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# A Comprehensive Review of Laser-Compton Scattering Process Based on the Klein-Nishina Cross-section

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## 1. Introduction

The high intensity and energy tunable gamma rays sources are being widely used in various researches including medical [1], biological [2], and industrial fields [3]. In addition to the aforementioned research areas the gamma rays have massive utilization in nuclear industry. The scope of utilization is very broad covering various aspects of nuclear industry including nuclear medicine, nuclear materials, nuclear safety & security and nuclear waste transmutation [4].

These high intensity gamma rays can be generated from various methods including laser-induced bremsstrahlung (LIB) and conventional bremsstrahlung. However, the processes lack energy tunablity and have low yield of γ-rays in the energy regions to be useful for some of the nuclear applications. Presently, synchrotron radiation (SR) sources satisfies the required brilliance and tunablity. Though, the limitations of SRs are high cost, large size, and low energies of X/γ-rays [5].

One another method of desired γ-rays are linac based laser-Compton scattering (LCS) process. Currently, the photon flux from such sources is modest up to the level of $10^{13}$ γ/s. However, with the advancements in the field of high repetition rate lasers and innovative concepts, it is expected that photon flux will increase to a level of $10^{18}$-$10^{20}$ γ/s in near future. Therefore, LCS process has significant importance in the future generation light sources.

## 2. Laser-Compton Scattering Process

In this section, a comprehensive review of LCS process, brief description of Klein-Nishina differential scattering cross-section and detailed kinematic study (based on energy and angle) of LCS process is presented. The LCS process was originally proposed by Milburn and Arutyunyan [6, 7]. In LCS, low energy photons (on the order of a few eV) are scattered from an energetic electron beam to produce highly energetic gamma rays. Figure 1 further describes the concept of the LCS process.

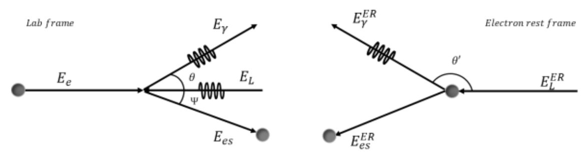


Fig. 1. Laser-Compton scattering (LCS) process.

### *2.1 LCS Photons energy and Scattering angle*

The scattering angle can be transformed from laboratory frame of reference to the electron rest frame of reference or vice versa using Lorentz transformation.

$$\cos\theta' = \frac{\cos\theta - \beta}{1-\beta\cos\theta}, \tag{1}$$

where θ′ and θ are the backscattering angles in electron rest frame and laboratory frame of references. The energy of the scattered photon in the laboratory frame of reference is given by the Eq. (2):

$$E_\gamma = \frac{(1+\beta)\,\mathrm{E}_L}{1-\beta\cos\theta + \frac{E_L}{mc^2}\sqrt{1-\beta^2}\,(1+\cos\theta)}, \tag{2}$$

where $E_\gamma$ is the scattered photon energy in laboratory frame of reference, θ is the scattering angle in laboratory rest frame of reference, $E_L$ is the incident laser photon energy, and β is the ratio of electron velocity to the speed of light and is given by Eq. (3):

$$\beta = \frac{\sqrt{E_e(E_e+2mc^2)}}{E_e+mc^2}, \tag{3}$$

where $E_e$ is the electron kinetic energy and $mc^2$ is the electron rest mass energy i.e. 0.511 MeV. Figures 2 and 3 show the energy distribution of the scattered photons in terms of ER and lab frame of references for the incident laser energy of 1.165 eV.

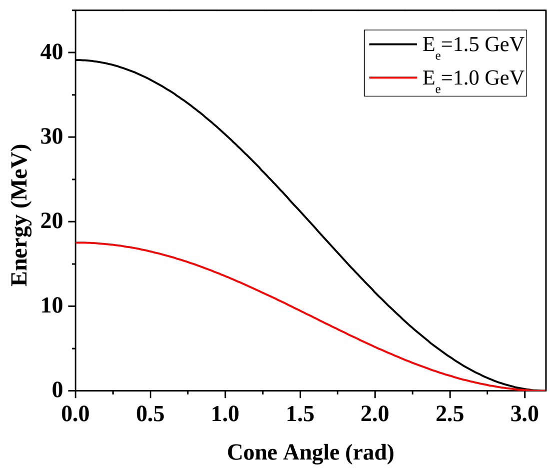


Fig. 2. Scattered γ-rays energies in ER frame.

