Effects of Uncertainties on Response Spectrum Calculated by Fast Fourier Transform Algorithm in Seismic Monitoring System of NPPs

Tongkyu Park[a*], Jeong Hun Lee[a], and Byeong Hyeok Ha[a]
[a]*FNC Technology, Heungdeok IT Valley Bldg. 32F, 13, Heungdeok 1-ro, Giheung-gu, Yongin-si, 16954, Korea*
[*]*Corresponding author: tongkyu@fnctech.com*

1. Introduction

After the Fukushima-Daiichi Nuclear Power Plants (hereafter, NPPs) were disabled due to the massive earthquake and tsunami in Japan, the safety of the NPPs became one of the major social and public policy issues in Korea. Furthermore, the earthquakes that jolted Gyeongju in 2016 prompted and accelerated concerns about the safety of NPPs even though no NPP was damaged in safety related systems at that time.

To protect the safety related systems in NPP and NPP itself by preventing a badly damage caused by earthquakes in advance, the seismic monitoring system is being currently operated for all the NPPs in Korea. Since the current US NRC Regulatory Guide 1.12 [1] presents the frequency range should be 0.20 Hz to 50 Hz for acceleration sensors and thus the bandwidth for recorder should be at least from 0.20 Hz to 50 Hz, the damage from the high frequency earthquake (> 50 Hz) has not been considered in the system. Note that the sample rate for recorder should be at least 200 samples per second in each of the three direction in Ref. 1. However, the Regulatory Guide has been recently revised [2] even though it is just draft version until now. The revised Regulatory Guide says that the frequency range and bandwidth should be from zero Hz to a minimum of 100 Hz, and the sample rate should be at least 250 samples per second in each of the three direction. Therefore, we can expect that the high frequency effects will be considered in seismic analyses of NPPs in the near future. Note that the upper limit is a minimum of 100 Hz and thus higher frequencies than 100 Hz may be considered in the analysis procedure. Additionally, researches on effects of uncertainties that may be generated during measurements (or analyses) on analysis results [3] have been rarely performed in Korea.

In this work, the uncertainty effects on analysis results, especially, frequency response spectrum has been mainly investigated with high frequencies. As a first step, a simple fast Fourier transform [4,5,6] (hereafter, FFT) code which can convert a signal from its original time domain to the frequency domain was implemented. To verify the effectiveness of this simple code, three sample functions consisting of simple sine and cosine sub-functions were defined and tested as well. As a next step, a series of sensitivity analyses were performed using a wide range of uncertainty which is easily added to the three sample functions to investigate the uncertainty effects on response spectrum calculated by the FFT algorithm in seismic monitoring system of NPPs.

2. Methodology

Since the equations for discrete Fourier transform (hereafter, DFT) and FFT are well-known and discussed in more detail elsewhere [4,5,6], a brief description is presented here to facilitate the discussion of the uncertainty effects on response spectrum calculated by the FFT algorithm in seismic monitoring system.

The main purpose of the Fourier transform is to convert a signal from its original domain (usually time domain) to a representation in the frequency domain and vice versa as mentioned in introduction. Namely, if $X_j, j = 0,1,\cdots,N-1$, is a sequence of complex numbers, then the DFT of X_j is the sequence as

$$H_n = (1/N)\sum_{j=0}^{N-1} X_j \exp^{-2\pi ijn/N}; n = 0,1,\cdots,N-1 \quad (1)$$

where $i = (-1)^{1/2}$.

Then the inverse DFT can be expressed as

$$X_j = \sum_{n=0}^{N-1} H_n \exp^{2\pi ijn/N}. \quad (2)$$

By factorizing the DFT matrix into a product of sparse factors, the FFT can rapidly compute this kind of transformations, and thus it can reduce the complexity of computing the DFT from $O(N^2)$ to $O(N\log N)$, where the $O(x)$ means the required number of operations is x to complete the calculation. It can be easily proven that the DFT of length N can be rewritten as the sum of two DFTs, each of length $N/2$. One of the two is formed from the even-numbered points of the original N while the other from the odd-numbered points of the original N. Consequently, in the FFT algorithm, Eq. (1) can be rewritten as

$$F_k = (1/N)\left[\sum_{j=0}^{N/2-1} e^{2\pi ik(2j)/N} f_{2j} + \sum_{j=0}^{N/2-1} e^{2\pi ik(2j+1)/N} f_{2j+1}\right] = (1/N)[F_k^e + W^k F_k^o], \quad (3)$$

where $W = \exp^{-2\pi i/N}$. Here, F_k^e denotes the k-th component of the DFT of length $N/2$ formed from the even components of the original f_j's (i.e., f_{2j}) while F_k^o is the corresponding transform of length $N/2$ formed from the odd components (i.e., f_{2j+1}).

