Study on the most applicable critical flow models in the current system codes

Tran Thanh Tram [a], Yeon-Sik Kim [a, b], and Hyun-Sik Park [a, b*]
[a] *University of Science and Technology, 217 Gajeong-ro, Yuseong-gu, Daejeon, 34113, Korea*
[b] *Korea Atomic Energy Research Institute, 111-989 Deadeokdaero, Yuseong, Daejeon, 34057, Korea.*
[*]*Corresponding author: hspark@kaeri.re.kr*

1. Introduction

Choking condition or critical flow can occur when liquid, gas or mixture leaks from a system at a high pressure to the ambient at a lower pressure through a break. It could happen through a break of safety valves or safety injection lines during LOCA. When this condition could be reached, it means that the maximum possible discharge flow through the broken exit occurs and the flow rate becomes independent from the downstream pressure.

Among many models for critical flow nowadays, Henry-Fauske (HF) and Trapp-Ransom (TR) are two main applicable models for critical flow calculation in the current safety analysis codes. However, as the application of those models is different, these models should be used carefully by users. In this paper, we try to make clear the applicability of these models, reproduce them using MATLAB. Our reproduced models for HF and TR show a good prediction on the selected database of experimental data.

2. Reproducing HF and TR models

In this section, the description of HF and TR models will be given first in detail. The technical notes on these models will be provided in the last part of this section.

2.1 HF model

This model was suggested in Henry-Fauske [1] for a non-equilibrium model, using two continuity and one momentum equations without considering the wall shear stress and heat exchange, for one component flow only (water–vapor system) with some following approximations:

- Phase velocities are identical (k=1)
- Mass transfer in the expansion is negligible
- Thermal equilibrium exists
- The expansion is isentropic, constant entropy (s_g=s_l)
- The liquid temperature keeps unchanged.
- Polytropic expansion of vapor at the exit, the equation of gas is treated as the ideal gas.
- Critical mass flow rate reaches a maximum value concerning the throat pressure, $dG/dp_t = 0$.

Using these approximations, with the polytropic exponent, n, reflects the heat transfer rate at the throat,

$$n = \frac{\frac{(1-x)c_{pft}}{c_{pgt}} + 1}{\frac{(1-x)c_{pft}}{c_{pgt}} + \frac{1}{\gamma}},$$

the flow rate can be determined as follows [1]:

$$G_{HF}^2 = \left[\frac{x_0 v_{gt}}{nP} + (v_{gt} - v_{f0})\left\{\frac{(1-x_0)N}{s_{gt} - s_{ft}}\frac{ds_{ft}}{dP} - \frac{x_0 c_{Pg}\left(\frac{1}{n} - \frac{1}{\gamma}\right)}{P(s_{g0} - s_{f0})}\right\}\right]^{-1} \tag{2.1.1}$$

Where N = min{ x_t/0.14,1} is an experimental parameter which represents the partial phase change at the throat. By integrating the momentum equation from the stagnant (subscript 0) to the throat (subscript t) locations, we can obtain:

$$(1-x_0)v_{f0}(P_0 - P_t) + \frac{x_0\gamma}{\gamma - 1}(P_0 v_{g0} - P_t v_{gt}) = \frac{\left[(1-x_0)v_{f0} + x_0 v_{gt}\right]^2}{2} G_{HF}^2 \tag{2.1.2}$$

Substitution eq. (2.1.1) into the eq. (2.1.2) moreover, rearrange Eq. (2.1.2) we can get the compact form as follows:

$$\eta = \left[\frac{\frac{(1-\alpha_0)}{\alpha_0}(1-\eta) + \frac{\gamma}{\gamma - 1}}{\frac{1}{2\beta\alpha_t^2} + \frac{\gamma}{\gamma - 1}}\right]^{\frac{\gamma}{\gamma - 1}} \tag{2.1.3}$$

