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Robust and Optimal Nuclear Reactor Power Tracking using Integral Sliding Mode Control

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1. Introduction

The nuclear reactor is a complex, nonlinear system. The system evolves over time as the system parameters vary as the power level changes, the fuel burns up, etc. This makes it difficult to use a single linear controller for the nuclear reactor power control over the entire operating ranges. Recently, in addition, the load-following capacity of nuclear power plants draws much attention as there are cases where the electricity market requires nuclear power plants to response to the frequent variations in the electricity demand, as opposed to the conventional role of nuclear power plant as the baseload supplier. These circumstances necessitate robust and optimal power tracking controllers for nuclear reactor operation.

This study presents the application of integral sliding mode control-based optimal tracking controller to secure both robustness and optimality in nuclear reactor power control. The validity and effectiveness of the proposed control scheme are demonstrated by simulations.

2. Problem Statement

Consider a single-input single output (SISO), control-affine non-linear dynamic system represented by the following state-space equation

$$\dot{x} = f(x) + g(x)u, y = h(x) \quad (1)$$

where $x \in \mathbb{R}^{n\times 1}$ is the state vector, $u \in \mathbb{R}$ the control input, f, g smooth vector fields on $\mathbb{R}^{n\times 1}$ and $\mathrm{h} \in \mathbb{R}$ a scalar, smooth output function.

Suppose there exists a feedback control law $\mathrm{u} = \mathrm{u}_0(\mathrm{x})$ such that system (1) can be controlled in a desired way. We denote this ideal system as

$$\dot{x}_0 = f(x_0) + g(x_0)u_0 \, , y_0 = h(x_0) \quad (2)$$

where $x_0 \in \mathbb{R}^n$ represents the state trajectory of the ideal system under the ideal control u_0.

However, in practice, system (1) is perturbed by uncertainties such as parameter variations, model errors and external disturbances, and the real trajectory of the closed-loop control system may be represented by

$$\dot{x} = \hat{f}(x) + \hat{g}(x)u + d(x,t) \quad (3)$$

where vector $\hat{f}$, $\hat{g}$ represent known model of f and g, respectively, and $d(x,t)$ comprises the uncertainties. It is also assumed that there is no modelling error or measurement noise in output, $h(x)$.

The control design aims to find a control law u such that the state trajectories or the output of system (3) satisfy $\mathrm{x(t)} = \mathrm{x}_0(\mathrm{t})$ or $\mathrm{y(t)} = \mathrm{y}_0(\mathrm{t})$. This study focuses on the tracking of desired output trajectories.

3. Controller Design

The control design approach in this study follows the input-output linearization scheme. After the system is linearized in terms of output and a new control input, say ν, we will design an optimal and robust tracking controller for ν.

3.1 Input-Output Linearization of SISO system using Feedback Linearization

Our objective is to make the output $\mathrm{y(t)}$ track a desired trajectory $\mathrm{y}_0(\mathrm{t})$, and it is assumed that $\mathrm{y}_0(\mathrm{t})$ and its time derivatives up to a sufficiently high order are assumed to be known and bounded, and the system's relative degree γ is equal to one. Using the notations of differential geometry, the differentiation of the output is represented by

$$\dot{y} = \nabla h \cdot (\hat{f} + \hat{g}u) = \mathcal{L}_{\hat{f}}h + (\mathcal{L}_{\hat{g}}h)u \quad (4)$$

where $\mathcal{L}_{\hat{f}}h = \nabla h \cdot \hat{f}$ and $\mathcal{L}_{\hat{g}}h = \nabla h \cdot \hat{g}$

It is also assumed that $\mathcal{L}_{\hat{g}}h(\mathrm{x}) \neq 0$ for some $\mathrm{x} = \mathrm{x}_0$ in an open connected set Ω_x in the state space. Then, in a finite neighborhood Ω of x_0, the input transformation

$$u = \frac{1}{\mathcal{L}_{\hat{g}}h}(-\mathcal{L}_{\hat{f}}h + \nu) \quad (5)$$

yields a first-order linear relation between y and a new control input ν, namely $\dot{y} = \nu$