3. Numerical Results

To verify the effectiveness of the FFT algorithm and investigate the effects of uncertainties on response spectrum, a series of sensitivity analyses were performed for three established test problems with a wide range of uncertainty randomly generated.

3.1 Description of Test Problems

The three test problems are listed in Table I. The primary objective of the first test problem is to verify the effectiveness of the FFT algorithm and the newly developed code. The target function consists of five cosine sub-functions. Note that the five cosine sub-functions have different periods and thus the corresponding frequencies are 1, 2, 3, 4, and 5 Hz, respectively, as shown in Table I. On the other hand, they have the same amplitude as 1. Therefore, the five frequencies should be obtained from the FFT algorithm as the response frequencies and the response frequencies should have same magnitude.

The second target function has five low frequency cosine sub-functions and five very high frequency sine sub-functions. The corresponding frequencies of the very high frequency sub-functions are 110, 120, 130, 140, and 150 Hz, respectively, as shown in Table I. In this test problem, the amplitude for all the sub-functions set to 1 as well. N is specified as 256 for these two test functions. Due to so-called aliasing effect which is the effect that causes different signals to become indistinguishable when sampled, it is expected that the effective response frequency would be less than 128 Hz in these problems.

The third test problem has the exact same ten sub-functions with the second test problem, but the number of samples is much larger (i.e., eight times) than the counterpart of the second problem as shown in Table I. It should be noted that these problems may be non-feasible, but the main purpose of these tests is just to show the uncertainty effects on response spectrum.

3.2 Sensitivity Analyses

Each problem was analyzed by considering randomly generated uncertainties at each time point. Three different ranges for the uncertainties were considered, i.e., [-1,1], [-3,3], and [-5.5]. For example, the uncertainty range of [-1,1] means a random number between -1 and 1 is generated at each time point and the randomly generated number is added to the original target function consisting of cosine and sine sub-functions. Thus a total of twelve cases were defined for the FFT calculations including cases without uncertainty for the three problems. To easily compare the magnitude at each frequency each other, the magnitude at each frequency was normalized using the maximum calculated magnitude within the domain. Namely, the maximum magnitude is always 1 for all the cases tackled in this study.

Fig. 1.b.1 shows that the expected frequencies (i.e., 1, 2, 3, 4, and 5 Hz) were obtained from transformations for the first problem. However, it seems about 40 Hz was one of the effective frequencies in this problem when the relatively large uncertainties were considered in the target function at each time point as shown in Fig. 1.b.4. From this observation, it can be concluded that this misanalyzed frequency may affect the other analysis results and the corresponding follow-up action in NPPs.

Even though the expected corresponding frequencies for the second problem are 1, 2, 3, 4, 5, 110, 120, 130, 140, and 150 Hz, the high frequencies that are larger than 128 Hz cannot be precisely obtained due to the aliasing effect as mentioned above. Fig. 2.b.1 shows that instead of 130, 140, and 150 Hz, 126, 116, and 106 Hz, respectively, were obtained as the response frequencies in this problem. Note that the values at each time point for the 130, 140, and 150 Hz sub-functions are exactly same with the counterparts for the 126, 116, and 106 Hz sub-functions, respectively, when 256 samples per second are considered.

Unlike the second problem, the response frequencies for the third problem were exactly same with what we expected as shown in Fig. 3.b.1 because the maximum effective response frequency is about 500 Hz in this problem if the aliasing effect is considered. Fig. 3.b.4 shows the fluctuation level in middle range frequencies is much smaller compared to that of Fig. 2.b.4. From these observations, it can be concluded that much more samples should be used to reduce uncertainty effects.

