the complicatedness of equation developments[6]. Moreover, nomenclatures for that have not been unified yet, and it may add additional

difficulties in understanding and exploration of the formula. Of course it makes difficult to compare the formula between literatures.

This paper is to provide a detailed derivation procedure for the mass condensation flux, and to give exact meaning of each term and symbol. This will help to understand the mathematical formula right and compare each other between formula.

2. Fundamental Definitions for the Mass Diffusion

For the strict understanding and derivation of the steam and air diffusion around the condensing film, several symbols should be well defined. Most of them were taken from Bird et al., and Incropera et al.[7,8].

Absolute mass flux and absolute molar flux at the fixed coordinated are defined

$$\mathbf{n}_s'' \equiv \rho_s \mathbf{v}_s \quad (1a)$$

$$\mathbf{N}_s'' \equiv C_s \mathbf{v}_s \quad (1b)$$

And the absolute mixture mass flux and molar flux can be expressed by

$$\rho \mathbf{v} = \mathbf{n}'' = \rho_s \mathbf{v}_s + \rho_a \mathbf{v}_a = \mathbf{n}_s'' + \mathbf{n}_a'' \quad (2a)$$

$$C \mathbf{v}^* = \mathbf{N}'' = C_s \mathbf{v}_s + C_a \mathbf{v}_a = \mathbf{N}_s'' + \mathbf{N}_a'' \quad (2b)$$

Thus, the mass averaged velocity and molar averaged velocity are given by

$$\mathbf{v} = m_s \mathbf{v}_s + m_a \mathbf{v}_a \quad (3a)$$

$$\mathbf{v}^* = x_s \mathbf{v}_s + x_a \mathbf{v}_a \quad (3b)$$

, where the mass fractions and molar fractions are defined as followings

$$m_a = \frac{\rho_a}{\rho} \quad (4a)$$

$$m_s = \frac{\rho_s}{\rho} \quad (4b)$$

$$x_a = \frac{C_a}{C} \quad (5a)$$

$$x_s = \frac{C_s}{C} \quad (5b)$$

Of course, the sum of each fraction is unity.

$$m_a + m_s = 1 \quad (6a)$$

$$x_a + x_s = 1 \quad (6b)$$

And relative steam mass flux and relative steam molar flux to the averaged velocities can be defined

$$\mathbf{j}_s \equiv \rho_s (\mathbf{v}_s - \mathbf{v}) \quad (7a)$$

$$\mathbf{J}_s^* \equiv C_s (\mathbf{v}_s - \mathbf{v}^*) \quad (7b)$$

Above two equations can be applied to the air, and then summing the fluxes yields following relations

$$\mathbf{j}_s + \mathbf{j}_a = 0 \quad (8a)$$

$$\mathbf{J}_s^* + \mathbf{J}_a^* = 0 \quad (8b)$$

And the absolute steam mass flux and the absolute steam molar flux are then arranged into

$$\mathbf{n}_s'' = \mathbf{j}_s + \rho_s \mathbf{v} \quad (9a)$$

$$\mathbf{N}_s'' = \mathbf{J}_s^* + C_s \mathbf{v}^* \quad (9b)$$

3. Fick's Law and Gas Mass Flux

Original statement of Fick's law is that the rate of flow of the solute is proportional to the negative gradient of the solute concentration, according to Lamarch(1983)[9, 10]. In order to be faithful this statement, the diffusion equations for steam to air can be expressed like this[7, 8, 9, 10, 11]

$$\mathbf{j}_s = -D_{sa}\nabla\rho_s = -\rho D_{sa}\nabla m_s \quad (10a)$$

$$\mathbf{J}_s^* = -D_{sa}\nabla C_s = -CD_{sa}\nabla x_s \quad (10b)$$

In above equations the last equal sign implies that the mixture gas mass density (ρ) and the mixture molar concentration (C) are constant regardless of the location. And it is valid if total pressure and temperature is uniform in the domain.

