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A Simulation of Tsunami Propagation near Nuclear Power Plant Using Incompressible SPH Method and Comparison with Lab-scale Tsunami Run-up Experiment

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1. Introduction

After Fukushima accident, it has been important to evaluate the impact of a tsunami on the nuclear power planta. If a tsunami occurs near the nuclear power plant site it may smash against the barrier wall or flood the power plant. The inflow into the plant site may lead to several accidents such as failure of drains, station black out, and loss of emergency power supplier. Therefore, it is important to understand the tsunami propagation phenomena and access the effects on the watertight barrier of power plant in order to improve the response capability to the tsunami.

Smoothed Particle Hydrodynamics (SPH) is a Lagrangian particle-based method for fluid simulation [1]. As the fluid particles are moving in accordance with the governing equations SPH method can handle problems with free surface such as tsunami simulation. Taking the advantages of SPH, we have developed SOPHIA, a new multi-dimensional, multi-physics code using Weakly Compressible SPH (WCSPH) method. WCSPH method adopts Equation Of State (EOS) which calculates the pressure proportional to the density variation [2]. Since WCSPH allows compressibility of the fluid, a stiff coefficient is required for EOS in order to simulate incompressible fluid flow. A stiff EOS induces the fluctuation on the pressure field so that the time step should be small enough to accommodate sensitive pressure changes, resulting in the decrease of computation speed [3].

Recently, several Incompressible SPH (ISPH) methods have been proposed to simulate incompressible fluid flows with large time step. Bender et al. [4] proposed Divergence Free SPH (DFSPH) method. DFSPH method enforces incompressibility on the fluid by calculating pressure forces iteratively. Unlike the WCSPH method, DFSPH method satisfies both density invariant condition and divergence free velocity condition of incompressibility without the stiff EOS, which makes the simulation more stable and well converged [4].

In this paper, we applied DFSPH method to study tsunami propagation nearshore by simulating a set of physical experiments. At first, we implemented DFSPH algorithm to SOPHIA code to solve incompressible flow. Second, we simulated the tsunami propagation near Kori Nuclear Power Plant to compare with lab-scale tsunami run-up experiments. The comparison was conducted on the free surface motion of waves.

2. Methodology

In this section, the fundamentals of SPH with WCSPH are reviewed and Divergence Free SPH method is introduced.

2.1 Fundamentals of SPH method

Smoothed Particle Hydrodynamics (SPH) method is formulated by approximated integral interpolant on a computational domain [1]. Since the computational domain is discretized into particles, the integral interpolant is converted to the summation interpolant. For example, an arbitrary function $f(\boldsymbol{x})$ can be approximated as follow [1, 5].

$$f(\boldsymbol{x}_i) = \sum_j \frac{m_j}{\rho_j} f(\boldsymbol{x}_j) W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{1}$$

Where i, j denote center particle and neighbor particle and m and ρ denote particle mass and density. h is Smoothing length determining the influence area of weighting function W. The function W, called Smoothing kernel, calculates the contribution of neighbor particles through the distance between center position $\boldsymbol{x}$ and neighbor position $\boldsymbol{x}'$. It ensures that the closer to the center particle the higher values neighbor has when calculating center particle's property. In this paper, we use the Wendland C6 kernel [6].

According to [5], the spatial derivative approximation is obtained by taking a differential operation on the smoothing function.

$$\nabla f(\boldsymbol{x}_i) = \sum_j \frac{m_j}{\rho_j} f(\boldsymbol{x}_j) \nabla W(\boldsymbol{x}_i - \boldsymbol{x}_j, h) \tag{2}$$

By using Eq. (1) and (2), SPH method formulates the governing equations in a discretized form: The Navier-Stokes equation and continuity equation.

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \boldsymbol{u} = 0 \tag{3}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{\nabla p}{\rho} + \boldsymbol{g} + \nu \nabla^2 \boldsymbol{u} \tag{4}$$

Where $\rho, \boldsymbol{u}, p, \boldsymbol{g}$ and ν are material density, velocity, pressure, gravitational acceleration and kinematic viscosity.

In the SPH method, there are two approaches to compute the density of particles [5].

One is mass summation,

$$\rho_i = \sum_j m_j W(\boldsymbol{x}_{ij}, h) \tag{5}$$

The other is calculating time derivative of density.

$$\left(\frac{D\rho}{Dt}\right)_i = \sum_j m_j (\boldsymbol{u}_i - \boldsymbol{u}_j) \nabla W(\boldsymbol{x}_{ij}, h) \tag{6}$$

Where $\boldsymbol{x}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$.