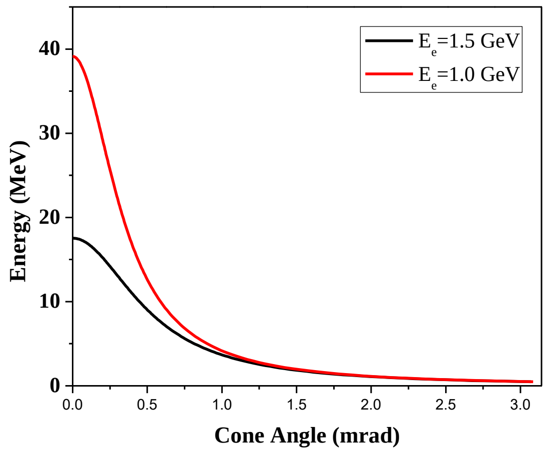


Fig. 3. Scattered γ-rays energies in lab frame.

It is to be noted that most of the energetic γ-rays fall in to a very narrow backscattering angle in case of lab frame in comparison with ER frame of reference. Figure 4 explains the reason based on the relationship given in Eq. (1).

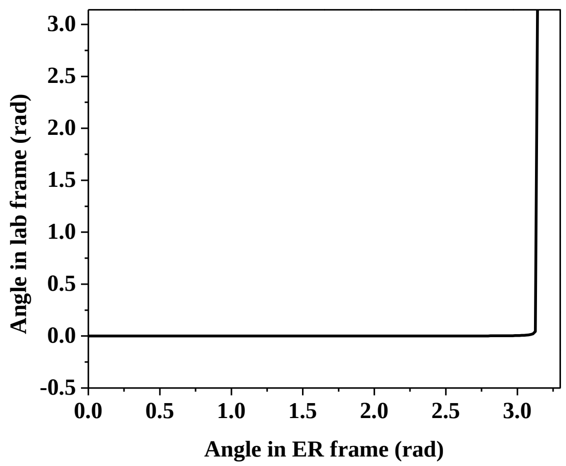


Fig. 4. Relationship between ER and lab frame angles.

This clearly shows that even for a very broad ER frame of reference angle the corresponding angle in laboratory frame of reference is very narrow and this is the reason, that despite very broad angular distribution of scattered gamma rays energy in ER frame, the corresponding laboratory frame of reference distribution is quite narrow.

*2.2 Energy based Kinematics of LCS photons*

The dependence of the scattered γ-rays energy on the backscattering angle for various incident laser and electron energies are shown in Figs. 5 and 6.

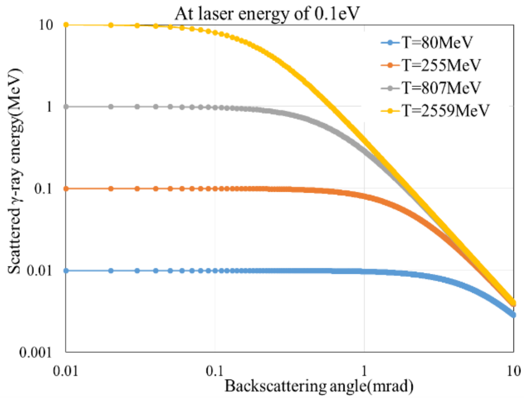


Fig. 5. Scattered γ-rays energy as function scattering angle at a laser energy of 0.1 eV.