4. Derivation of Condensation Mass Diffusion Flux

Let's consider the one-dimensional situation like Fig. 1. Air molar concentration at the film interface is surely higher than that at the bulk. And the absolute air molar flux cannot penetrate the film. Thus, the air flow must be zero due to the continuity. The steam condenses only at the film interface, so the steam flow along the y axis must be negative and constant.

$$N_a'' = 0 \quad (11a)$$

$$N_s'' = \text{constant} \quad (11b)$$

4.1 Constant Mixture Molar Concentration Case

Constant or uniform mixture molar concentration does not look appropriate as shown in Fig. 1, where the partial pressures of the air and the steam vary in the diffusion layer, and moreover the mixture temperature also varies. However, Collier et al. (1994) [1] is believed to assume constant in their derivation. Using the second 'equal mark' in Eqs. (10a) and (10b) of Fick's law together with Eqs. (3a) to (4b) yields following expressions for the absolute mass and molar fluxes of Eqs. (9a) and (9b)

$$\mathbf{n}_s'' = -\rho D_{sa}\nabla m_s + m_s(\mathbf{n}_s'' + \mathbf{n}_a'') \quad (12a)$$

$$\mathbf{N}_s'' = -CD_{sa}\nabla x_s + x_s(\mathbf{N}_s'' + \mathbf{N}_a'') \quad (12b)$$

Applying Eq. (12b) for the air in one-dimensional case leads to

$$N_a'' = -CD_{sa}\frac{dx_a}{dy} + x_a N'' = 0 \quad (13)$$

Similarly, molar flux equation for the steam can be setup, except that there is an effective absolute molar flux for the steam

$$\begin{aligned} N_s'' &= -CD_{sa}\frac{dx_s}{dy} + x_s N'' \\ &= CD_{sa}\frac{dx_a}{dy} + x_s N'' \end{aligned} \quad (14)$$

Summing Eqs. (13) and (14)

$$N'' = N_s'' \quad (15)$$

Above result is very reasonable, since the absolute air flow is zero as shown in Eq. (12a). Eliminating N'' in Eq. (14) using Eq. (15)

$$\begin{aligned} N'' = N_s'' &= CD_{sa}\frac{dx_a}{dy}\left(\frac{1}{1-x_s}\right) \\ &= CD_{sa}\frac{dx_a}{dy}\left(\frac{1}{x_a}\right) \\ &= CD_{sa}\frac{dx_a}{dy}\left(\frac{x_s}{x_a} + 1\right) \end{aligned} \quad (16)$$

The absolute flow of steam is adverse direction to the y coordinate. And the gradient of the air mole fraction ($\frac{dx_a}{dy}$) must be negative. Eq. (16) well matches this fact.

Integrating Eq. (16) between interface ($y=0$) and the edge of the diffusion layer ($y=\delta$)¹ with the consideration of constant steam flow as in Eq. (11b)

$$\begin{aligned} N_s'' &= \frac{CD_{sa}}{\delta}\ln\left(\frac{x_a(\delta)}{x_a(0)}\right) \\ &= \frac{CD_{sa}}{\delta}\ln\left(\frac{1-x_s(\delta)}{1-x_s(0)}\right) \\ &= -\frac{CD_{sa}}{\delta}\ln\left(\frac{1-x_s(0)}{1-x_s(\delta)}\right) \end{aligned} \quad (17)$$

Last expression in above equation is to make the natural log positive and to show that the total value is negative.

Now, let's convert the mole to mass unit for the above equation. Following relation is available

$$N_s'' = \frac{n_s''}{M_s} \quad (18)$$

$$C = C_a + C_s = \frac{\rho_a}{M_a} + \frac{\rho_s}{M_s} = \frac{\rho_s M_a + \rho_a M_s}{M_s M_a} \quad (19)$$

Thus, Eq.(17) is rearranged into

$$\begin{aligned} n_s'' &= -\left(\rho_s + \rho_a \frac{M_s}{M_a}\right)\frac{D_{sa}}{\delta}\ln\left(\frac{1-x_s(0)}{1-x_s(\delta)}\right) \\ &= -\left(\rho_s + \rho_a \frac{M_s}{M_a}\right)\frac{D_{sa}}{\delta}\frac{p_{so}-p_{si}}{p_{am}} \end{aligned} \quad (20)$$

