

Validation of Two-dimensional Natural Convection using CUPID code in Application with SFR Properties

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1. Introduction

CUPID (Component Unstructured Program for Interfacial Dynamics) code is a multi-dimensional thermal-hydraulics analysis code for Pressurized Water Reactors (PWRs). Up till now CUPID has been used for light water (H_2O) thermal hydraulic issues. For extending the use of CUPID, properties of various fluids have to be included and develop correlation models of different fluids (i.e. sodium and light water).

In the present study, sodium properties are applied to CUPID and the validation of two-dimensional natural convection problems. Natural convection has been widely researched in many engineering areas. Also, it is an important phenomenon for the development of safety systems in the nuclear field. The natural convection analysis by CUPID simulation results are compared with De Vahl Davis' (1986) solution. The analysis comparison parameters include Nusselt number, temperature and velocity fields. In the future, CUPID hopes to perform safety analysis for Sodium Fast Reactors (SFRs).

2. Methods and Results

For applying sodium properties, data of MARS-LMR are implemented on this study.

2.1 Verification of properties application.

Before the analysis of natural convection, the SFR properties calculated by CUPID must be verified. Density, temperature, and saturation temperature of each phase are a function of pressure and internal energy in CUPID. This relationship can be presented as:

$$\rho_g = \rho_g(P, e_g, X_n) \quad (1)$$

$$\rho_l = \rho_l(P, e_l) \quad (2)$$

$$T_v = T_v(P, e_g, X_n) \quad (3)$$

$$T_l = T_l(P, e_l) \quad (4)$$

$$T^s = T^s(P_s) \quad (5)$$

In other words, density and temperature are calculated by given pressure and internal energy, as shown in Eqs. (1) to (5) [1]. Also, the thermal conductivity, viscosity, and surface tension are necessary for calculations. These properties are function of temperature. Therefore,

exact estimation of temperature is important for calculating accurate properties. Therefore, density and temperature are calculated for verifying that the properties of sodium are in accordance with given pressures and internal energies.

The melting point of sodium is 371K and boiling point is 1156K at 1 bar. Generally, operating temperatures of SFRs are 700~800K, which means that sodium exists as a sub-cooled state during operation. This is due to sodium having a very high boiling point. Therefore, this study calculates the temperature and density of sodium at a sub-cooled liquid state along the given pressure and internal energy, depending on Eqs. (2) and (4). Obtained values are compared with sodium properties data of Argon National Lab (ANL) [2] for verification of application.

Figure 1 and Table 1 shows the densities of CUPID and ANL data, with respective to computed temperatures. The CUPID computed densities and ANL data are in good agreement and values are within 1% error. However, two density values deviate each other when temperatures reach 2000K, and errors are around 3%. Despite of these differences, 2000K is much higher than typical operation temperatures. So, comparing results at 2000K is not important. Therefore, CUPID confirms that sodium properties are implemented well for operating conditions of SFRs.

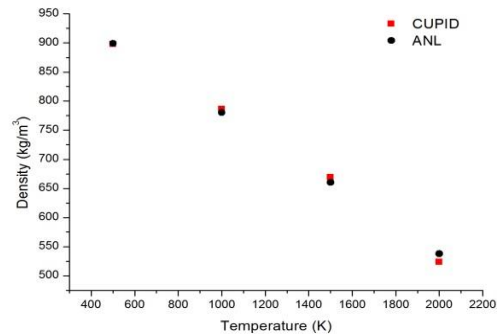


Fig. 1. Comparison of density of CUPID and ANL

Table. 1. Density data of CUPID and ANL [2]

Pressure	Temperature	Density		Error(%)
		CUPID	ANL	
10MPa	500K	898.04	899.2	0.129004
	1000K	786.28	780.3	0.76637
	1500K	669.38	660.4	1.36022
	2000K	523.67	538.0	2.663569

2.2 Natural convection outline

Natural convection is encountered in many thermal engineering applications. In the nuclear field, natural convection is important to the emergency core cooling of a reactor, passive safety systems, and post-accident scenarios in the lower plenum for SFRs [3]. In this study, CUPID performs natural convection analysis of liquid sodium in a two-dimensional square cavity (Aspect ratio=1). Vertical walls are maintained at an isothermal condition where the left wall is high temperature (T_h) and right wall is low temperature (T_c). Horizontal walls are at an adiabatic condition [4]. The liquid sodium fills the square cavity and all of properties inside are calculated from the reference temperature (T_m). Figure 2 shows a schematic drawing of natural convection in a cavity with relevant notation and boundary conditions.