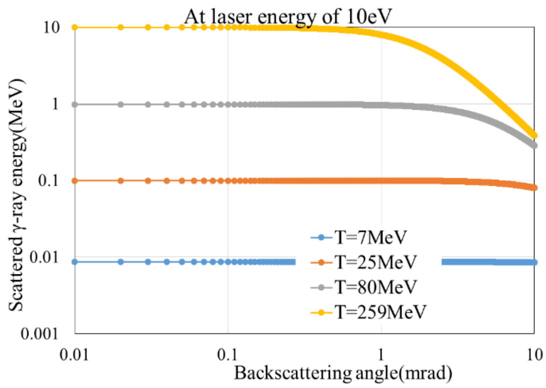


Fig. 6. Scattered γ-rays energy as function scattering angle at a laser energy of 10 eV.

It is evident that the scattered gamma-ray energies in all cases are flat up to certain backscattering angles until rapid decrease. It can also be observed that largest backscattering angle in the flat γ-ray energy domain increases as the electron energy decreases. Apparently, there is no relation between the largest backscattering angle in the flat γ-ray energy domain and laser energy. However, an important conclusion can be drawn from the above analysis is that, higher laser energies in combination with relatively lower electron energies are better in terms of keeping the scattered photon energies constant for relatively broad scattering angles.

*2.3 Angle based Kinematics of LCS photons*

The angular distribution of LCS photons can simply be expressed by the well-known Klein-Nishina (KN) cross-section that is used to describe the conventional Compton scattering process. In this section brief derivation of KN cross-section and then the characteristics of LCS photons based on this cross-section are presented.

The general Compton scattering process can be expressed by the following two lowest order Feynman diagrams. Table I gives the nomenclature of the terms used in Fig. 7.

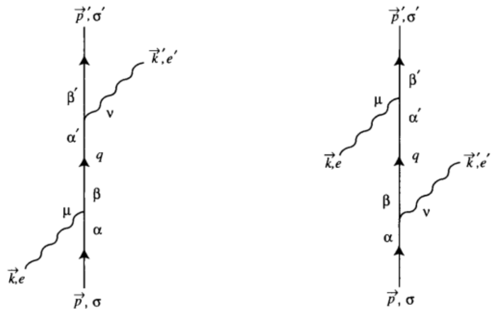


Fig. 7. Lowest order Feynman diagrams for Compton scattering process.

Table I. Nomenclature.

| | |
|---|---|
| $k^{\mu}$, $k'^{\mu}$ | Initial and final photon momenta. |
| $e^{\mu}$, $e'^{\mu}$ | Initial and final photon polarization vector. |
| $p^{\mu}$, $p'^{\mu}$ | Initial and final electron momenta. |
| $\sigma^{\mu}$, $\sigma'^{\mu}$ | Initial and final electron spin z-components. |
| $\alpha,\beta$ | Four component Dirac indices. |

Based on the rules of the quantum electrodynamics (QED) the scattering matrix for these Feynman diagrams can be given by Eq. (4):

$$S(\mathbf{p},\sigma+\mathbf{k},e\rightarrow \mathbf{p}',\sigma'+\mathbf{k}',e')=$$
$$\frac{\bar{u}(\mathbf{p}',\sigma')_{\beta'}}{(2\pi)^{3/2}}\frac{e_{\nu}^{*'}}{(2\pi)^{3/2}.\sqrt{2k^{0'}}}\frac{u(\mathbf{p},\sigma)_{\alpha}}{(2\pi)^{3/2}}\frac{e_{\mu}}{(2\pi)^{3/2}.\sqrt{2k^{0}}}$$
$$\times\int d^4q\left[\frac{-i}{(2\pi)^4}\right]\left[\frac{-i\,\not{q}+m}{q^2+m^2-i\varepsilon}\right]_{\alpha'\beta} \tag{4}$$
$$\times\left\{\begin{array}{l}\left[e(2\pi)^4\gamma^{\nu}_{\beta'\alpha'}\delta^4(q-p'-k')\right]\\ \left[e(2\pi)^4\gamma^{\mu}_{\beta\alpha}\delta^4(q-p-k)\right]\\ +\left[e(2\pi)^4\gamma^{\mu}_{\beta'\alpha'}\delta^4(q+k-p')\right]\\ \left[e(2\pi)^4\gamma^{\nu}_{\beta\alpha}\delta^4(q+k'-p)\right]\end{array}\right\}$$