The pressure term of Eq.(4) can be formulated as follow [1,5].

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)_i = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right) \nabla W(\boldsymbol{x}_{ij}, h) \tag{7}$$

Where P_i and P_j stand for pressure of center particle and neighbor particle.

For the viscous force, we applied Morris's viscous force model [7].

$$\left(\frac{D\boldsymbol{u}}{Dt}\right)_i = \sum_j m_j \frac{\mu_i + \mu_j}{\rho_i \rho_j} \frac{\boldsymbol{x}_{ij} \cdot \boldsymbol{u}_{ij}}{|\boldsymbol{x}_{ij}|^2} \nabla W(\boldsymbol{x}_{ij}, h) \tag{8}$$

Where $\boldsymbol{x}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$, $\boldsymbol{u}_{ij} = \boldsymbol{u}_i - \boldsymbol{u}_j$, and μ denotes dynamic viscosity of fluid.

The governing equations are closed by Equation Of State (EOS) which is modified by Monaghan [2].

$$P = \frac{c^2 \rho_0}{\gamma} \left(\left(\frac{\rho}{\rho_0} \right)^\gamma - 1 \right) \tag{9}$$

Where $\gamma = 7$, ρ_0 is the reference(initial) density and c is the speed of sound. According to [2], the speed of sound is taken as 10 times maximum velocity or $M{\sim}0.1$ which is large enough to solve incompressible fluid flow.

We additionally implemented artificial viscous force [5] and XSPH method [8] to enhance the stability of simulation. These methods control the excessive particle motion.

2.2 Divergence Free SPH

Enforcing incompressibility means satisfying the density invariant condition and the divergence free velocity condition. The density invariant condition makes the first term of Eq. (3) zero and the divergence free velocity condition makes the second term of Eq. (3) zero. For the density invariant condition, the zero-time derivative of density is described as the density having zero variation for the initial density. The final form of conditions of incompressibility is as follow.

$$\rho - \rho_0 = 0 \tag{10}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{11}$$

Bender et al. [4] proposed DFSPH method to simulate incompressible fluid flow efficiently and stably. DFSPH method introduces two implicit pressure solvers to satisfy Eq. (10) and Eq. (11) respectively: Constant Density solver and Divergence Free solver. The Constant Density solver enforces density invariant condition $(\rho - \rho_0 = 0)$ by calculating velocity iteratively to reduce the density variation. The Divergence Free solver enforces divergence free velocity $(\nabla \cdot \boldsymbol{u} = 0)$ by correcting velocities and pressure forces until the velocity field becomes divergence-free. Since two solvers calculate pressure forces implicitly, DFSPH method can handle large time step while avoiding pressure fluctuation of WCSPH [4].

Algorithm 1 outlines DFSPH method. The algorithm is divided into three steps. First, the particle velocities are predicted for the non-pressure forces (Line 4). In the second step, Constant Density solver corrects these unconstrained velocities until the density variation becomes zero. The obtained velocities determines the new position (Line 7). As the particles moves, we search nearest neighbor particles and calculate smoothing kernel (Line 9). Using the updated kernel values, particle density and stiffness coefficient(δ) are computed (Line 11, 12). At last, Divergence Free solver projects the velocities over the divergence free field (Line 13). The resultant velocities are updated to the next time step (Line 15).

2.2.1. Divergence Free solver

Divergence Free solver aims at obtaining divergence free velocity field $(\nabla \cdot \boldsymbol{u} = 0)$ by computing pressure forces and integrating velocities iteratively [4].

Algorithm 1. Simulation algorithm of DFSPH method

1: **While** animating **do**
2:     **for all** particles **do**
3:         $f_i^{adv} = g + f_i^{viscosity} + f_i^{artificial}$
4:         $u_i^{adv} = u_i + \Delta t f_i^{adv}$
5:     Density Invariant Solver $u_i^*(t)$
6:     **for all** particles **do**
7:         $x_i(t+\Delta t) = x_i(t) + \Delta t u_i^*(t+\Delta t)$
8:     **for all** particles **do**
9:         Find Neighbors $N_i(t+\Delta t)$
10:     **for all** particles **do**
11:         $\rho_i(t+\Delta t) = \sum_j m_j W(x_{ij}, h)$
12:         $\delta_i(t+\Delta t) = \frac{\rho_i}{|\sum_j m_j \nabla W_{ij}|^2 + \sum_j |m_j \nabla W_{ij}|^2}$
13:     Divergence Free Solver $u_i^*(t+\Delta t)$
14:     **for all** particles **do**
15:         $u_i(t+\Delta t) = u_i^*(t+\Delta t)$

