Development of a Monte Carlo Deflation Method without Weight Cancellation for the First Order Eigenfunction Calculation

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1. Introduction

In the Monte Carlo (MC) eigenvalue calculations, the higher order eigenvalues and eigenfunctions can be useful to evaluate nuclear criticality safety, accelerate the fission source convergence and estimate the real variance.

There have been several studies [1–4] to obtain the higher order eigenfunctions in the MC power iteration method [5]. Booth [2] proposed a modified power iteration method that simultaneously determines the dominant and subdominant eigenvalues and eigenfunctions. Zhang et al. [4] proposed a general solution strategy which extends Booth's modified power iteration method, and it is applied to continuous energy Monte Carlo simulation.

Hotelling's deflation method [6] is a well–known technique to calculate the higher order eigenfunctions which removes lower order components from the fission source distribution by adjoint solution. However there are two obstacles in implementing this method in the MC neutron transport simulation – adjoint flux calculation and pointwise subtraction of lower order components. Recently, a method to estimate adjoint flux during the MC forward calculations was developed and successfully applied to kinetic parameter estimations and sensitivity and uncertainty analyses. To overcome the pointwise subtraction issue, Booth [1] and Yamamoto [3] proposed a point detector procedure and mesh based deflation algorithm, respectively.

The objective of this paper is to present a MC deflation algorithm without the source weight cancellation by scoring deflation responses in the fundamental mode eigenvalue calculation.

2. Methodology

2.1 Hotelling's Deflation Method

In Hotelling's deflation method, the first order eigenfunction is calculated by removing the fundamental mode component as

$$S_1 = \frac{1}{k_1}\left[\mathbf{H}S_1 - \mathbf{H}S_0 \cdot \frac{\langle \phi_0^\dagger, S_1 \rangle}{\langle \phi_0^\dagger, S_0 \rangle} \right], \qquad (1)$$

where S_0 is the fundamental mode eigenfunction, S_1 the first order eigenfunction, k_1 the first order eigenvalue and $\phi_0^\dagger$ the fundamental mode adjoint flux. Fission operator $\mathbf{H}$ is defined by [7]

$$\mathbf{H} = \mathbf{F}\mathbf{T}^{-1} , \qquad (2)$$

where operator $\mathbf{F}$ and $\mathbf{T}$ are fission production operator and net loss operator, respectively. The bracket in Eq. (1) indicates integration over volume and neutron energy. Also $\mathbf{H}S_N$ in Eq. (1) can be written as

$$\mathbf{H}S_N = \int H(\mathbf{r}' \to \mathbf{r}) S_N(\mathbf{r}') d\mathbf{r}', \quad N = 0,1; \qquad (3)$$

where $H(\mathbf{r}' \to \mathbf{r})$ means the number of first-generation fission neutrons born per unit phase space volume about $\mathbf{r}$ due to a parent neutron located at $\mathbf{r}'$.

However, pointwise subtraction in Eq. (1) is difficult to conduct without some techniques such as point detector procedure or discretization of space and energy since eigenfunctions are represented by point particles in MC power iteration method.

2.2 MC Deflation Method without source weight cancellation

In order to estimate the first order eigenfunction in the middle of iterative updates of S_0, Eq. (1) is rewritten as

$$S_1(\mathbf{r}) = \frac{1}{k_1}\left[\int d\mathbf{r}' H(\mathbf{r}' \to \mathbf{r}) \frac{S_1(\mathbf{r}')}{S_0(\mathbf{r}')} S_0(\mathbf{r}') \right.$$
$$\left. - \frac{\langle \phi_0^\dagger, S_1 \rangle}{\langle \phi_0^\dagger, S_0 \rangle} \int d\mathbf{r}' H(\mathbf{r}' \to \mathbf{r}) S_0(\mathbf{r}') \right]$$
$$= \frac{1}{k_1}\left[\int d\mathbf{r}' H(\mathbf{r}' \to \mathbf{r}) \left(\frac{S_1(\mathbf{r}')}{S_0(\mathbf{r}')} - \frac{\langle \phi_0^\dagger, S_1 \rangle}{\langle \phi_0^\dagger, S_0 \rangle} \right) S_0(\mathbf{r}') \right]$$

$$(4)$$

By applying the power iteration method to Eq. (4), the first order fission source is updated cycle–by–cycle as

$$S_1^{(m)}(\mathbf{r}) = \frac{1}{k_1^{(m-1)}} \left[\int d\mathbf{r}' H(\mathbf{r}' \to \mathbf{r}) \left(\frac{S_1^{(m-1)}(\mathbf{r}')}{S_0^{(m-1)}(\mathbf{r}')} - \frac{\left\langle \phi_0^\dagger, S_1^{(m-1)} \right\rangle}{\left\langle \phi_0^\dagger, S_0^{(m-1)} \right\rangle} \right) S_0^{(m-1)}(\mathbf{r}') \right] \tag{5}$$

where $S_1^{(m)}(\mathbf{r})$ is the first order fission source located at $\mathbf{r}$ in m–th cycle.