, where p_{am} is the log mean of air partial pressure given by

$$p_{am} \equiv \frac{(p_{ai}-p_{ao})}{\ln\frac{p_{ai}}{p_{ao}}} \quad (21)$$

Strictly speaking, the mixture molar concentration (C) is not a constant value along the y axis. The mixture molar concentration decreases as approaching the film interface, because the total pressure is uniform but the mixture temperature decreases as shown in Fig. 1. In order to implement the integration of Eq.(16), let's consider following relations. Linear variation of the air partial pressure and gas temperature in the diffusion layer was assumed for the easy calculation, even though exact profile will be slightly different. Otherwise the integration may be impossible in analytic manners.

$$x_a = \frac{p_a}{p} \quad dx_a = \frac{1}{p} dp_a \quad (22)$$

$$p_a(y) = \frac{p_{ao}-p_{ai}}{\delta}y + p_{ai} \quad dp_a = \frac{p_{ao}-p_{ai}}{\delta}dy \quad (23)$$

$$T(y) = \frac{T_o-T_i}{\delta}y + T_i \quad (24)$$

$$p = CR_u T \quad (25)$$

Integration of Eq. (16) considering above relations yields

$$\int_0^\delta N_s'' dy = D_{sa} \int_{x_a(0)}^{x_a(\delta)} C \frac{1}{x_a} dx_a \quad (26)$$

Or

¹ Strictly speaking, the mixture molar concentration is not constant. Here, it is assumed constant for simplicity. The effect is soon discussed.

$$\begin{aligned}
 N_s'' &= \frac{D_{sa}}{\delta} \frac{1}{R_u} (p_{ao} - p_{ai}) \int_0^\delta \frac{1}{(T_o - T_i)y + \delta T_i} dy \\
 &= \frac{D_{sa}}{\delta} \frac{1}{R_u} \frac{(p_{ao} - p_{ai})}{(T_o - T_i)} \ln \frac{T_o}{T_i} \\
 &= -\frac{D_{sa}}{\delta} \frac{1}{R_u} \frac{(p_{ai} - p_{ao})}{T_m} \\
 &= -\frac{D_{sa}}{\delta} \frac{1}{R_u} \frac{(p_{so} - p_{si})}{T_m} \quad (27)
 \end{aligned}$$

, where T_{am} can be said as the log mean of gas temperature given by

$$T_m \equiv \frac{(T_o - T_i)}{\ln \frac{T_o}{T_i}} \quad (28)$$

Eq. (27) can be converted into mass based unit.

$$\begin{aligned}
 \frac{n_s''}{M_s} &= -\frac{D_{sa}}{\delta} \frac{1}{R_u} \frac{(p_{so} - p_{si})}{T_m} \\
 \text{Or} \\
 n_s'' &= -\frac{D_{sa}}{\delta} \frac{(\rho_{so} T_o - \rho_{si} T_i)}{T_m} \quad (29)
 \end{aligned}$$

Eq. (20) and Eq. (29) looks far different each other in their form and the in their driving potential. And both equations are different from the equations (10.37) and (10.38) of Collier et al.

4.2 Variable Mixture Molar Concentration Case

Using the second 'equal mark' in Eq. (10b) of Fick's law together with Eq. (3a) to (4b) yields following expressions for the absolute molar fluxes of Eq. (9b)

$$\mathbf{N}_s'' = -D_{sa} \nabla C_s + x_s (\mathbf{N}_s'' + \mathbf{N}_a'') \quad (30)$$

Applying Eq. (30) for the air in one-dimensional case leads to

$$N_a'' = -D_{sa} \frac{dC_a}{dy} + x_a N'' = 0 \quad (31)$$

Similarly, for the steam

$$N_s'' = -D_{sa} \frac{dC_s}{dy} + x_s N''$$

Then, $N'' = N_s''$ results tin

$$\begin{aligned}
 N_s'' &= \left(\frac{D_{sa}}{1 - x_s} \right) \left\{ \frac{d(C_s)}{dy} \right\} \\
 &= \left(\frac{D_{sa}}{x_a} \right) \left\{ \frac{d(C - C_a)}{dy} \right\} \quad (32)
 \end{aligned}$$

For the integration of above equation, following form is derived

$$\begin{aligned}
 N_s'' dy &= \left(\frac{D_{sa}}{1 - x_s} \right) dC_s \\
 &= D_{sa} \left(\frac{C}{C - C_s} \right) dC_s \quad (33)
 \end{aligned}$$

Integrating above equation from $y=0$ to $y=\delta$,

$$N_s'' = \frac{D_{sa}}{\delta} \int_{y=0}^{y=\delta} \left(\frac{C}{C - C_s} \right) dC_s \quad (34)$$

This integration is possible only when the profiles of C and C_s are known. What is clear is that C and C_s both decrease as approaching to the film.