Table I: Description of Three Test Problems

Problem Number	Target Function	Corresponding Frequencies (Hz)	Number of Samples
1	$X_j = \sum_{x=1}^{5} \cos(2\pi xj / N)$	1, 2, 3, 4, 5	256
2	$X_j = \sum_{x=1}^{5} \cos(2\pi xj / N) + \sum_{x=1}^{5} \sin((200 + 20\pi)xj / N)$	1, 2, 3, 4, 5, 110, 120, 130, 140, 150	256
3	$X_j = \sum_{x=1}^{5} \cos(2\pi xj / N) + \sum_{x=1}^{5} \sin((200 + 20\pi)xj / N)$	1, 2, 3, 4, 5, 110, 120, 130, 140, 150	1024

4. Conclusions

To investigate the effects of uncertainties on frequency response spectrum, a FFT code was developed and tested using three test problems by considering randomly generated random values at each sampled point. The analysis results say that the FFT algorithm may misanalyze the response frequencies when the number of samples is relatively smaller than the effective high frequencies in which the severe earthquake may include, and thus this misanalysed frequency may affect the other safety analyses and the corresponding follow-up actions in NPPs. In other words, the high frequency earthquake needs to be carefully analyzed. The easiest way to avoid this misanalysis and to reduce the uncertainty effects is to increase the number of samples per second. However, it is quite limited to enhance the sampling capability of sensors or detectors. Therefore, to resolve this problem effectively and efficiently, all the causes of uncertainties need to be considered when the FFT algorithm works.

REFERENCES

[1] "Nuclear Power Plant Instrumentation for Earthquakes," U. S. Nuclear Regulatory Commission, Regulatory Guide 1.12, Revision 2, March (1997).
[2] "Nuclear Power Plant Instrumentation for Earthquakes," U. S. Nuclear Regulatory Commission, Regulatory Guide 1.12, Proposed Revision 3, September (2016).
[3] G. Betta et al., "Propagation of uncertainty in a discrete Fourier transform algorithm," *Measurement*, **27**, 231, (2000).
[4] J. W. Cooley and J. W. Tukey, "An Algorithm for the Machine Calculation of Complex Fourier Series," *Math. Comp.*, **19**, 297, (1965).
[5] H. J. Nussbaumer, Fast Fourier Transform and Convolution Algorithms, Springer Verlag, New York, 1982.
[6] E. O. Brigham, The Fast Fourier Transform, Prentice-Hall, New Jersey, (1974).

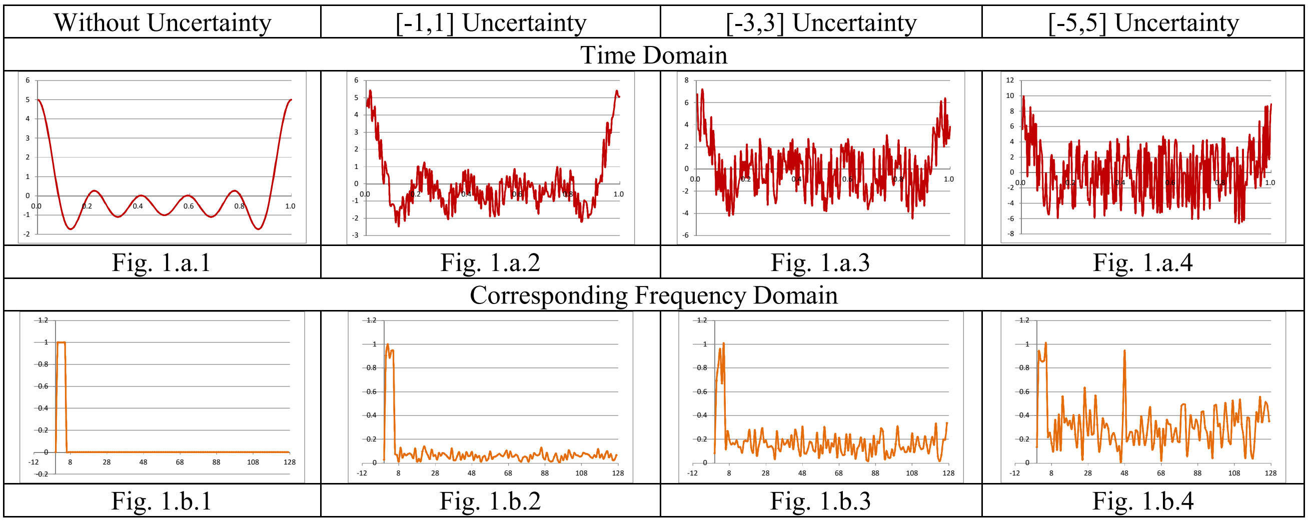

Fig. 1. Analysis Results for the First Test Problem.