3.2 Tracking Control

Consider the problem of tracking a given desired trajectory $\mathrm{y}_0(\mathrm{t})$, and define the tracking error vector by $\mathrm{y_e(t)} = \mathrm{y(t)} - \mathrm{y}_0(\mathrm{t})$. Then it is known that, by using the following control law,

$$u = \frac{1}{\mathcal{L}_{\hat{g}}h}\left(-\mathcal{L}_{\hat{f}}h + \dot{y}_0 + Ky_e\right), K < 0 \quad (6)$$

the output remains bounded and the tracking error y_e converges to zero exponentially [4,6]

3.3 Robust Control

3.3.1 Sliding Mode Control

From (3) and (5), the following output error dynamics is obtained,

$$\dot{y}_e = \nu + \mathcal{L}_d h \quad (7)$$

Since the system under consideration is SISO, the "observable" uncertainty, $\mathcal{L}_\mathrm{d}\mathrm{h}$, lies in the range space of the input. Thus the uncertainty term can be considered as matched uncertainty

For the development of the controller, consider a generalized form of the linear output error dynamics with matched uncertainty,

$$\dot{y}_e = Ay_e + Bw + M\xi \quad (8)$$

where $A \in \mathbb{R}$, $B \in \mathbb{R}$. $w \in \mathbb{R}$ is a new control input. B is assumed to be non-zero and $M \in \mathbb{R}$ is known. The function $\xi(y_e, t)$ represents an uncertainty which is unknown but has a known upper bound for all y_e and t.

Then it is possible to write $M = BD$ for some $D \in \mathbb{R}$. Now the uncertain system (8) can be rewritten as

$$\dot{y}_e = Ay_e + Bw + BD\xi \quad (9)$$

As a first step, define a sliding surface as

$$\mathcal{S} = \{y_e \in \mathbb{R} : \sigma(t) = Gy_e(t) = 0, \ G \neq 0\} \quad (10)$$

Consider the time derivative of the sliding surface given by

$$\dot{\sigma}(t) = G\dot{y}_e(t) \quad (11)$$

Substituting the error dynamics equation (9) into (11) gives

$$\dot{\sigma}(t) = G(Ay_e + Bw + BD\xi) \quad (12)$$

It is assumed that the system states are forced to reach the sliding surface at time t_s so that for all $t \geq t_s$ an ideal sliding motion can be obtained i.e.

$$\sigma(t) = \dot{\sigma}(t) = 0 \text{ for } \forall t \geq t_s$$

The equivalent control can be obtained by equating equation (11) to zero,

$$w_{eq}(t) = -(GB)^{-1}G\big(Ay_e(t) + BD\xi(t, y_e)\big) \ \forall t \geq t_s \quad (13)$$

The equivalent control can be thought of as the average value which the control signal must take to maintain the sliding motion on the sliding surface [1,2,3]

In order to obtain an expression for the sliding motion (i.e. the motion while the system is in the sliding mode), substituting the value of $w_{eq}(t)$ from (13) into (9)

$$\dot{y}_e = \underbrace{(I - B(GB)^{-1}G)}_{\Gamma} Ay_e + \underbrace{\big(I - B(GB)^{-1}G)\big)}_{\Gamma} BD\xi \quad (14)$$

Since the projection term Γ has the property that $\Gamma B = 0$, by definition, equation (14) reduces to

$$\dot{y}_e(t) = \Gamma Ay_e(t)$$

i.e. while the system is in the sliding mode, the effect of the uncertainty ξ is completely removed. The stability of the sliding motion depends on the choice of the sliding surface, i.e. G.

