A Novel Approach to Investigation of Axial Fluid Conduction Effect in Low Prandtl Number Fluids

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1. Introduction

Liquid metals have been considered to be useful working fluids in nuclear and non-nuclear applications associated with thermal-hydraulic aspects. For non-nuclear fields, liquid metals are broadly used in refinery and casting processes of any metals like carbon steel, copper, tin, aluminum, etc. Moreover, in solar plants, the sunlight is reflected by numerous mirror panels onto a heat exchanger operated with liquid metals [1] as well. In nuclear fields, liquid metals are widely used in fusion as well as fission areas that need effective cooling means without pressurization. In the particular fields of fast breeder nuclear reactors [2], sodium and lead (or lead-alloys) have been practically used as the primary heat transfer medium or coolant.

Among the common applications of liquid metals, for proper design and operation of liquid metal system, it needs to be taken into account that basic nature of liquid metal fluids distinguished from that of ordinary water in thermal-hydraulic aspects. In most fluids as a heat transfer medium, a ratio of the effectiveness of energy transfer through momentum as compared to the transfer through thermal diffusion in a fluid boundary layer can be defined as the Prandlt number. For highly conductive liquid metals, the Prandtl number becomes quite low due to their high thermal conductivity and the thermal boundary layer becomes quite large. Therefore, thermal diffusion would be an effective mode of heat transfer.

In broad fields of thermal-hydraulic applications dealing with higher Prandtl number fluids than ordinary water, axial fluid conduction along with flow stream can be negligible. This is mainly because the magnitude of longitudinal convection term is much larger than that of fluid conduction along with flow. However, in many application fields of liquid metal fluids, it has been also known that axial fluid conduction along with flow direction should not be negligible in some thermal-hydraulic applications. In general, conditions that the effect of axial fluid conduction can be negligible have been theoretically suggested with a dimensionless number, e.g. Peclet number, which is defined as a ratio of axial fluid conduction term over that of convection [3]. However, relatively few research papers appear in the literature dealing with the practical criteria on being negligible of axial fluid conduction lucidly. So far, no general methods have been available for obtaining a solution to the practical problems dealing with axial fluid conduction effect in low Prandtl number fluids.

In the present study, a noble approach using a commercial CFD tool has been carried out to prove and examine the axial fluid conduction effect limited in highly conductive laminar flow cases. The main objective of this work is to solve numerically the transient problem of heat transfer in a closed loop piping system including volumetric heat source and sink. Transient performance of the thermal system was investigated with respect to the conditions of various Prandtl number fluids that postulated in this study. The results will be used to drive the specific conditions that thermal system designers dealing with low Prandtl number fluids can decide whether they can disregard the effect of axial fluid conduction in practical use.

2. Methods and Results

2.1 Theoretical backgrounds

By applying the first law of thermodynamics to an open system, the equation of energy can be derived. A rigorous derivation is presented by Bird et al. [4], and the final equation of energy for axial flow in cylindrical coordinates (r, θ, z) is as follows;

$$\rho c_p V_z \frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r \cdot q_r) - \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z} + \tau_{rz}\frac{\partial V_z}{\partial r} + G \tag{1}$$

The r-momentum flux in the z-direction (τ_{rz}) is defined as

$$\tau_{rz} = -\mu \frac{\partial V_z}{\partial r} \tag{2}$$

Similarly, the heat fluxes are defined by

$$q_r = -k\frac{\partial T}{\partial r}, \quad q_z = -k\frac{\partial T}{\partial z}, \quad q_\theta = -\frac{k}{r}\frac{\partial T}{\partial \theta} \tag{3}$$

Assuming radial symmetry ($q_\theta = 0$), then Eq.(1) becomes

$$\rho c_p V_z \frac{\partial T}{\partial z} = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] - \mu\left(\frac{\partial V_z}{\partial r}\right)^2 + G \tag{4}$$

The last two terms in Eq.(4) are designated as a heat source from viscous dissipation and a uniform heat generation source or sink, respectively. Viscous dissipation occurs with all fluids with finite viscosity. However, it is usually of significance only in highly viscous fluids. An example of a uniform heat generation source is evenly distributed nuclear particles giving off energy during radioactive decay.