where $k^0 = |\mathbf{k}|$, $k'^0 = |\mathbf{k}'|$, $p^0=\sqrt{\mathbf{p}^2+m^2}$, $p'^0=\sqrt{\mathbf{p}'^2+m^2}$, μ and υ are space-time labels. After performing trivial q-integration and some simple simplification the S-matrix can be written as Eq. (5):

$$S=-2\pi i\delta^4(p'+k'-p-k)M, \tag{5}$$

where $M$ is the Feynman amplitude and is given by:

$$M=\frac{e^2}{4(2\pi)^3\sqrt{k^0k'^0}}\bar{u}(\mathbf{p}',\sigma')\left\{\begin{array}{l}\not{e}'^{*}\left[-i(\not{p}+\not{k})+m\right]\not{e}/p.k\\ -\not{e}\left[-i(\not{p}-\not{k}')+m\right]\not{e}'^{*}/p.k'\end{array}\right\}u(\mathbf{p},\sigma)$$

Therefore, the differential cross-section in terms of $M$ can be represented by Eq. (6):

$$d\sigma=(2\pi)^4u^{-1}|M|^2\,\delta^4(p'+k'-p-k)d^3p'd^3k' \tag{6}$$

As one of the particles is massless and initial velocity is gives $u=|p\cdot k|/p^0k^0$ . For simplicity, electron in rest frame of reference is considered with $\mathbf{p}$=0 and $p^0$=m. The velocity will then simply be equal to u =1. Further we denote initial and final photon energies as E and E' which can be defined as $E=k^0=|\mathbf{k}|=-p.k/m, E'=k^{0'}=|\mathbf{k}'|=-p.k'/m.$ Also, $\mathbf{p}'=\mathbf{k}-\mathbf{k}'$ the resulting three momenta delta function can be rewritten after $d^3p'$ integration as:

$$\delta(p'^0+k'^0-p^0-k^0)=\delta(\sqrt{(k-k')^2+m^2}+E'-m-E) \tag{7}$$

The integral $d^3k'$ can be written as $d^3k'=E'^2dE'd\boldsymbol{\Omega}$. The integration over $dE'$ can be eliminated by the delta function given in Eq. (7). The rest of the differential scattering cross section can be given by Eq. (8):

$$d\sigma=(2\pi)^4|M|^2\frac{p'^0E'^3}{mE}d\boldsymbol{\Omega} \tag{8}$$

Eq. (8) is simplified using complicated QED rules and tracer properties to give the final expression of Compton scattering KN cross section given in Eq. (9):

$$\frac{d\sigma}{sin\theta d\theta}=\pi r_0^2\frac{(1-\beta^2)}{(1-\beta cos\theta)^2}R^2\times\left(R+\frac{1}{R}-1+\cos^2\theta'\right), \tag{9}$$

where $r_0$ is the classical electron radius (2.818 fm), $\beta$ is the ratio of the electron velocity to the speed of light, $R$ is the ratio of the scattered gamma ray energy to incident laser energy in the electron rest frame of reference, $\theta$ is the backscattering angle in the laboratory frame of reference, and $\theta'$ is the backscattering angle in the electron rest frame of reference. The total Compton scattering cross-section can be calculated using Eq. (10):

$$\sigma(\theta_c)=\int_0^{\theta_c}\frac{d\sigma}{d\theta}d\theta. \tag{10}$$

Figure 8 shows the variation of the Compton scattering cross-section with the change in electron energy for various laser energies with a fixed cone angle of 1 mrad.