The fundamental mode adjoint solution $\phi_0^\dagger$ is calculated by scoring the fission neutrons after sufficient number of generations. In this study, 10 is chosen as the waiting generation. For tallying deflation response, a variable to store the weight of the first order source is introduced where the fundamental mode source is located. Then, the first order fission source for m–th cycle can be obtained by Eq. (5).

3. Numerical Result

A verification of the method is performed by a one–dimensional two–group test problem [3], then the calculated first order fission source distribution is compared with reference solution from fission matrix method. Fig. 1 and Table I show the configuration and two-group parameters of the problem, respectively.
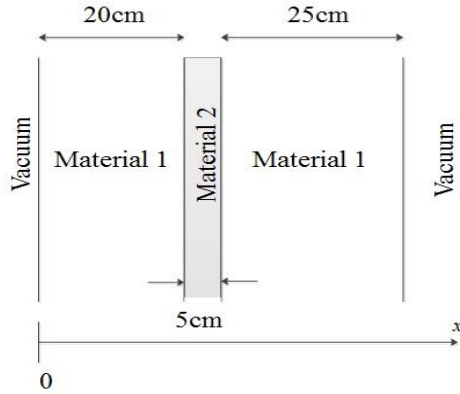
Table I: Two-group parameters for test problem.

Cross section	Material 1	Material 2
Σ_t^1 (cm^{-1})	0.20	0.20
Σ_t^2 (cm^{-1})	0.40	0.40
Σ_a^1 (cm^{-1})	0.07	0.05
Σ_a^2 (cm^{-1})	0.12	0.05
Σ_f^1 (cm^{-1})	0.02	0.00
Σ_f^1 (cm^{-1})	0.07	0.00
$\Sigma_s^{1\to1}$ (cm^{-1})	0.06565	0.07575
$\Sigma_s^{1\to2}$ (cm^{-1})	0.06435	0.07425
$\Sigma_s^{2\to1}$ (cm^{-1})	0.00280	0.00350
$\Sigma_s^{2\to2}$ (cm^{-1})	0.27720	0.34650
ν	2.50	-
χ^1	1.00	-
χ^2	0.00	-

The number of histories per cycle is set to 1 million. The geometry of the problem is equally divided into 200 tally regions. The initial weight of the first order fission source is negative (-1) on the left side and positive (+1) on the right side. Fig.2 shows the cycle–by–cycle first order fission source distributions.

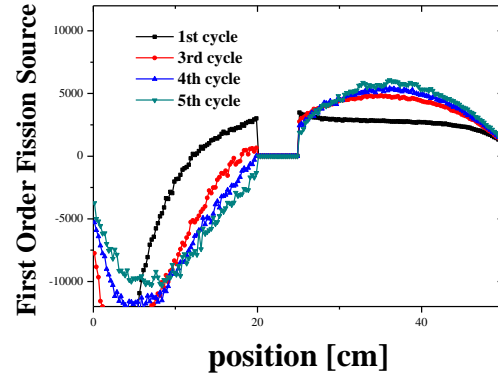
Fig. 1. One–dimensional slab for test problem.

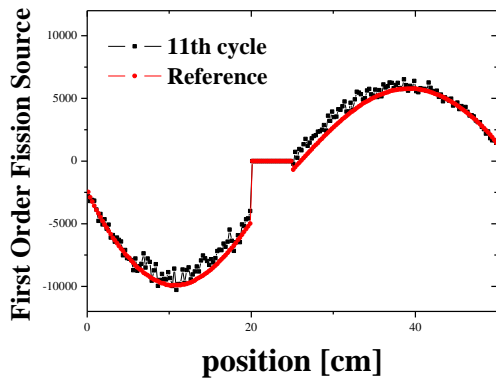Fig. 2. Cycle–by–cycle first order fission source distributions.

Fig. 3. The first order fission source distribution compared with reference source distribution.

Fig. 3 shows the first order fission source distribution of 11−th cycle compared with reference solution. One can see that the first order fission source is converged to the reference source distribution.

4. Conclusions & Future Work

In this study, the MC deflation method without source weight cancellation for the first order eigenfunction calculation is developed and verified by the one-dimensional two-group problem, which shows good agreement with the reference solution from fission matrix method. In the method, source weight cancellation which is necessary for the previous studies is not needed by tallying deflation response in the fundamental mode eigenvalue calculation.

The preliminary results are encouraging, but more research for reliability of this method will be done.

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