For low Peclet numbers, when the axial fluid conduction term ($k\frac{\partial^2 T}{\partial z^2}$) is neglected, the fluid temperature approaches the temperature of the conduit as the Peclet number approaches zero. The order of magnitude of the Peclet number at which the conduction effect becomes important can be estimated from the relative magnitude of the longitudinal conduction and convection terms [5]. The axial fluid conduction and that of convection are $k\frac{\partial^2 T}{\partial z^2}$ and $\rho c_p \vec{V}_z \frac{\partial T}{\partial z}$, respectively. The order of magnitude of these two terms are k/L and $\rho c_p \vec{V}_z/L$, where L is a characteristic order of magnitude length such as tube diameter. Therefore, their ratio is

$$\frac{axial\ \ fluid\ conduction}{axial\ \ convection} = \frac{1}{Pe} = \frac{1}{Peclet\ number}$$

This suggests that the effect of axial fluid conduction is generally negligible for Peclet numbers exceeding 100 for which the present theoretical analyses will be valid. The axial fluid conduction effect should be examined primarily in the range of $0 \le Pe \le 100$. It is interesting to note that for liquid metals, the laminar flow region would occur generally at Peclet numbers of 100 or less, and therefore, the axial fluid conduction term should be considered for such heat transfer calculations.

For extension of the theoretical results to the practical application case and for more understanding of the actual criterion being negligible of the effect of fluid conduction in very low Prandtl number fluids, some transient analysis for a closed loop piping system with a simplified geometry were made in the present study. The detailed descriptions of the analysis are provided in the following section.

2.2 CFD analysis for a simplified loop system

In order to examine the practical effect of axial fluid conduction in a low Prandtl number fluid along the flow direction, a closed loop piping system with a volumetric heat source and sink parts was set up in this work. For simplicity of the analysis, the loop piping was set to have a circular-shape cross-sectional area without flow area change. For a proper control of the boundary conditions at the inlet part of the heat source, an infinite reservoir was postulated to be employed in the system such that the transient effect of fluid conduction along the backward direction can be meaningfully examined. The problem treated in the present study is represented schematically in Fig.1.

The dimension of the loop piping geometry was set to be 2.0 meters long and 1.5 inches in inner diameter. Equivalent heat transfer rates both in heat source and sink pars were set to be 3.0kW, and the inlet and exit temperatures at the heater part were respectively set to be 390°C and 545°C, which are obtained from the design conditions of STELLA-2 test section [6]. The flowrate conditions of low Prandtl number fluids implemented in the analysis were also set to be the specified values such that the temperature rise at the heater part can be kept at 155°C being consistent with the reference operating conditions of STELLA-2 [6].

The domain for CFD analysis fully covers the entire flow region in all loop piping system such that flow distributions as well as temperature profiles along the flow direction in the loop piping can be clearly obtained. Since the solid structure of pipe wall was not modeled in this problem, conjugate heat transfer from fluid to solid and heat loss through pipe wall in radial direction were not considered. The material properties (e.g., density, viscosity, specific heat, thermal conductivity, etc.) were implemented at the specified temperature, and all exterior parts of the loop piping were assumed to be adiabatic.