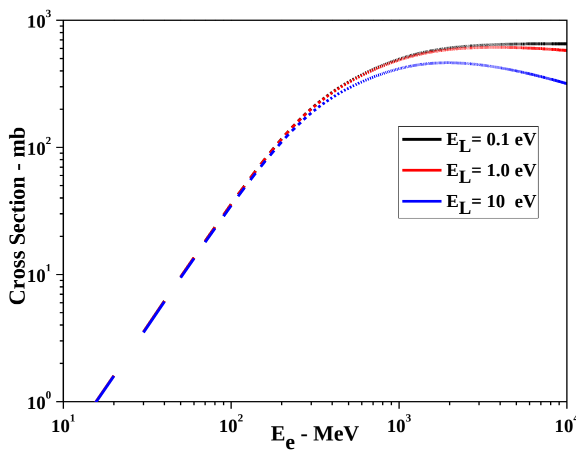


Fig. 8. Compton cross-section variations with the electron energy for various laser energies.

It can be seen, for laser beam incident energies up to the reasonable value of 10 eV and electron energies of up to 1 GeV, the Compton scattering cross-section increases sharply. For electron energies after that, the cross-section becomes saturated for low laser beam incident energies (i.e. 0.1 eV and 1 eV) and decreases significantly for higher energies (i.e. 10 eV).

In Section 2.2, the higher energies of incident laser beams are recommended with the suitable electron energies to keep the scattered gamma ray energy constant throughout the scattering angle. However, in case of head on collision the scattering cone angle is very narrow, hence broad energy distribution is not a big issue. On the other hand, the increase in laser energy can significantly reduce the Compton scattering cross-section as shown in Fig. 8. This may limit the increase in laser energy to some optimum value in practical studies.

## 3. Effects of Energy and Angle based Kinematics on LCS Photon Flux

The total LCS photon flux is calculated by the Eq. (11) as follows:

$$N_{\gamma}\,(\gamma/\mathrm{s}) = \frac{f\,(Hz)\,N_e\,N_p\,\sigma(\mathrm{cm}^2)}{A(cm^2)}, \qquad (11)$$

where $f$ is the collision frequency in Hz, $N_e$ is the total number of electrons in an electron bunch, $N_p$ is the total number of photons in a single photon pulse, $\sigma$ is the total Compton scattering cross- section that was previously introduced, and $A$ is the effective collision area. The effective collision area depends on the RMS (root means square) size of the laser power density profile (for an electron beam of the same RMS size). Figure 9 shows the change in the photon flux or gamma ray intensity with the change in laser energy.

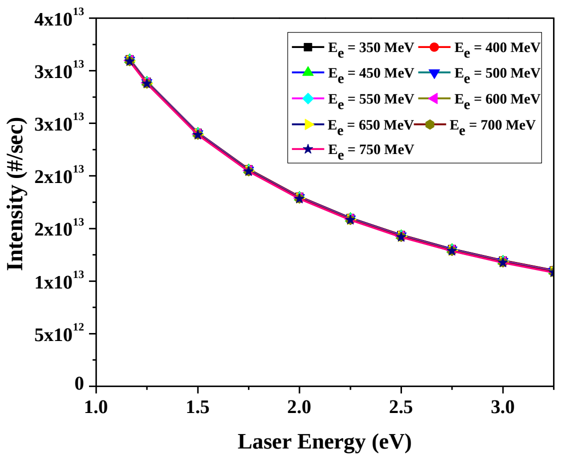


Fig. 9. Gamma ray intensity as a function of laser energy for various electron energies.

It clearly shows that the electron energy has very less impact on the total gamma ray intensity but the increase in laser energy reduces the total gamma ray intensity significantly.

## 4. Conclusions

The paper presents a comprehensive review of laser-Compton scattering (LCS) process in terms of the energy of scattered LCS photons and total LCS flux. The parametric study based on the Klein-Nishina Compton scattering cross-section shows that the laser energy should be increase to an optimum level such that the scattered gamma rays energy remain constant within suitable narrow cone angle and also must not reduce the total gamma ray intensity. In view of the study presented in this paper, detailed optimization of the various LCS facility related parameters can be performed to achieve the desirable LCS gamma ray spectrum and intensity for an intended application.

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