Since we don't know the uncertainty ξ, the equivalent control cannot be applied to the system to induce the sliding mode. To ensure the sliding motion on the surface in finite time, following sliding mode control law is suggested [1]

$$w = -(GB)^{-1}GAy_e - \varepsilon(t, y_e)(GB)^{-1}\, sgn\ (\sigma) \quad (15)$$

where $sgn\ (\cdot)$ is the signum function, $\varepsilon(t, y_e)$ a scalar gain chosen large enough to enforce the sliding motion,

$$\varepsilon \geq \|GDB\|\|\xi\| + \eta$$

where η is some positive scalar that satisfies the η-reachability condition [1,2,3],

$$\sigma^T(t)\dot{\sigma}(t) \leq -\eta\|\sigma(t)\|.$$

3.3.2 Integral Sliding Mode Control

It is well known that the standard sliding mode control loses robustness in the reaching phase. As a remedy for the issue, this study adopts the integral sliding mode control method. Let's consider a new sliding surface defined as

$$\mathcal{S} = \{y_e \in \mathbb{R} : \sigma(t) = Gy_e(t) + z(t) = 0, G \neq 0, \ z \in \mathbb{R}\}$$

where z is to be specified later.

It is also assumed that the system states are forced to reach the sliding surface at time t_s so that for all $t \geq t_s$ an ideal sliding motion can be obtained i.e. $\sigma(t) = \dot{\sigma}(t) = 0$ for $\forall t \geq t_s$ and therefore

$$\dot{\sigma}(t) = G\dot{y}_e(t) + \dot{z}(t) = 0 \quad (16)$$

Furthermore, consider the following form of the control input

$$w = w_o + w_s$$

where w_o is a nominal control input to make the system behave as we want when there are no uncertainties or disturbances, and w_s is a control input from the sliding surface approach to reject uncertainties.

Substituting the control input w into the equation (16) and enforcing the ideal sliding motion give:

$$\dot{\sigma} = G(Ay_e + Bw_o + Bw_s + BD\xi) + \dot{z} = 0 \quad (17)$$

Consider a control input

$$w_{temp}(t) = -D\xi(t, e) \ \forall t \geq t_s \quad (18)$$

By setting $w_s = w_{temp}$ in equation (16), we get

$$G(Ay_e + Bw_o) + \dot{z} = 0. \quad (19)$$

If we define the dynamics of z(t) by (19), i.e.,

$$\dot{z} = -(GAy_e + GBw_o), \ z(0) = -Gy_e(0) \quad (20)$$

then following control input satisfies the sliding motion condition:

$$w = w_o + w_s = w_o - D\xi$$

Substituting $w = w_o + w_s$ into equation (8) gives

$$\dot{y}_e = Ay_e + Bw_o \quad (21)$$

The integral sliding function σ is represented by

$$\sigma = Gy_e + Gz(0) - \int_0^t G\big(Ay_e(\tau) + Bw_o(\tau)\big)d\tau \quad (22)$$

One thing to note is that the reaching phase is eliminated and $\sigma(0) = 0$ is guaranteed by definition.

Finally we can define an integral sliding mode controller that is robust against matched uncertainties by

$$w = w_o + w_s = w_o - \varepsilon(t, e)(GB)^{-1}\, sgn\ (\sigma(t)) \quad (23)$$

where ε should be greater than any disturbance or uncertainty in the system, i.e.,

$$\varepsilon \geq \|D\|\|\xi\| + \eta$$

and η is some positive scalar that satisfies the η-reachability condition,

$$\sigma^T(t)\,\dot{\sigma}(t) \leq -\eta\|\sigma(t)\|$$

The discontinuous part w_s is to enforce a sliding mode along the sliding surface, and the continuous, nominal control w_o can be any stabilizing controller as discussed in 3.2.

