# Some Uncertainties of DeCART2D/MASTER4.0 for 17x17 PWR Core

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### 1. Introduction

Korea Atomic Energy Research Institute (KAERI) has developed DeCART2D1.1 [1]/MASTER4.0 [2] (hereafter noted as DeCART2D/MASTER) core nuclear design system for PWR core design and analysis. Before DeCART2D/MASTER, KAERI used the CASMO-3/MASTER system of which uncertainties were evaluated based on Hanbit Unit 1, Hanbit Unit 3/4, and Palo Verde Unit 1. [3] In this paper, the uncertainties of some nuclear design parameters, axially integrated power peaking factors of the fuel assembly and the hottest fuel rod, and individual and total control rod worth of the DeCART2D/MASTER system are evaluated based on Hanbit Unit 1 of which the core consists of 17x17 fuel assemblies.

The uncertainty of the parameter is defined using the tolerance limits with 95% probability and 95% confidence. For the parameters with insufficient measured data, the applicability of the uncertainties of the CASMO-3/MASTER system is demonstrated by showing that the differences between calculated and measured data are bounded.

## 2. Methods and Results

### 2.1 Definition of the Uncertainties

The uncertainty of the power peaking factors is defined as the multiplicative factor such that the calculated peak value is less than measured data with not less than 95% probability with 95% confidence. On the other hand, the uncertainty of the individual rod worth or total rod worth is defined such that the actual rod worth will be between lower limit and upper limit with at least 95% with 95% confidence, which are obtained by multiplying the uncertainty to the calculated value.

If the distribution of parameter X is normal, the tolerance interval can be evaluated using the tolerance factors [4] as follows:

$$\mu - KS < X < \mu + KS \tag{1}$$

where  $\mu$  is the sample mean, K is the appropriate factor for tolerance limits for the one-sided tolerance limit or two-sided tolerance limits with 95% probability with 95% confidence, and S is the standard deviation of the samples.

If the distribution is not normal, distribution free tolerance limits can be obtained by the nonparametric statistics. From Reference [5], the distribution free tolerance limits can be determined from the largest (r+m)that satisfies the following relation:

$$\alpha \ge \sum_{i=0}^{r+m-1} \binom{n}{i} (1-q)^{i} q^{n-i}$$
(2)

where 1 - a is the confidence coefficient, n is the sample size, q is the proportion of the population, r and m is the number of position, or rank from the top and bottom if the sample parameter is orderly arranged. If one-sided tolerance limit is of interest, either r or m can be assigned 0.

#### 2.2 Fuel Assembly Peak Power Uncertainty

For the peak power uncertainty evaluation of DeCART2D/MASTER, measured power distributions of Hanbit Unit 1 cycles 1, 2, 3, 5, 6 and 7 are available and are used. Core follow calculations are performed and restart file is generated at each burnup point. To obtain the power distribution to compare with the measured one, the MASTER restart file at the nearest burnup is used and the same control rod position is also used for the MASTER calculation. MASTER output provides the reaction rate at each selected detector position. Each calculated reaction rate is compared with the normalized measured power.

There are total 51 burnup point data used for the analysis. The total degrees of freedom are more than 2,000. Using the cycle-wise sample standard deviation, Bartlett's test [6] is performed for the poolability check. Note that the normalized distributions result in same sample mean of zero. However, the test fails and the cycle-wise standard deviations are not from the same population. Therefore, cycle-wise 95/95 tolerance limits are obtained by employing the distribution free nonparametric statistics. However, to obtain the conservative tolerance limit, another tolerance limit is calculated assuming normal distribution. Between two tolerance limits, the more limiting value is chosen as the final tolerance limit. The distribution of differences between calculated fuel assembly powers and measured one is shown in Fig. 1. It shows that the difference distribution is away from the normal distribution. Since we are interested in the peak value uncertainty of the DeCART2D/ MASTER system, lower side tail is of interest. The one-sided tolerance limit obtained by the nonparametric statistics method is clearly lower than the one obtained from the normal distribution curve. Since the estimated tolerance limit is in absolute value, it is divided by the minimum of the peak assembly power during the cycle resulting in the relative fuel assembly peak power (Fr) uncertainty. The fuel assembly Fr uncertainty of DeCART2D/MASTER is evaluated less than 4%.