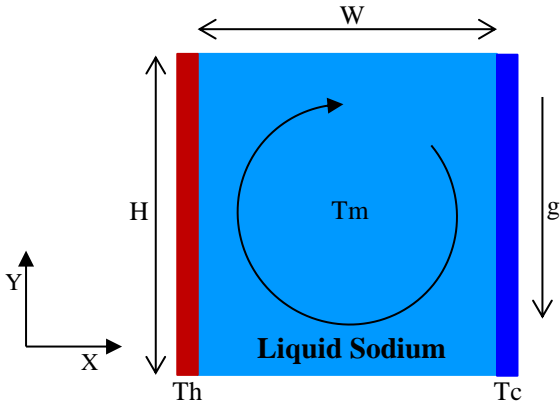


Fig. 2. Schematic drawing of natural convection in a cavity with relevant notation and boundary conditions

De Vahl Davis (1986) had previously studied natural convection phenomenon in an air filled square cavity. The results show Nusselt number (Nu) according to Rayleigh number (Ra) up to 10^6 , when the value of Prandtl number (Pr) equals to 0.71. The Nusselt number is being used as a criteria on natural convection for comparing validation results, because the Nusselt number is an important dimensionless number for deciding if the flow is more active convection or conduction. So, for checking validation, results of CUPID are compared with this analysis.

2.3 Effects of dimensionless number

The initial condition is that T_h is 400.5K and T_c is

399.5K. The reference temperature of sodium is given as 400K, and all of the used properties for calculating Ra and Pr value are taken on this temperature. The computed geometry is 2-D square cavity that has 40×40 and 60×60 meshes. Figure 3 shows the geometry shape of 2-D square cavity for two cases. Left side is 40×40 meshes, right side forms 60×60 meshes. Keeping a temperature difference of 1K, the length of cavity changes while Rayleigh number is set to 10^3 , 10^4 , 10^5 and 10^6 .

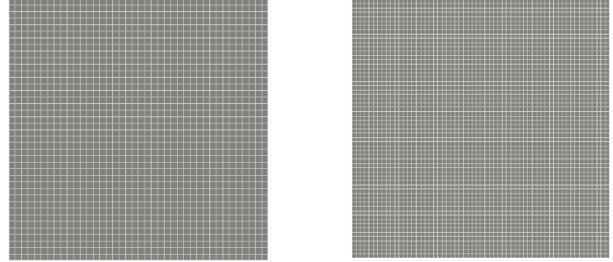


Fig. 3. 40×40 meshes (left side) and 60×60 meshes (right side) of 2-D square cavity

Sodium has a very low Pr value, because the thermal conductivity of sodium is higher than that of normal liquids such as water and air. Pr is represented by Eq. (9). Because of large thermal conductivity, Pr becomes low value. Owing to this reason, Pr value is much lower than 0.71. But, Pr is one of the important factors affecting the natural convection. So, it has to be fixed for adjusting solution condition. The governing equations for incompressible sodium are consisted of a mass, momentum, and energy conservation equations. For confirming a relationship between Ra (or Pr) and natural convection, the momentum and energy conservation equations can be converted into dimensionless equations [5]. These equations in two dimensions are described as follows:

-mass conservation

$$\frac{D\rho}{Dt} = \nabla \cdot \mathbf{u} = 0, \quad (6)$$

-momentum conservation

$$\begin{aligned} \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\frac{\partial P}{\partial X} + Pr \nabla^2 U \\ \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial Y} + Pr \nabla^2 V + Ra Pr T, \end{aligned} \quad (7)$$

-energy conservation

$$\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \nabla^2 T, \quad (8)$$

where U, V is dimensionless velocity in X and Y direction, and X, Y is dimensionless length.

