Core Parameter Uncertainty Analysis on HTGR UAM Benchmark with DeCART/MUSAD/CAPP

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1. Introduction

Recently, MUSAD (Modules of Uncertainty and Sensitivity Analysis for DeCART) [1,2] code has been developed by the Korea Atomic Energy Research Institute (KAERI) for quantifying a sensitivity and uncertainty induced by the covariance of the nuclear data. The code can produce a sensitivity and uncertainty for few-group cross sections (XS) based on the generalized perturbation theory (GPT) using generalized adjoint solutions and group constants generated by DeCART (Deterministic Core Analysis based on Ray Tracing) [3].

In this study, a two-step procedure [4] for an uncertainty analysis of VHTR core parameters was established with DeCART/MUSAD/CAPP code system. In the first step, homogenized few-group XSs and their uncertainties are prepared based on a generalized perturbation theory. DeCART/MUSAD code was applied in this step. In the next step, CAPP code [5] produces various core parameters and their uncertainties can be obtained from the outputs with statistical processing.

For the core simulation step, modules for producing covariance matrix and random sampling of few-group XSs have been implemented into the MUSAD code. The performance of the DeCART/MUSAD/CAPP code system was confirmed with the benchmark proposed by IAEA CRP on HTGR UAM [6].

2. Methods and Results

The methodology used in the lattice physics step for a core parameter uncertainty analysis will be simply reviewed in the section 2.1 and a random sampling method with a few-group XS covariance matrix will be described in the next sections. The last section presents core parameter uncertainty analysis results on the benchmark problem.

2.1 Generalized Perturbation Theory Review

The general response for a few-group XS, its sensitivity, and uncertainty are expressed as Eqs. (1), (2), and (3), respectively.

$$R = \frac{\langle H_1 \psi \rangle}{\langle H_2 \psi \rangle}, \tag{1}$$

$$S_{R,X} = \frac{\delta R}{\delta X}\frac{X}{R} = X\left(\frac{\left\langle \dfrac{\delta H_1}{\delta X}\psi \right\rangle}{\langle H_1\psi\rangle} - \frac{\left\langle \dfrac{\delta H_2}{\delta X}\psi \right\rangle}{\langle H_2\psi\rangle} - \left\langle \Gamma^* \left(\frac{\delta A}{\delta X} - \lambda \frac{\delta F}{\delta X} \right)\psi \right\rangle \right), \tag{2}$$

$$u_R^2 = \mathbf{S}_R \mathbf{C} \mathbf{S}_R^T, \tag{3}$$

where A, F, ψ, and λ are the neutron transport operator, the fission source operator, the angular flux, and the eigenvalue of the system, respectively. Also, $\mathbf{C}$ is a multi-group covariance data processed from the evaluated nuclear data files.

To obtain the generalized adjoint flux of Eq. (2), Γ^*, the following generalized adjoint equation for the general response can be defined as follows:

$$\left(A^* - \lambda F^*\right)\Gamma^* = S_\Gamma^* \equiv \frac{H_1}{\langle H_1\psi\rangle} - \frac{H_2}{\langle H_2\psi\rangle}, \tag{4}$$

where S_Γ^* is the generalized adjoint source for the response.

In addition, to perform a sensitivity and uncertainty analysis for doubly heterogeneous (DH) region of a VHTR fuel, a generalized adjoint transport equation with the explicit DH treatment [2] was made based on the Sanchez-Pomraning method [7]. The generalized adjoint source was defined with two terms for compact matrix and TRISO region separately and they were added into the effective source. The detailed equations can be found in reference [2].

2.2 Covariance Matrix for Few-Group Cross Sections

To generate random sampled few-group XS sets, we need a covariance matrix including both variance and covariance between few-group XSs. The covariance matrix of few-group XSs can be defined as a matrix form (5).

$$\mathbf{V_R} = \begin{bmatrix} u^2_{\sigma^1_{x11}} & C_{\sigma^1_{x11},\sigma^1_{x21}} & \cdots & C_{\sigma^1_{x11},\sigma^n_{LGZ}} \\ C_{\sigma^1_{x21},\sigma^1_{x11}} & \ddots & & \\ \vdots & & u^2_{\sigma^i_{xgz}} & \vdots \\ & & \ddots & \\ C_{\sigma^n_{LGZ},\sigma^1_{x11}} & & \cdots & u^2_{\sigma^n_{LGZ}} \end{bmatrix}, \tag{5}$$

where $\mathbf{R} = (R_{\sigma^1_{x11}}, ..., R_{\sigma^i_{xgz}}, R_{\sigma^n_{LGZ}})$, n is the number of nuclides, L is the number of XS types, G is the number of few-groups, and Z is the number of assemblies or block types.