Firstly, the pressure force is computed as follow.

$$\boldsymbol{f}_i^{pressure} = -\sum_j m_j \left(\frac{\kappa_i}{\rho_i} + \frac{\kappa_j}{\rho_j}\right) \nabla W(\boldsymbol{x}_{ij}, h) \qquad (12)$$

The stiffness parameter (κ) is obtained from density time derivative,

$$\kappa_i = \frac{1}{\Delta t}\frac{D\rho_i}{Dt} \cdot \delta_i \qquad (13)$$

and density time derivative is calculated as below.

$$\frac{D\rho_i}{Dt} = \sum_j m_j(\boldsymbol{u}_i - \boldsymbol{u}_j)\nabla W(\boldsymbol{x}_{ij}, h) \qquad (14)$$

The pressure forces of Eq. (12) are integrated to the particle velocities.

$$\boldsymbol{u}_i = \boldsymbol{u}_i + \Delta t \boldsymbol{f}_i^{pressure} \qquad (15)$$

These updated velocities are again used in stiffness parameter calculation so that the iteration loop is closed. The DF solver performs iteration until the density time derivative becomes zero, which mean that the velocities fulfills divergence free condition.

2.2.2. Constant Density solver

Constant Density solver aims at eliminating density variation $(\rho - \rho_0 = 0)$ by computing pressure forces and correcting velocities iteratively. The formulation of CD solver is analogous to DF solver except the stiffness parameter(κ). The stiffness parameter is derived from the density deviation.

$$\kappa_i = \frac{(\rho_i - \rho_0)}{\Delta t^2}\delta_i \qquad (16)$$

The particle density is integrated as follow.

$$\rho_i = \rho_i^t + \Delta t \sum_j m_j(\boldsymbol{u}_i - \boldsymbol{u}_j)\nabla W(\boldsymbol{x}_{ij}, h) \qquad (17)$$

Analogous to the DF solver, CD solver loop is closed by using the resulting velocities as the input values. The CD solver performs iteration until the density variation becomes zero.

3. Experimental Set up and Simulation Model

We conducted the tsunami propagation near Kori Nuclear Power Plant using the lab-scale tsunami run-up experimental equipment. The equipment consisted of a water tank, a wave generator, and a terrain model. A water tank is 8m long, 0.5m high, and 0.3m wide. The wave generator generated three different waves in Table 1. A terrain model represented the topography around Kori Nuclear Power Plant site and it was placed at the end of the water tank.

Table 1. Generated wave condition of experiment

Case	Period (sec)	Displacement (m)
Low frequency	2	0.058
Medium frequency	1.3	0.058
High frequency	1	0.058

We performed a 3D simulation of tsunami propagation under the same conditions as the experiment. First, we instance the topography of the nuclear power plant to particles using Hypermesh. The total number of particles was 485,642 composing water, the water tank, the wave generator and the terrain model. The wave generator was represented by moving boundary wall which moved back and forth with the same frequency and wave length of the experiments. For the initial condition, the total simulation time was 10 ~ 15 sec and the time step was 5×10^{-5} sec. The average number of iterations of DFSPH solvers was 4 and it ensured that maximum density errors were below 0.5%.

4. Results

In this section, we compare free surface motion of simulation and experimental results. Both waves propagated and collided with the barrier of the nuclear power plant. Here we show comparison of the colliding wave motion for three different frequency waves: low, medium, and high frequency

Figure 1 presents the surface motion of the simulation and the experimental results for the low frequency wave. The simulated wave reached to the barrier of nuclear power plant at time t = 8.1 sec, but there was no inflow into the plant as the experimental results showed.

Figure 2 compares the flow motion of simulation and that of experiment at time t = 8.2 sec for the medium frequency wave. The simulated waves smashed against the barrier but there was no inflow into the plant, whereas the water flooded in the experiments. This difference comes from the fact that the simulation had a smaller amplitude right before colliding with the barrier due to the loss of energy.