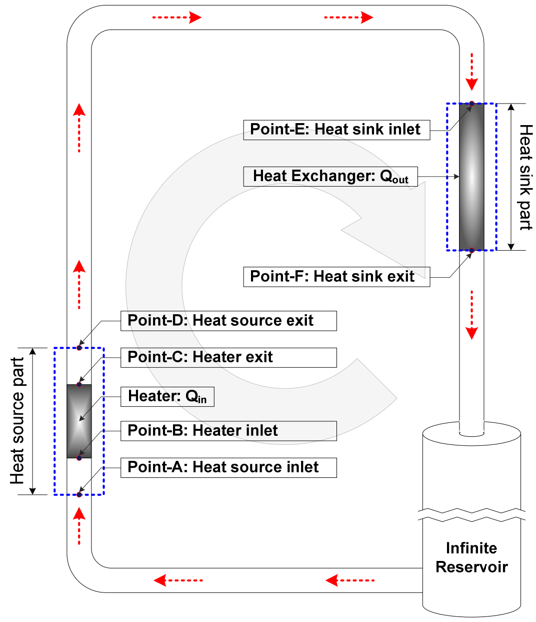


Fig.1 Schematic of the postulated closed loop system for numerical investigation

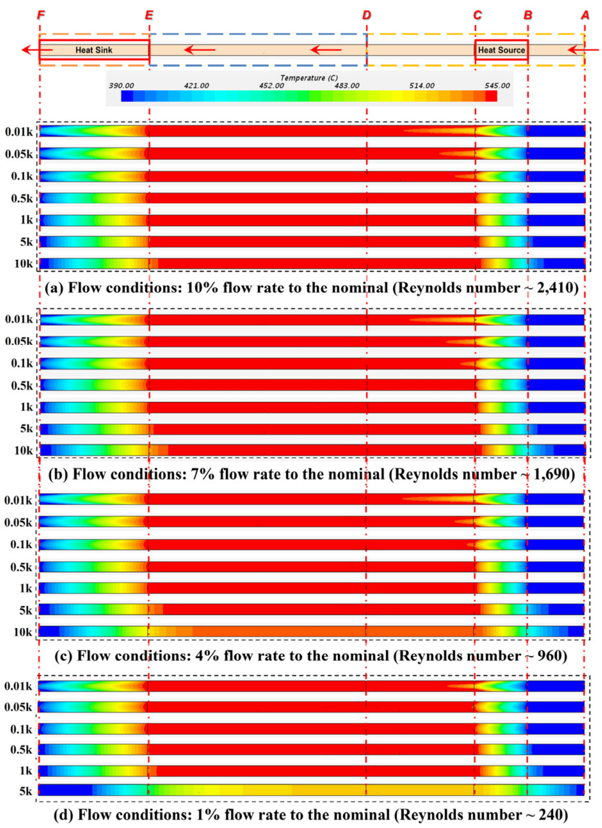
Total seven postulated fluids were considered in this analysis to examine the effect of axial fluid conduction with respect to the variations of their Prandtl numbers. This means seven different thermal conductivities were assigned to each postulated fluid. The reference fluid was picked as liquid sodium since it is the primary coolant of a sodium-cooled fast reactor as well as STELLA-2 test section [4]. Thermal conductivities of the other six fluids were set to have different thermal conductivities, which are 1% (0.01k), 5% (0.05k), 10% (0.1k), 50% (0.5k), 500% (5k), and 1000% (10k) of that of liquid sodium (1k). That is, liquid sodium is the reference (1k) and the ordinary water is corresponding to the case of 1% (0.01k). Moreover, to figure out the relative flow rate effect on the above seven cases of Prandtl numbers variations, the flow rate conditions were also set to be ranged from 1% to 10% of the nominal flowrate [6]. Since we have actually dealt with all extreme conditions, such as very low flow conditions that potential effect of axial fluid conduction could be easily seen, Reynolds numbers in all test cases were carefully considered as well to check the flow regimes of extreme conditions in this work. Table 1 summarizes the boundary conditions for the CFD analysis performed in the present work, which are obtained from the design conditions of STELLA-2 [6].

Table 1. Boundary conditions for the CFD analysis to examine axial fluid conduction effect

Flow conditions	Ratio of thermal conductivities (k/k_{Ref}, %)	1% (0.01k, Ordinary water)	5% (0.05k)	10% (0.1k)	50% (0.5k)	100% (1k, Ref.: sodium)	500% (5k)	1000% (10k, Super-conductive fluid)
1% flow rate to the nominal	Re number	241	241	241	241	241	241	N/A
	Pr number	0.4941	0.0988	0.0494	0.0099	0.0049	0.0010	N/A
	Pe number	119.5	23.8	11.9	2.4	1.2	0.2	N/A
	$T_{heater,in}$	390.4	391.6	392.9	403.4	416.5	455.3	N/A
	$T_{heater,ex}$	544.6	544.1	543.6	538.5	532.0	459.4	N/A
4% flow rate to the nominal	Re number	964	964	964	964	964	964	964
	Pr number	0.4941	0.0988	0.0494	0.0099	0.0049	0.0010	0.0005
	Pe number	476.2	95.2	47.6	9.5	4.8	1.0	0.5
	$T_{heater,in}$	390.0	390.4	390.8	393.5	396.8	422.7	445.1
	$T_{heater,ex}$	544.9	544.8	544.6	543.3	541.6	528.4	507.7
7% flow rate to the nominal (Nominal case)	Re number	1686	1686	1686	1686	1686	1686	1686
	Pr number	0.4941	0.0988	0.0494	0.0099	0.0049	0.0010	0.0005
	Pe number	833.3	166.7	83.3	16.7	8.3	1.7	0.8
	$T_{heater,in}$	390.1	390.2	390.4	392.0	394.0	409.0	426.8
	$T_{heater,ex}$	545.0	544.9	544.8	544.0	543.0	535.6	525.8
10% flow rate to the nominal	Re number	2409	2409	2409	2409	2409	2409	2409
	Pr number	0.4941	0.0988	0.0494	0.0099	0.0049	0.0010	0.0005
	Pe number	1190.5	238.1	119.0	23.8	11.9	2.4	1.2
	$T_{heater,in}$	390.0	390.2	390.3	391.4	392.8	403.3	416.5
	$T_{heater,ex}$	545.0	544.9	544.9	544.3	543.6	538.4	531.8

2.3 Results and discussions

The results of transient analyses using commercial CFD tool, STAR-CCM+ V9 [7] are shown in Fig. 2, which is especially for examining the effect of fluid conduction to the forward and backward region adjacent to the heat source (i.e. heater) and heat sink parts along the flow direction. All temperature data were obtained with mass flow averaging process at the interesting locations.