The diagonal elements are the variance or uncertainty of few-group XS and the off-diagonal elements mean the covariance between few-group XSs, which can be calculated from two sensitivities of different XS types as follows.

$$C_{\sigma^i_{xgz},\sigma^{i'}_{x'g'z'}} = \mathbf{S}_{\sigma^i_{xgz}} \mathbf{C} \mathbf{S}^T_{\sigma^{i'}_{x'g'z'}} \quad (6)$$

2.3 Random Sampled Few-group Cross Section Sets

Once a covariance matrix of a few-group XS is obtained, random sampled few-group XS sets should be prepared for a core simulation code. It is noted that random sampling should be conducted considering the few-group XS covariance matrix in Eq. (5). For this, the covariance matrix should be decomposed using the eigenvalue decomposition method as Eq. (7).

$$\mathbf{V_R} = \mathbf{QDQ}^T. \quad (7)$$

Generally, covariance matrix is positive semi-definite matrix allowing zero and positive eigenvalue. Because Cholesky decomposition can, however, be applied to only positive definite matrix, the eigenvalue decomposition method was applied in this calculation.

Random sampling can then be applied using the following equation.

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 (1 + \mathbf{QD}^{1/2}\mathbf{Q}^T \mathbf{x}), \quad (8)$$

where $\boldsymbol{\alpha}_0$ is average few-group XS and $\mathbf{x}$ is a random number with a standard normal distribution, which elements can be generated using the Box Muller method as follows.

$$x = \sqrt{-2 \cdot \log(s)} \cdot \cos(2\pi t) \quad (0 < s, t < 1 : random\ number) \quad (9)$$

2.4 Core Simulation

CAPP code is a 3D diffusion code for a block type VHTR. In the second step, CAPP can produce various core parameters with random sampled few-group XS sets. Then, core parameter uncertainties can be obtained from CAPP outputs using a statistical post-processing.

Fig. 1 shows a flow chart of the DeCART/MUSAD/CAPP code system for a two-step procedure of an uncertainty analysis. MUSAD uses multi-group unshielded covariance matrix using ERRORR(J) of NJOY and generalized adjoint fluxes from DeCART. It then produces covariance matrix for few-group cross section and random sampled few-group cross section sets. PXSGEN is a XS format conversion code for CAPP code.

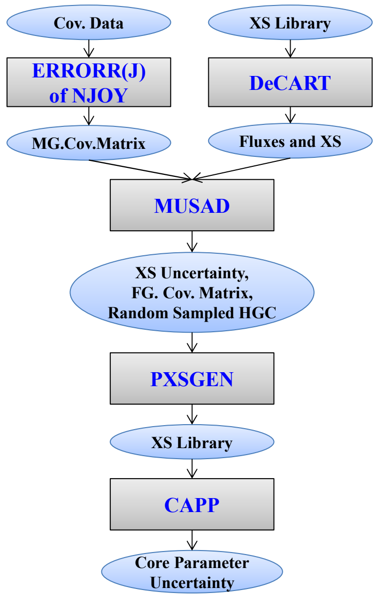

Fig. 1. Flow chart of uncertainty analysis code system.

2.5 Numerical Results

Fig. 2 shows a 2D core configuration based on the MHTGR-350 benchmark by IAEA HTGR UAM [6]. It has the Ex.I-2a single block of the benchmark. For few-group XS generation, two super cell models were set up, which are a fuel block type and a reflector block type, as shown in the right side of Fig. 2. DeCART/MUSAD code used the 190 group cross sections originated from ENDF/B-VII.0 and the covariance data processed from ENDF/B-VII.1, because the ENDF/B-VII.0 contains covariance for only a few materials and a new VHTR library for DeCART based on the ENDF/B-VII.1 is not currently optimized. However, the effect by the difference of the version between cross section library and covariance data is very small, because covariance data is relative value. Three major isotopes (U235, U238, C12), and four XS types (capture, fission, nu, total scattering) with a 10-group structure were used for core parameter uncertainty analysis. The covariance data for total scattering XS can be obtained by aggregating the elastic and inelastic scattering covariance data using covariance properties [1].

The number of few-group XS sets can be about 100 from the Wilks' formula which is widely used in a safety parameter uncertainty analysis. In this study, various sample sizes were, however, examined for determining a reasonable sample size due to the complex correlation between the few-group XSs.