Fig. 1. Histogram of differences between calculated and measured fuel assembly powers and assumed normal distribution for Hanbit Unit 1 cycle 6.

#### 2.3 Fuel Rod Peak Power Uncertainty

The fuel rod power in the core is calculated by synthesizing the fuel assembly power and heterogeneous formfunction in the DeCART2D/MASTER system. Therefore, the tolerance limit of the peak fuel rod power is the result of synthesizing the fuel assembly power and the pin-to-box factor which are independent each other. The overall standard deviation can be obtained by the following relation:

$$S^{2} = S_{B}^{2} + S_{P}^{2}$$
(3)

where subscript B is for the fuel assembly, subscript P is for the pin-to-box factor. Moreover, the overall degrees of freedom, v, can be obtained by the relation:

$$\frac{S^4}{v} = \frac{S_B^4}{v_B} + \frac{S_P^4}{v_P}$$
(4)

The standard deviation of the fuel assembly power normalized by the cycle minimum of the peak assembly power can be obtained from the analysis in section 2.2. The equivalent degrees of freedom corresponding to the one-sided 95/95 tolerance limit can be found from the Reference [4].

To obtain the standard deviation and the degrees of freedom of the pin-to-box factor, calculations by DeCART2D1.1 with the ENDF/B-VII.1 based library [7] are performed for the fuel rod power distributions about 5 CE benchmark [8], 5 KRITZ benchmark [9], and 6 B&W benchmark [10].

The analysis result is that the pin-to-box factor uncertainty is evaluated sufficiently less than 2%, and fuel rod peaking factor, Fr, uncertainty is much less than 5%.

# 2.4 Individual Control Rod Worth

For the individual rod worth uncertainty evaluation, 22 individual rod worth measurement data of Hanbit Unit 1

are used from Cycle 1 through Cycle 7. Fig. 2 shows the relative differences vs. measured rod worth. The figure shows the range from -4.1% to 13.2%, and it indicates that DeCART2D1.1/MASTER4.0 slightly overestimates individual rod worths in general. Analysis results in the average,  $\mu$  and the standard deviation, *S* of 3.18% and 4.18%, respectively. By the W-test [11], the relative differences are normally distributed. Two-sided tolerance factor is used for the individual rod worth uncertainty. In order to use "zero bias" in the individual rod worth calculation, the larger side of the tolerance interval of Eq. (1) which is less than 15%, is taken as the DeCART2D/MASTER individual rod worth uncertainty.



Fig. 2. Relative differences between calculated and measured individual control rod worth.

### 2.5 Total Control Rod Worth

Fig.3 shows the relative differences between calculated and measured total control rod worths in conjunction with the CASMO-3/MASTER total rod worth uncertainty of 10%. The range is from -0.8% to 6.3%. Although the W-test indicates normal distribution, the short of data results in an unacceptably wide tolerance interval as shown in Fig. 4. The normal distribution curve has the same mean and standard deviation of the histogram. Therefore, the CASMO-3/MASTER total rod uncertainty of 10% will be used for the DeCART2D/MASTER system as the range of the data is well bounded by 10%.



Fig. 3. Relative differences between calculated and measured total control rod worth with the traditional uncertainty.



Fig. 4. Histogram and the normal distribution curve for the total rod worth.

# 3. Conclusions

The uncertainties of DeCART2D/MASTER nuclear core design and analysis code system are evaluated for the axially integrated fuel assembly power and fuel rod power, Fr and for the individual and total rod worths. One-sided tolerance limit is taken for the Fr and larger limit of the two-side tolerance limits is taken for the individual rod worth. For the total rod worth, data range is compared with the CASMO-3/MASTER uncertainty 10%. The results demonstrated that of the DeCART2D/MASTER system is suitable for the design of PWR nuclear reactor core that consists of 17x17 fuel assemblies with reasonable uncertainties for the power peaking factor, Fr and control rod worth.

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