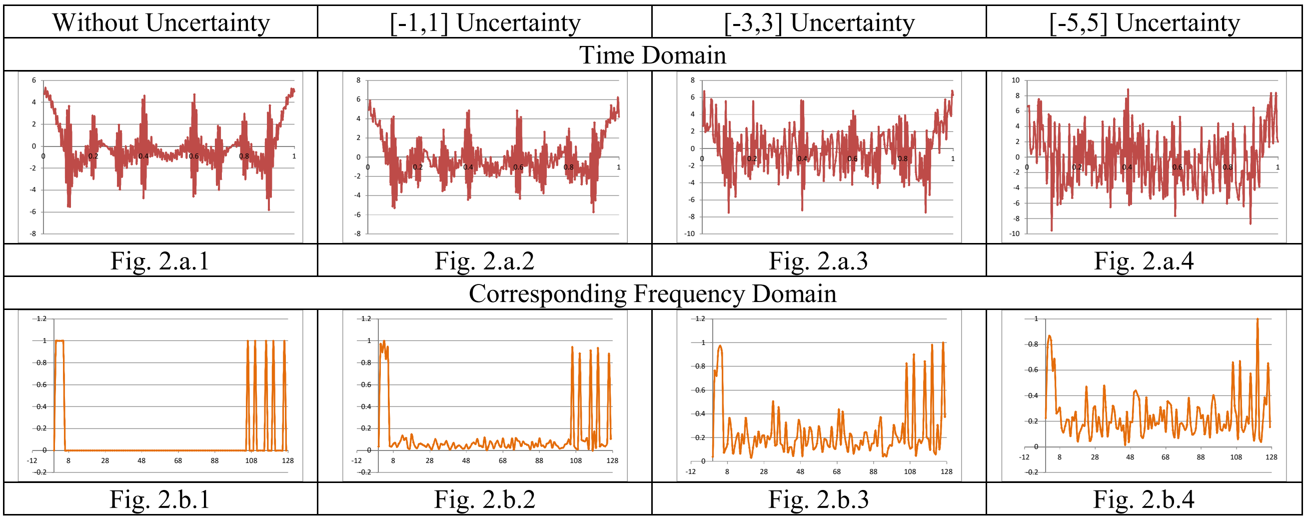

Fig. 2. Analysis Results for the Second Problem.

Without Uncertainty	[-1,1] Uncertainty	[-3,3] Uncertainty	[-5,5] Uncertainty
Time Domain			
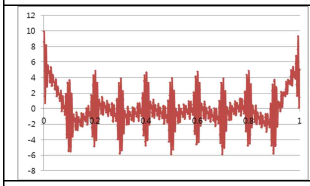	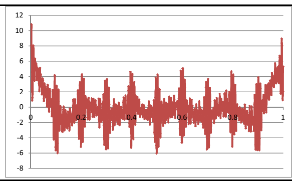	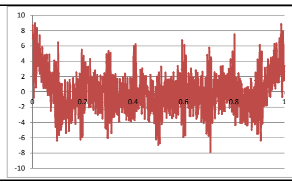	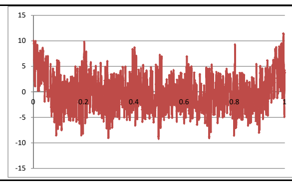
Fig. 3.a.1	Fig. 3.a.2	Fig. 3.a.3	Fig. 3.a.4
Corresponding Frequency Domain			
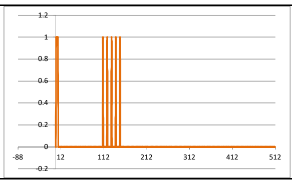	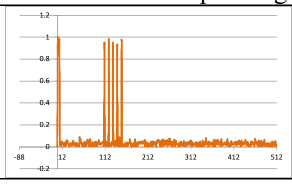	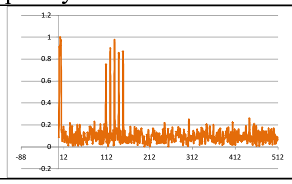	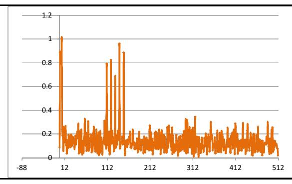
Fig. 3.b.1	Fig. 3.b.2	Fig. 3.b.3	Fig. 3.b.4

Fig. 3. Analysis Results for the Third Test Problem.