3.4 Optimal Control

In this study, the nominal control w_o is designed using infinite time linear quadratic optimal control method with the following cost function,

$$J = \int_0^\infty y_e{}^T Q y_e + w_o^T R w_o \ d\tau$$

and, thus, the control law is defined by

$$w_o = K y_e$$

where $K = -R^{-1}B^T P$ and P is the solution of Algebraic Riccati equation

$$0 = PBR^{-1}B^T P - A^T P - PA - Q$$

3.5 Solution to Discontinuous Control Input and Chattering

The main drawback of the sliding mode control is chattering effect which is mainly due to the fast-switching, discontinuous input. The continuous and non-switching equivalent control can be obtained by averaging the discontinuous switching component in (22), via a low pass filter [1], i.e.,

$$w_s = w_{\text{ave}}$$

$$\mu\ \dot{w}_{\text{ave}} + w_{\text{ave}} = -\varepsilon(t,e)(GB)^{-1}\ \text{sgn}\ (\sigma(t))$$

4. Application

4.1 Simulation Model

Consider following reactor model with 1-group delayed neuron precursor,

$$\dot{x} = f(x) + g(x)u$$

where

$$x = [P, \mathbb{C}, T_F, T_C]^T$$

P is the reactor power, $\mathbb{C}$ the delayed neutron precursor, T_F the fuel temperature, and T_C the coolant temperature,

$$\mathrm{f(x)} = \begin{bmatrix} \left(\dfrac{\rho_0 + \alpha_T^C T_C + \alpha_T^F T_F - \beta}{\Lambda}\right)\mathrm{P} + \lambda\mathbb{C} \\ -\lambda\mathbb{C} + \dfrac{\beta}{\Lambda}\mathrm{P} \\ \dfrac{f}{m_F c_{pF}} P - \dfrac{1}{\tau_F}\left(T_F - T_C\right) \\ \dfrac{(1-f)}{m_C c_{pC}} P + \dfrac{1}{\tau_C} T_F - \left(\dfrac{1}{\tau_C} + \dfrac{2}{\tau_R}\right) T_C + \dfrac{2}{\tau_R} T_{Cin} \end{bmatrix}$$

$$\mathrm{g(x)} = \begin{bmatrix} \frac{P}{\Lambda} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathrm{u} = \rho_c(\text{control reactivity})$$

Values of all parameters in the simulation model were obtained from [5], which establishes a PWR NPP model for 1300 MWe Palo Verde Nuclear Generating Station.

4.2 Simulation Results and Discussion

Three controllers are considered in the simulation study,

- SMC(Sliding mode controller derived in 3.3.1)
- ISMC(Integral sliding mode controller derived in 3.3.2)
- LQ(Optimal controller without w_s)

To show the robustness of controller, $\pm$ 1% parameter uncertainties in α_T^C, α_T^F are considered.

Figure 1 is the simulation result with LQ controller without parameter uncertainties. LQ controller performs well in this case but the cases with parameter uncertainties the controller fails to track the desired reference power trajectory.