(a) Flow conditions: 10% flow rate to the nominal (Reynolds number ~ 2,410)
(b) Flow conditions: 7% flow rate to the nominal (Reynolds number ~ 1,690)
(c) Flow conditions: 4% flow rate to the nominal (Reynolds number ~ 960)
(d) Flow conditions: 1% flow rate to the nominal (Reynolds number ~ 240)

Fig.2 Temperature distribution along the flow direction in the test loop configuration

As shown in Fig.2, the effect of fluid conduction along the flow stream becomes stronger as the thermal conductivity of the working fluid increases. That is, the inlet temperature at the heater source part is getting higher as the thermal conductivity of the working fluid increases. This is mainly because heat generation at the heater part does not fully transfer forward along the flow stream but transfer partially backward direction due to an essential temperature gradient from the heater region to its inlet. This nature also means that the exit temperature at the heater part would be lower than we expected theoretically and the advection process of heat flow does not adequately work as other thermal-hydraulic systems using ordinary water as working fluid. In particular, when the flow rates are getting small (See Fig.2(d)), the advection term becomes weaker and weaker as compared to the stronger thermal conduction term in low Prandtl number fluids. This also means the interesting phenomenon is actually getting manifest as we figured out from the theoretical understanding of the effect of axial fluid conduction. Figs.3 and 4 show the quantitative trends of unexpected heat dissipation or transfer caused by the effect of axial fluid conduction.

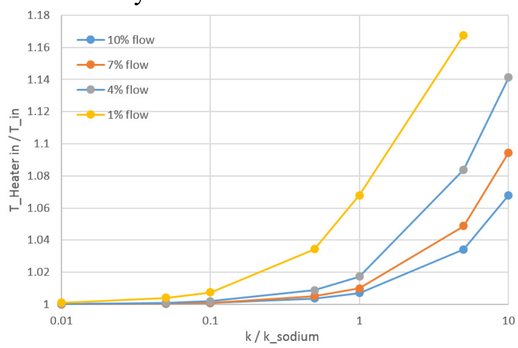


Fig.3 Variation of temperature ratios at the heater inlet (point-B) to the heater source part (point-A, ideal)

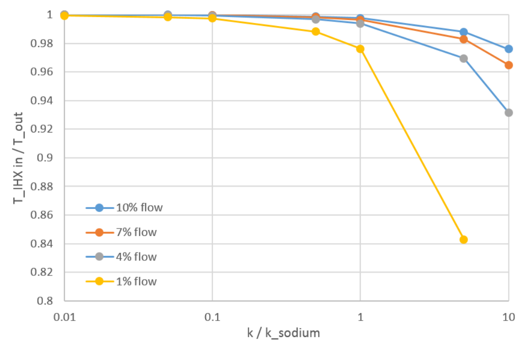


Fig.4 Variation of temperature ratios at the heat sink inlet (point-E) to the heat source exit (point-D)

The variation of the ratios of mass-flow-averaged temperatures at the heater inlet (point-B) to the ideal temperature condition of 390^{o}C (point-A) are shown in Fig.3 with respect to the thermal conductivity variations of working fluids. Based on the analysis results, it was figured out that approximately 7% deviation at its maximum was observed in the case of 10% flow rate condition to the nominal for the super-conductive fluid having 10 times thermal conductivity to the liquid sodium. The deviations are getting large as the flow rate decreases, and the quantitative analysis results showed that the ratios of calculated temperatures at the heater inlet to the ideal condition consistently increase up to more than 20% at the maximum in the super-conductive fluids. Fig.4 shows the similar trends described in Fig.3. In this case, it was dealt with the ratios of calculated temperatures at the heat sink inlet (point-E) to the heater exit (point-D). Similar to Fig.3, the inlet temperatures at the heat sink part are getting decrease due to the effect of higher thermal conductivity, which results in strong forward heat transfer along the flow stream took place fast in super-conductive fluids more than 1k. This also means that the system heat load can be easily dissipated backward as well as forward direction along the flow stream. Therefore, when the system designers make choice of working fluids except ordinary water coolant, they should take into account the basic nature of Prandtl numbers of working fluids for appropriate system design.