Figure 3 depicts the comparison of the smashing waves at time t = 7.0sec for the high frequency wave. In the experiment, the waves violently collided with the barrier resulting water flood. However, it was confirmed that the waves never overflowed the barrier in the simulation. As mentioned in medium frequency case, the amplitude of waves was not large enough to reach the top of barrier since the waves were smoothed out during propagation.

In summary, the simulation results showed a good agreement with the experimental results although there was a slight difference on the height of smashing waves.

5. Conclusions

In this paper, we applied Divergence Free SPH (DFSPH) method to simulate a set of tsunami propagation and compare the surface motion with the experimental results. First, we implemented DFSPH method, one of the Incompressible SPH (ISPH), to simulate fully incompressible fluid flows. DFSPH method enforces incompressibility using two solvers. One is Divergence Free solver which projects the velocity to the divergence free field, and the other is Constant Density solver which constrains the density constant. The combination of two solvers leads to a stable simulation with larger time steps. Then we conducted a simulation of tsunami propagation near Kori Nuclear Power Plant for comparison with the lab-scale tsunami run-up experiments. A set of simulations were conducted under the same condition as the experiments for the low, medium, high frequency waves. The surface motion around the barrier of nuclear power plant was compared for each case. Overall, the simulation results were agreed well with the experimental results.

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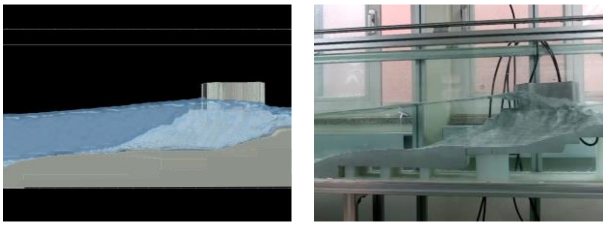

(a)

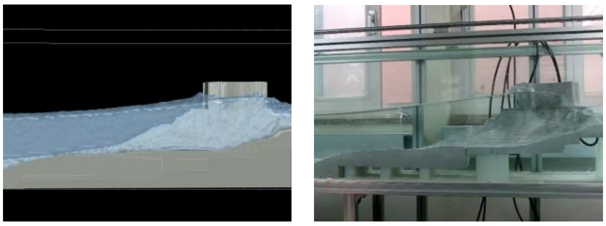
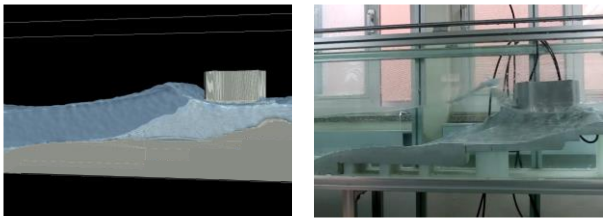

(b)

Figure 1. Comparison of simulated and recorded surface motion of the low frequency wave: (a) t=8.1s; (b) t=8.2s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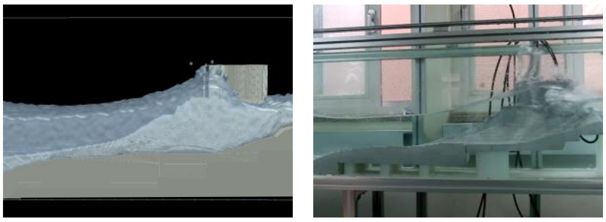
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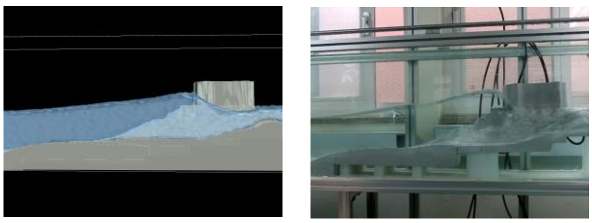
Figure 2. Comparison of simulated and recorded wave for the medium frequency wave: (a) t=7.3s; (b) t=7.4s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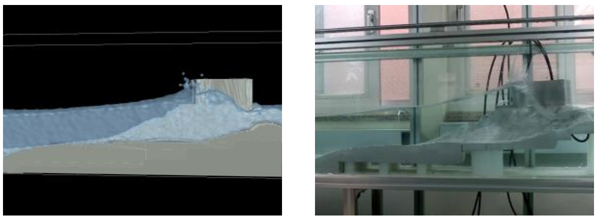
(b)

Figure 3. Comparison of simulated and recorded wave for the high frequency wave: (a) t=7.0s; (b) t=7.1s