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Development of a New Point Kinetic Equation with α-Adjoint Weighted Kinetics Parameters

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1. Introduction

Nowadays the accelerator-driven subcritical system (ADS) has been widely studied as a candidate of transmutation reactor.[1] Applications of the conventional point kinetics equation (PKE) [2] using the *k*-adjoint weighted kinetics parameters can be invalid for the time-dependent ADS analysis because it assumes that the reference system is critical.[3,4] In order to increase the accuracy of the point kinetics analysis for an ADS, Gandini and Salvatores [3] suggested a PKE using an importance function associated with the relative power level in a subcritical system and Nishihara et al. [4] proposed a PKE using kinetics parameters weighted by Green's function [5].

In this paper, we propose a new PKE with kinetics parameters weighted by the α-adjoint flux, solution to the adjoint α-mode eigenvalue equation, because the α-mode eigenvalue equation can accurately represent an off-critical system. In addition, algorithms to calculate the α-adjoint weighted kinetics parameters in the Monte Carlo (MC) α iteration method [6] are presented and tested in an infinite homogeneous 2-group problem.

2. Methods

2.1 New point kinetic equation

The time-dependent neutron transport equation and the delayed neutron precursor density equation can be expressed as

$$\frac{1}{v}\frac{\partial\phi}{\partial t} = -\mathbf{L}\phi + \mathbf{F}_p\phi + \sum_i \lambda_i c_i + Q \,, \tag{1}$$

$$\frac{\partial c_i}{\partial t} = \beta_i \mathbf{F}\phi - \lambda_i c_i \,; \tag{2}$$

$$\mathbf{L}\phi = \left[\mathbf{\Omega}\cdot\nabla + \Sigma_t(\mathbf{r},E,t)\right]\phi(\mathbf{r},E,\mathbf{\Omega},t) - \int dE'\int d\mathbf{\Omega}'\Sigma_s(E',\mathbf{\Omega}'\to E,\mathbf{\Omega}\,|\,\mathbf{r},t)\phi(\mathbf{r},E',\mathbf{\Omega}',t), \tag{3}$$

$$\mathbf{F}_p\phi = \int dE'\int d\mathbf{\Omega}'\frac{\chi_p(E)}{4\pi}\nu_p(E')\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',\mathbf{\Omega}',t), \tag{4}$$

$$\mathbf{F}\phi = \int dE'\int d\mathbf{\Omega}'\frac{\chi(E)}{4\pi}\nu(E')\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',\mathbf{\Omega}',t). \tag{5}$$

$c_i(\mathbf{r},E,\mathbf{\Omega},t)$ is defined as $\frac{\chi_i}{4\pi}C_i(\mathbf{r},t)$ where $C_i(\mathbf{r},t)$ denotes the delayed neutron precursor density of group *i*. Other notations follow standard.

For the further derivation, the *i*-th delayed neutron production operator, $\mathbf{F}_i$ is defined by

$$\mathbf{F}_i\phi = \int dE'\int d\mathbf{\Omega}'\frac{\chi_i(E)}{4\pi}\nu(E')\Sigma_f(\mathbf{r},E',t)\phi(\mathbf{r},E',\mathbf{\Omega}',t), \tag{6}$$

where $\chi_i(E)$ is the fission spectrum for the *i*-th precursor group.

The adjoint form of the α-mode eigenvalue equation for a reference state of the subcritical system can be expressed as [2]

$$\frac{\alpha_0}{v}\phi_0^\dagger = -\mathbf{L}_0^\dagger\phi_0^\dagger + \mathbf{F}_0^\dagger\phi_0^\dagger \,; \tag{7}$$

$$\mathbf{L}_0^\dagger\phi_0^\dagger = \left[-\mathbf{\Omega}\cdot\nabla + \Sigma_{t0}(\mathbf{r},E)\right]\phi_0^\dagger(\mathbf{r},E,\mathbf{\Omega}) - \int dE'\int d\mathbf{\Omega}'\Sigma_{s0}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}'\,|\,\mathbf{r})\phi_0^\dagger(\mathbf{r},E',\mathbf{\Omega}'), \tag{8}$$

$$\mathbf{F}_0^\dagger\phi_0^\dagger = \int dE'\int d\mathbf{\Omega}'\frac{\chi(E')}{4\pi}\nu(E)\Sigma_{f0}(\mathbf{r},E)\phi_0^\dagger(\mathbf{r},E',\mathbf{\Omega}'), \tag{9}$$

where $\phi_0^\dagger$ denotes the α-adjoint flux and the subscript "0" indicates the reference state of the subcritical system.