As mentioned previously, the present work dealt with the practical problem to figure out the actual effect of axial fluid conduction along the flow direction in very low Prandtl number fluids as extreme conditions. To meet the main objective of this work beyond the theoretical understanding, CFD analyses for transient mode were made and the time for reaching quasi-steady conditions were investigated. Basically, if there is unexpected heat dissipation or fast transfer along the flow stream forward and/or backward, a steady-state to meet the system heat balance is hardly achieved when compare to the opposite situations in larger Prandtle number fluids. The unexpected phenomena occur mainly with the effect of axial fluid conduction in low Prandtl number fluids as described in the previous section of this paper. As shown in Fig.5, normalized physical times taking for each steady-state reaching were examined with respect to the thermal conductivity variations in the working fluids. In this analysis, the judgment criterion of each steady-state reaching was defined as the variance of variables (e.g. temperature) to be less than 10^{-4}.

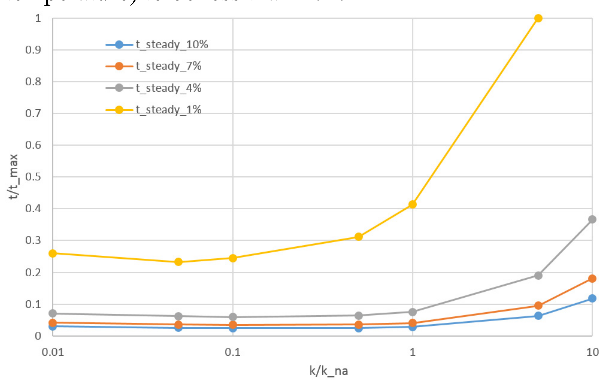


Fig.5 Comparison of normalized physical times taking for each steady-state reaching

Based on the analysis and comparison results, it has been figured out that the physical times taking for each steady-state reaching are getting longer and longer as thermal conductivity of working fluids increases and flow rate decreases. The results are very consistent with the analysis results described in Figs.2-4, which are mainly caused by the weakness of advection term of fluid flow in super-conductive fluids working in low Reynolds number range.

3. Conclusions

A noble approach using a commercial CFD tool was carried out to examine the axial fluid conduction effect limited in highly conductive laminar flow cases. Based on the analysis results obtained from the present work, we could figure out a couple of very interesting things. Firstly, there is obvious effect on axial fluid conduction in very low Prandtl number fluids, but its quantitative criteria are totally dependent on the conditions of fluid flow and its thermal properties. Secondly, the axial fluid conduction effect along the flow stream is getting stronger in lower Reynolds number flows and lower Prandtl number fluids. Finally, it was concluded that thermal designers can neglect the axial fluid conduction effect only for the Prandtl number fluids having its thermal conductivity of less than or similar to that of liquid sodium. Otherwise, the designers should take into account the axial fluid conduction effect in all kinds of thermal system design process as they have not done before in ordinary water system.

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REFERENCES

[1] Benemann, J., “Status Report on Solar Through Power Plants,” Pilkington Press (1996)

[2] Kamil Tucek, Johan Carlsson, Hartmut Wider, “Comparison of sodium and lead-cooled fast reactors regarding reactor physics aspects, severe safety and economical issues,” Nuclear Engineering and Design, Vol.236, pp.1589-1598 (2006)

[3] Todreas N.E., Kazimi M.S., “Nuclear Systems (I); Thermal Hydraulic Fundamentals,” Hemisphere Publishing Corp. (1990)

[4] Bird, R. Byron, Stewart, Warren E., and Lightfoot, Edwin N. Transport phenomena. New York, New York, John Wiley and Sons, Inc. (1960)

[5] Does de Bye, J.A.W. Van der and Schenk, J. Heat transfer in laminar flow between parallel plates. Appl. Sci.. Res. Sec. A, Vol.3, pp.308-316 (1953)

[6] Jaehyuk Eoh, et al., “Computer Codes V&V Tests with a Large-scale Sodium Thermal-hydraulic Test Facility (STELLA),” Transactions of the American Nuclear Society, Vol. 114, New Orleans, Louisiana, June 12-16 (2016)

[7] STAR-CCM+ 9.02.007 User Guide, CD-adapco (2014)