Where γ is isentropic exponent and η is critical pressure ratio:

$$\eta = \frac{P_t}{P_0} \tag{2.1.4}$$

$$\beta = \frac{1}{\eta} + \left(1 - \frac{v_{f0}}{v_{gt}}\right)\left(\frac{(1-x_0)NP_t}{x_0(s_{gt} - s_{ft})}\frac{ds_{ft}}{dP}\right) - \frac{c_{pg}\left(\frac{1}{n} - \frac{1}{\gamma}\right)}{(s_{g0} - s_{f0})} \tag{2.1.5}$$

With the upstream and throat void fractions are determined as:

$$\alpha_0 = \frac{x_0 v_{g0}}{(1-x_0)v_{f0} + x_0 v_{g0}}$$

$$\alpha_t = \frac{x_0 v_{gt}}{(1-x_0)v_{f0} + x_0 v_{gt}}$$

The specific volume of vapor at the throat is: $v_{gt} = v_{g0}(\eta)^{-\frac{1}{\gamma}}$

For a given stagnant conditions of pressure, P_0, and quality, x_0, by iteration until the η values in two Eqs. (2.1.3) and (2.1.4) are convergent, the critical pressure, P_t, can be obtained. Then the critical mass flow rate, G_{HF}, can be easily calculated.

2.2 TR model

This model was suggested by Trap and Ransom [2] for non-homogenous, non-equilibrium system using two continuity, two momentum, and two energy equations, for two-component system (air-water) with some assumptions as bellow:

- Mass is exchanged between phases
- No heat transfer occurred between the phases thermal equilibrium

- Phase temperature is different

Solving those six equations using the first order, quasi-linear, partial differential approach by finding the roots of characteristic equation: det(Aλ-B)=0. The real part of a root is the velocity of propagation. And the complex part of a root is the growth or attenuation. The main root for characteristic equation which gives the choked-flow criterion is [2]:

$$\lambda = u + D(u_G - u_L) \pm a \qquad (2.2.1)$$

where u_G and u_L are the phasic velocities. D and a are parameters are determined in the reference [2]. The choked criterion will exist if the signal propagating with maximum velocity relative to the fluid is stationary:

$$\lambda = 0 \ \text{ or } u + D(u_G - u_L) = \pm a \qquad (2.2.2)$$

The eq. (2.2.2) can be written in term of relative Mach numbers using new variables $M_v = \frac{u}{a}, M_r = \frac{D(u_G - u_L)}{a}$:

$$M_v + M_r = \pm 1 \qquad (2.2.3)$$

In their model, the virtual mass, C, is chosen to assure smooth transition between pure vapor and pure liquid. C equals 0, and 0.5 for separated and dispersed flows, respectively. K is a function of state properties which evaluated the non-equilibrium state of the mixture. K equals 0 and 1 for frozen and thermal equilibrium states.

2.3 Notes on HF and TR models

Nowadays, huge work on critical flow, both experimentally and theoretically [3, 4], is ongoing to improve the prediction capability of the safety analysis codes. Many models and correlations [3, 4] for critical flow are developed. Among them, TR and HF are two popular models in system codes such as MARS-KS [5], TRACE [6], RELAP5 [7], and CATHARE.

It is important to realize that each critical flow model has applicable limits because of the model approximations. For the HF model, it is only applicable for one component steam-water system. Furthermore, based on the assumption of HF model, it could be applied to a well-mixed condition, thermal equilibrium. This model was evaluated using the high-pressure experimental data. So in the authors' opinion, HF model could predict well for one component system at high pressure. TR model, however, is applied to the two-component system, air-water. Together with this, to predict the critical mass flow, HF model uses the upstream conditions, while TR model bases on the throat conditions. These are the major differences between TR and HF models that the authors would like to remind when users use them.

3. Reproducing and comparison of HF and TR models

Using the MATLAB program, we reproduce HF and TR models using the equations given in the original papers.