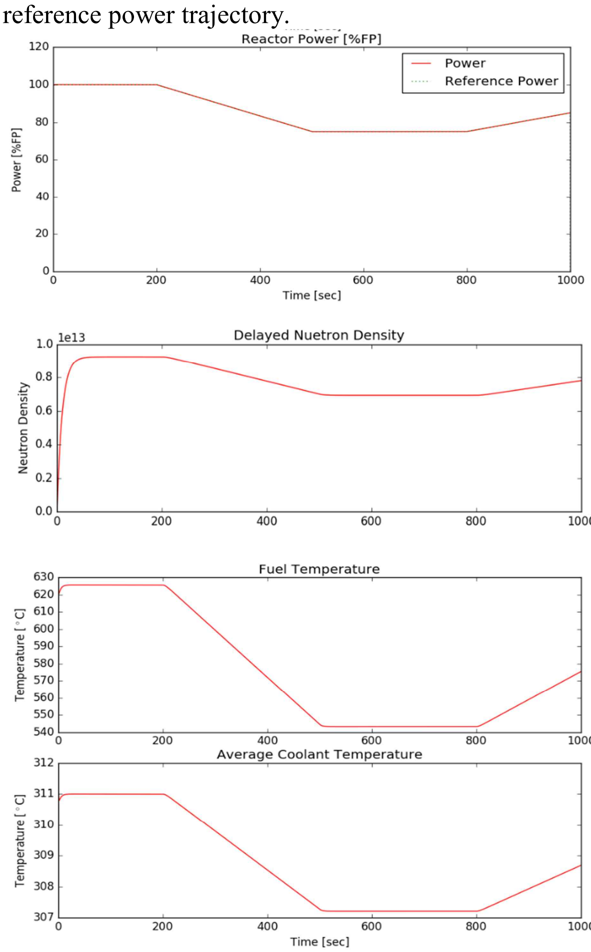

Figure 1. LQ control w/o parameter uncertainty

Unlike the LQ controller, both SMC and ISMC controllers succeed in reference tracking under $\pm 1\%$ parameter uncertainties.

Figure 2 illustrates the control input from a simulation with ISMC. From the start of simulation until 20sec, original ISMC (red color) is used and filtered control input is applied from 20sec (green color). Due to filtering, the discontinuity in control input is smoothed.

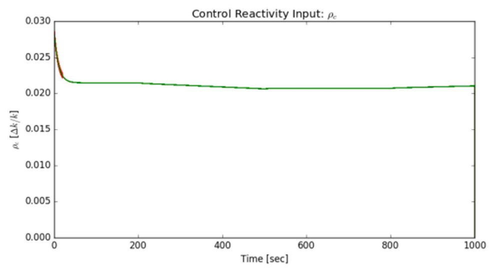

Figure 2. Control Input: ISMC

Figure 3 shows only the w_s part of the ISMC control input, where green line is switching control input and red line is the filtered control input.

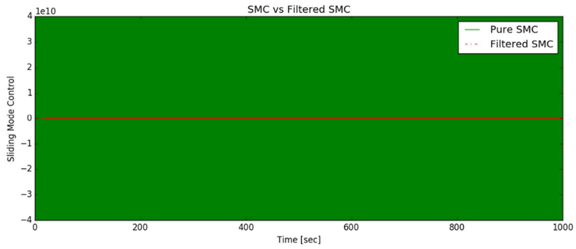

Figure 3. Control Input Filtering

The effectiveness of ISMC over SMC can be visualized by comparison of sliding variable phase portraits. Figure 4 and 5 are the phase portraits of ISMC and SMC, respectively. The volume of the phase portrait shows that ISMC tracks the reference trajectory more tightly than SMC

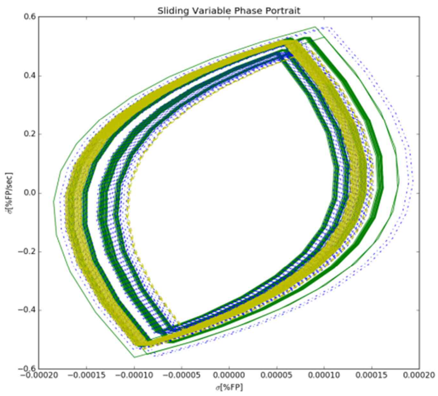

Figure 4. Phase Portrait: ISMC

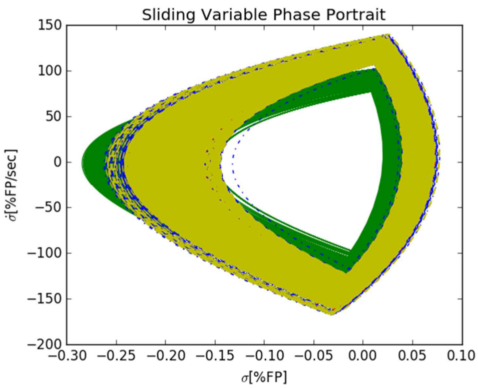

Figure 5. Phase Portrait: SMC

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