Pr and Ra are defined as:

-Prandtl number

$$Pr = \frac{\mu c_p}{k}, \quad (9)$$

-Rayleigh number

$$Ra = \frac{\rho^2 g \beta (T_h - T_c) H^3}{\mu^2} Pr. \quad (10)$$

Equation (7) shows that natural convection is dependent on Pr and Ra . So, for analyzing natural convection and comparing with solution of De Vahl Davis, Pr value has to be matched to 0.71. Therefore, it is necessary to multiply a factor to make Pr value 0.71. Three properties: viscosity, thermal conductivity and heat capacity can be chosen for revising Pr . In present study, thermal conductivity (k) are amended. Accordingly, thermal conductivity is multiplied by a conversion factor (0.0132281) in the case of $T_m=400K$.

2.4 Result of natural convection analysis.

The Nusselt number is the ratio of convective to conductive heat transfer at the boundary within a fluid. Typically, the Nusselt number for natural convection is expressed as a function of Ra and Pr as:

$$Nu = f(Ra, Pr). \quad (11)$$

As mentioned earlier, Ra and Pr are significant parameters in convection. Therefore, Nusselt number is also mainly used for determining how the convection affects fluid flows. So, this study compares Nu values on right wall (hot wall), according to the Rayleigh number. When the temperature field reached to a converged state, the local and mean Nusselt numbers are calculated on the hot wall.

Local Nusselt number is defined as:

$$Nu = -\frac{H}{\Delta T} \left(\frac{\partial T}{\partial x} \right)_h, \quad (12)$$

where H is total length, ΔT is 1K.

And, mean Nusselt number can be obtained from:

$$\overline{Nu} = \frac{1}{H} \int_0^H Nu dy. \quad (13)$$

Table. 2. Comparison of Nusselt number of CUPID and De Vahl Davis on hot wall [5]

	$Ra=10^3$	$Ra=10^4$	$Ra=10^5$	$Ra=10^6$
	Hot wall (right side wall)			
CUPID (40 × 40)	1.128 (0.93)*	2.268 (1.11)*	4.672 (3.38)*	9.668 (9.86)*
CUPID (60 × 60)	1.125 (0.60)*	2.258 (0.66)*	4.559 (1.78)*	9.246 (5.07)*
De Vahl Davis (FDM, 40 × 40)	1.118	2.243	4.519	8.8

* : error(%)

For analysis of natural convection by CUPID, the calculation time takes 300 seconds for convergence of both temperature and velocity fields. In Table 2, a comparison is given between the present results and results by De Vahl Davis. It is found that Nusselt

numbers from CUPID are in good agreement with the reference properly. This means that CUPID successfully predicts the temperature gradient and field. However, in the larger Ra case, uncertainty in the Nusselt number is bigger than other cases. The big Ra leads to the transition from laminar flow to turbulent flow. Until now, turbulent flow is hard to predict exactly. Owing to this reason, even the result by De Vahl Davis cannot be determined accurate at $Ra=10^6$. That is why errors increase over rising Ra values. Though it is natural results, 60×60 meshes are better than 40×40 meshes. Further error differences between 40×40 meshes and 60×60 meshes increase with the rise of Ra .

Temperature fields of Fig. 4 show how the dominant heat transfer mechanism changes as Ra increases. For low Ra conditions, heat is mainly transferred by conduction between hot and cold walls. As the temperature fields depart from both ends of wall, the heat transfer mechanism changes from conduction to convection. However, for high Ra values, convection is dominant and affects the formation of the temperature field and fluid flow. Figure 5 shows the velocity vectors at different Ra values. For low values of Ra , one central vortex is shown as the dominative characteristic of the flow. As increasing Ra , the vortex tends to be an oval shape and is broken up to two vortices when Ra becomes 10^5 [4]. The predicted temperature and velocity fields on CUPID compared with solutions of De Vahl Davis are in good agreement.