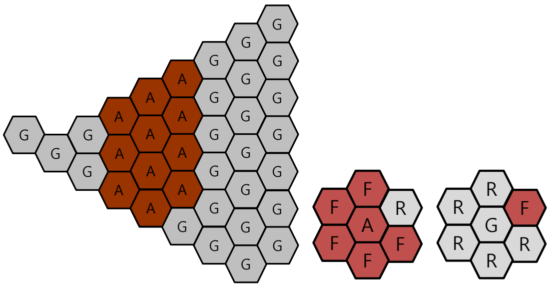

Fig. 2. 1/6 2D core model and super cell models for few-group cross section generation.

Table I shows the k_{eff} uncertainty by DeCART/MUSAD/CAPP on the 2D core model and the reference value is 0.768% by 1000 samples. Fig. 3 shows the change of the k_{eff} uncertainty according to the number of samples. Contrary to expectations, the uncertainty in the case of 100 samples is considerably different to those of the other cases. Therefore, it should be noted that the number of few-group XS sets should be over 200 in this problem.

The k_{eff} uncertainty by DeCART/MUSAD/CAPP can be compared to the Ex.I-2a single block problem result using only the GPT, DeCART/MUSAD. It can be seen that the uncertainty of the core model is slightly lower than that of the single block owing to the graphite reflectors. In addition, the table shows that there is the slight discrepancy between MUSAD and McCARD [8] in Ex.I-2a block problem. It might be caused by the implicit uncertainty.

Fig. 4 shows the block power distribution and their uncertainty, the values of which are under 0.19%. It is relatively lower than PWR cases. The reason is that the core problem of MHTGR-350 benchmark consists of only one block type.

Related reference values have not yet been reported in the HTGR UAM and a core parameter uncertainty analysis is scheduled next year.

Table I: k_{eff} and its uncertainty

	Code	No.samples	k_{eff}	*dk/k%*
Core Model	DeCART /MUSAD/ CAPP	100	1.06412	0.690
		200	1.06408	0.752
		300	1.06439	0.761
		400	1.06411	0.758
		500	1.06407	0.766
		600	1.06388	0.757
		700	1.06377	0.742
		800	1.06382	0.753
		900	1.06383	0.763
		1000	1.06388	0.768
Single Block (Ex.I-2a)	DeCART /MUSAD		1.05799	0.803
	McCARD		1.05525	0.727

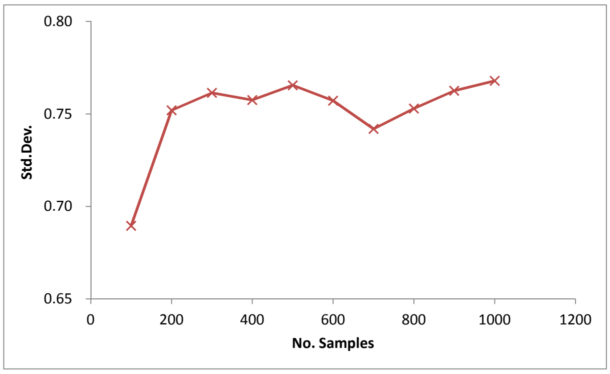

Fig. 3. k_{eff} uncertainty change according to the number of samples

Avg.Rel.Power	1.075	1.347	1.347	
Std.Dev(%)	0.023	0.137	0.137	
	0.873	0.808	0.822	0.808
	0.100	0.181	0.181	0.181
	1.026	0.935	0.935	1.026
	0.099	0.030	0.030	0.100

Fig. 4. Block average power distribution and uncertainties.

3. Conclusions

A two-step procedure with DeCART/MUSAD/CAPP was established for quantifying uncertainties of the VHTR core parameters induced by the covariance of the nuclear data, and the performance of the code system was evaluated using the MHTGR-350 benchmark.

In a lattice calculation step, the DeCART/MUSAD code system generated a covariance matrix of a few-group XSs based on the GPT. The covariance matrix was then decomposed using the spectral decomposition method, and was multiplied using a random number vector. Random samples of the few-group cross sections were then generated for the core simulation. CAPP performed core analysis with the few-group XS samples, and the uncertainties of the core parameters were obtained through statistical post processing. The number of samples was determined after case study with various sample sizes. The core parameter uncertainties were compared to that of the single block problem. Thus, it can be seen that the uncertainty is reasonable value and it was confirmed that two-step procedure with the code system is working on the MHTGR-350 core model.

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