3.1 HF model comparison

Comparisons have been made for one component steam-water. The critical mass flux data versus stagnation quality from Henry and Fauske [1] at high pressure is given as follows:

Figs 1 and 2 show the comparisons between reproducing HF model and the experimental data at high pressure [1]. The HF model predicts well the experimental data.

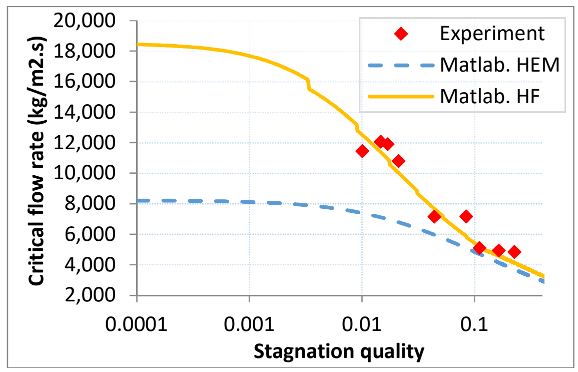

Fig. 1. Comparison of reproducing HF model with experimental data at 1.38 MPa (200 psia) [1].

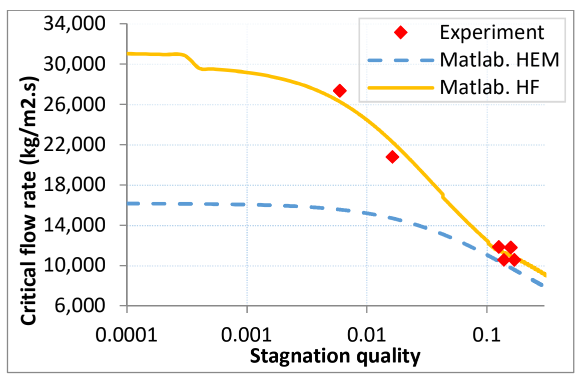

Fig. 2. Comparison of reproducing HF model with experimental data at 3.45 MPa (500 psia) [1].

3.2 TR model comparison

The TR model was first compared with the model prediction given in the original paper. This calculation is for a steam-water system at 7.5 MPa. Sound speed is a function of virtual mass, C, and thermal non-equilibrium property, k. The Mach number coefficient and sound speed predictions of reproducing HF model give similar results in comparing with those of the original paper as shown in Figs. 3 and 4.

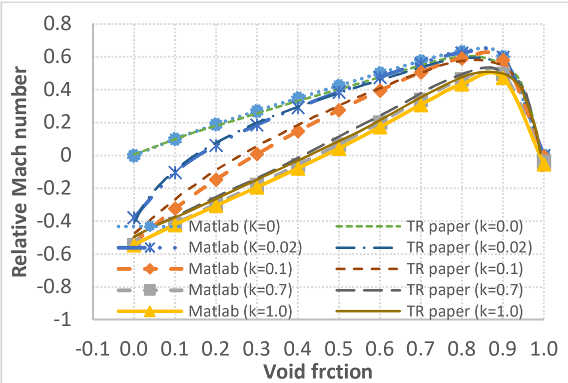

Fig. 3. Comparison of Mach number between the MATLAB and original calculations in case of C =0.5 and k varying.

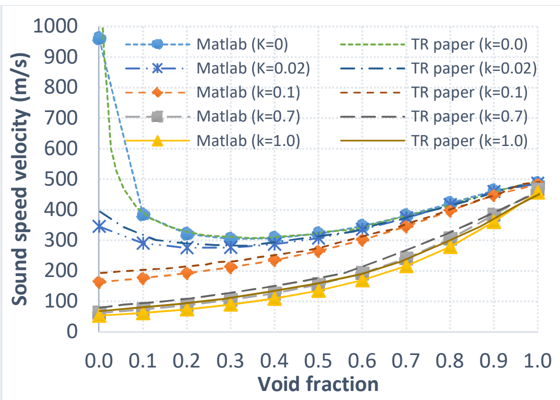

Fig. 4. Comparison of sound speed between the MATLAS and original calculations in case of C =0.5 and K varying.