5. Conclusions

Through strict and intensive review and derivation the exact formula of steam condensation mass flux was obtained. In Collier et al.'s derivation unrealistic assumption of the constant mixture molar concentration in the diffusion layer was used. And ρ_g which is not explicitly explained in their final result is believed steam density plus molar ratio weighted air density as

shown in Eq. (20). If the actual mixture molar profile is considered, somewhat different form of equation was derived.

Fundamentally, if Fick's law is expressed as the gradient of concentration rather than fraction, the analytic form seem very hard as shown in Eq. (34). However the final form of formula, if possible, may be far different from Collier et al..

Intensive exploration is highly required in further study.

ACKNOWLEDGMENTS

This work was supported by the Korea Institute of Energy Technology Evaluation and Planning(KETEP) and the Ministry of Trade, Industry & Energy(MOTIE) of the Republic of Korea (No. 20161510400120).

NOMENCLATURE

C	molar concentration of gas mixture [kmol/m ³], $C = \sum_i C_i$
C_a	molar concentration of air [kmol/m ³], $C_a = \frac{\rho_a}{M_a}$
C_s	molar concentration of steam [kmol/m ³], $C_s = \frac{\rho_s}{M_s}$
D_{sa}	binary diffusion coefficient or mass diffusivity between steam and gas [m ² /sec]
\mathbf{J}_a^*	relative molar flux vector of steam to the average mixture flux [kmol/m ² sec]
\mathbf{J}_s^*	relative molar flux vector of steam to the average mixture flux [kmol/m ² sec]
j_a	relative mass flux vector of air to the average mixture flux [kg/m ² sec]
j_s	relative mass flux vector of air to the average mixture flux [kg/m ² sec]
M_a	molecular weight of air [kg/kmol]
M_s	molecular weight of steam [kg/kmol]
m_a	mass fraction of air in air-steam mixture [-], $m_a = \frac{\rho_a}{\rho}$
m_s	mass fraction of steam in air-steam mixture [-], $m_s = \frac{\rho_s}{\rho}$
\mathbf{N}_s''	absolute molar flux vector of gas species s in fixed coordinates [kmol/m ² sec]
\mathbf{n}_s''	absolute mass flux vector of gas species s in fixed coordinates [kg/m ² sec]
p	pressure or total pressure of gas mixture [Pa]
p_{ai}	air partial pressure at the film interface [Pa]
p_{ao}	air partial pressure in the bulk [Pa]
p_{si}	steam partial pressure at the film interface [Pa]
p_{so}	steam partial pressure in the bulk [Pa]
R_u	universal gas constant, 8.31447kJ/lmol·K
T	temperature of gas mixture [K]
T_i	temperature of gas mixture at the film interface [Pa] (same temperature for each gas species in the mixture is assumed)

T_o	temperature of gas mixture in the bulk [K] (same temperature for each gas species in the mixture is assumed)
\mathbf{v}	mass weighted average velocity vector of gas mixture [m/s]
\mathbf{v}_s	mass weighted average velocity of gas species s [m/s]
\mathbf{v}^*	molar weighted average velocity vector of gas mixture [m/s]
x_a	mole fraction of air in air-steam mixture [-], $x_a = \frac{c_a}{c}$
x_s	mole fraction of steam in air-steam mixture [-], $x_s = \frac{c_s}{c}$
y	distance from film surface [m]
ρ	mass concentration (density) of gas mixture [kg/m ³], $\rho = \sum_i \rho_i$
ρ_a	mass concentration (density) of air [kmol/m ³], $\rho_a = M_a C_a$
ρ_s	mass concentration (density) of steam [kmol/m ³], $\rho_s = M_s C_s$

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