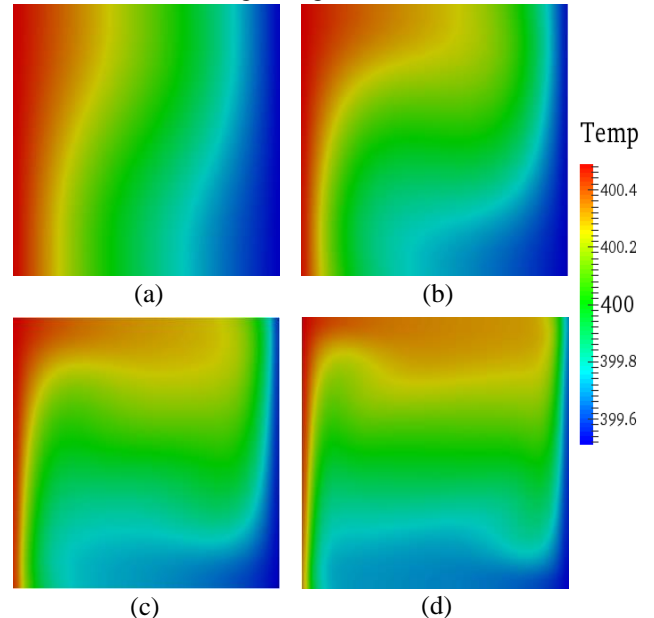


Fig. 4. Temperature fields at different Ra -values: (a) 10^3 , (b) 10^4 , (c) 10^5 , (d) 10^6

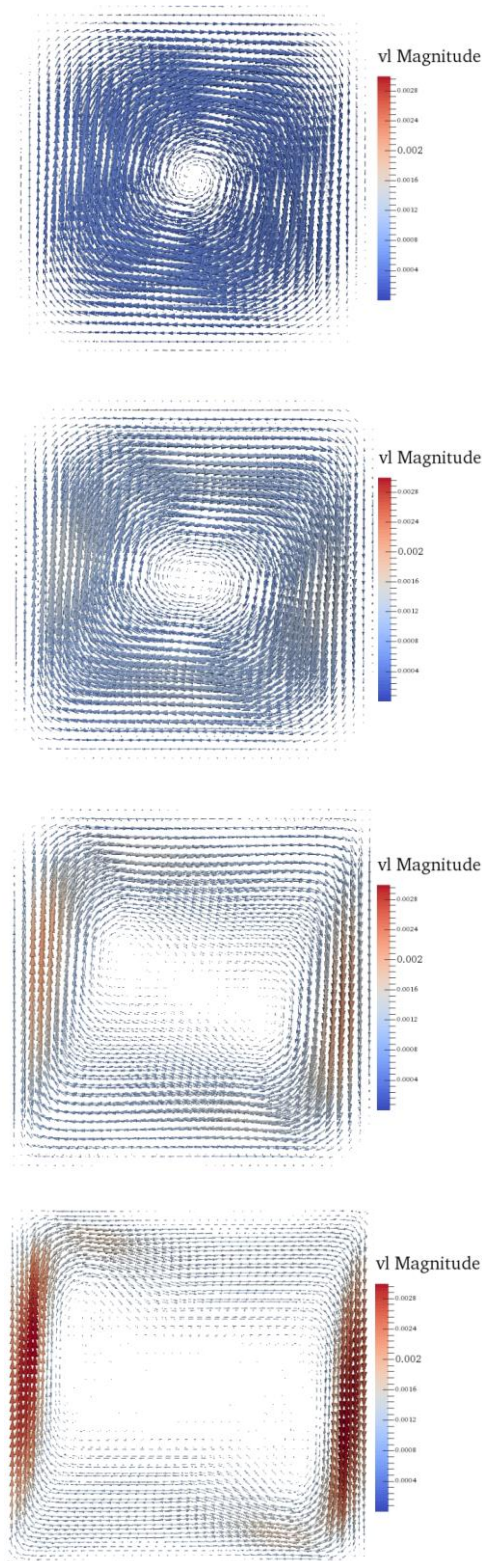


Fig. 5. Velocity vectors at different Ra -values: (a) 10^3 , (b) 10^4 , (c) 10^5 , (d) 10^6

3. Conclusions

For extended application of CUPID on SFR studies, sodium properties are implemented in the code. The

verification results show the expectation that CUPID can apply to not only light water, but also sodium. Furthermore, natural convection phenomenon is quite accurately predicted. However, sodium and light water have some different heat transfer correlation or model. So, these kinds of correlations have to be modified for exact analysis on SFRs and for validation of various examples in the future work.

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