The similar results were obtained for those models in case of C equals to 0 or infinity. Then more comprehensive comparisons have been done to evaluate the reproducing HF model. The steam-water velocity versus void fraction data from Karplus [9] and Henry [8] at low pressure is given as follows:

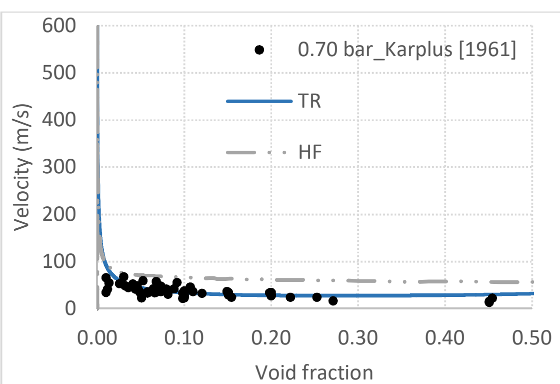

Fig. 5. Comparison of velocity prediction of TR and HF models using MATLAB with the steam water data from Karplus [9].

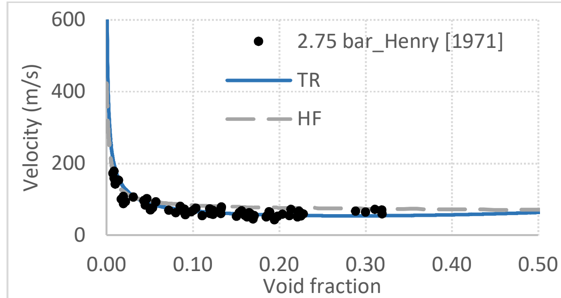

Fig. 6 Comparison of velocity prediction of TR and HF models using MATLAB with the steam water data from Henry [8].

From Figs. 5 and 6, we can see that TR model gives a better prediction compared with HF model. When the pressure increases, it seems to be that the prediction capability of two models tends to be closer.

4. Discussion and Conclusions

Based on MATLAB programming, the HF and TR models have been reproduced. While the HF model is applied to the one-component steam-water system, TR could be used for both one and two component systems. Furthermore, while HF model uses the up steam conditions to predict the throat pressure and the critical mass flow rate, TR model calculates them using the throat conditions.

The results given by MATLAB programming of both HF and TR models have been evaluated by comparing with the experimental data. From these calculations, we can conclude that HF model could predict well for one component system at high pressure and that TR model could predict well for the system around the atmospheric pressure. Because of the complexity of the thermal-hydraulic system, TR model is the most complex model which is closer to the real system. Therefore, it may be the most powerful but the most difficult model to use for critical flow prediction up to now.

REFERENCES

[1] R.E.Henry and H. K. Fauske, The two-phase critical flow of one-component mixtures in nozzles, orifices and short tubes, 1971.
[2] Trap and V.H. Ransom, A choked-flow calculation criterion for nonhomogeneous, non-equilibrium, two-phase flows, 1982.
[3] E. Elias and G. S. Lelleuche, Two-phase critical flow, Int. J. Multiphase flow, Vol. 20, pp 91-168, 1994.
[4] F. D'Auria et al., Critical modeling in nuclear safety, 1982.
[5] MARS manual Vol. 5, "Models and correlations", 2009.
[6] TRACE, V5.0, Theory manual, "Field Equations, solution methods, and physical models", 2010.
[7] RELAP5/MOD3.3 code manual Vol. 4, "Models and correlations", 2006.
[8] Henry, Pressure pulse propagation in two-phase one and two-component mixtures, 1971.
[9] Karplus, Propagation of Pressure Waves in a Mixture of Water and Stream, 1961.