By multiplying Eq. (1) by $\phi_0^\dagger$ and Eq. (7) by ϕ, subtracting the resulting equations and integrating it over $(\mathbf{r},E,\mathbf{\Omega})$, one can obtain

$$\left\langle \phi_0^\dagger, \frac{1}{v}\frac{\partial\phi}{\partial t}\right\rangle - \left\langle \phi_0^\dagger, \frac{\alpha_0}{v}\phi\right\rangle = -\left\langle\phi_0^\dagger,\mathbf{L}\phi\right\rangle + \left\langle\phi_0^\dagger,\mathbf{F}_p\phi\right\rangle + \left\langle\phi_0^\dagger,\sum_i\lambda_i c_i\right\rangle + \left\langle\phi_0^\dagger,Q\right\rangle + \left\langle\phi,\mathbf{L}_0^\dagger\phi_0^\dagger\right\rangle - \left\langle\phi,\mathbf{F}_0^\dagger\phi_0^\dagger\right\rangle. \tag{10}$$

Now let us separate the angular flux ϕ into the amplitude function $P(t)$ and the shape function $\psi(\mathbf{r},E,\mathbf{\Omega},t)$ as

$$\phi(\mathbf{r},E,\mathbf{\Omega},t) = P(t)\cdot\psi(\mathbf{r},E,\mathbf{\Omega},t). \tag{11}$$

Then insertions of Eq. (11) into the left hand side (LHS) of Eq. (10) give

$$\begin{aligned}&\left\langle \phi_0^\dagger, \frac{1}{v}\frac{\partial \phi}{\partial t}\right\rangle - \left\langle \phi_0^\dagger, \frac{\alpha_0}{v}\phi\right\rangle \\ &= \frac{dP}{dt}\cdot\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle + P\cdot\frac{\partial}{\partial t}\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle - P\cdot\alpha_0\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle \\ &= \frac{dP}{dt}\cdot\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle - P\cdot\alpha_0\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle,\end{aligned} \quad (12)$$

where the following normalization condition is applied to the second equality:

$$\frac{\partial}{\partial t}\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle = 0\,. \quad (13)$$

By inserting $\left\langle \phi_0^\dagger, \sum_i \beta_i \mathbf{F}_i \phi\right\rangle - \left\langle \phi_0^\dagger, \sum_i \beta_i \mathbf{F}_i \phi\right\rangle$ in the right hand side (RHS) of Eq. (10) and using Eqs. (10) and (12), a new $\phi_0^\dagger$ weighted PKE is obtained as

$$\begin{aligned}\frac{dP(t)}{dt} &= \left[\Delta\alpha(t) - \frac{\beta_{eff}(t)}{\Lambda_{eff}(t)}\right]P(t) + \alpha_0 P(t) \\ &\quad + \sum_i \lambda_i C_i(t) + Q(t);\end{aligned} \quad (14)$$

$$\Delta\alpha(t) \equiv \frac{1}{\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle}\left[-\left\langle \phi_0^\dagger, (\mathbf{L}-\mathbf{L}_0)\psi\right\rangle + \left\langle \phi_0^\dagger, (\mathbf{F}-\mathbf{F}_0)\psi\right\rangle\right], \quad (15)$$

$$\beta_{eff}(t) \equiv \sum_i \beta_{eff,i}(t), \quad (16)$$

$$\beta_{eff,i}(t) \equiv \left\langle \phi_0^\dagger, \beta_i \mathbf{F}_i \psi\right\rangle \Big/ \left\langle \phi_0^\dagger, \mathbf{F}\psi\right\rangle, \quad (17)$$

$$\Lambda_{eff} \equiv \left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle \Big/ \left\langle \phi_0^\dagger, \mathbf{F}\psi\right\rangle, \quad (18)$$

$$C_i(t) \equiv \left\langle \phi^\dagger, c_i\right\rangle \Big/ \left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle, \quad (19)$$

$$Q(t) \equiv \left\langle \phi_0^\dagger, Q\right\rangle \Big/ \left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle. \quad (20)$$

Note that the $\Delta\alpha(t)$ expression of Eq. (15) can be obtained by applying the perturbation theory for α-eigenvalue. [2]

2.2 Calculation of the α-adjoint weighted kinetics parameters

In the α iteration method [6] for a subcritical system, the time source distribution S_t is updated iteration-by-iteration as

$$S_t^{(i)} = -\alpha^{(i)}\mathbf{R}S_t^{(i-1)}; \quad (21)$$

$$\alpha^{(i)} = \frac{1}{\int d\mathbf{r}\int dE\int d\mathbf{\Omega}\mathbf{R}S_t^{(i-1)}}, \quad (22)$$

$$\mathbf{R}S_t = \frac{1}{v(E)}\mathbf{L}^{-1}(1-\mathbf{F}\mathbf{L}^{-1})S_t\,. \quad (23)$$

Then $\phi_0^\dagger$ implies the number of time sources produced in the *n*-th iteration due to a unit time source neutron located at $(\mathbf{r},E,\mathbf{\Omega})$ it as *n* approaches infinity. The time source probability after *n* iterations from a source of energy group *g* at iteration *i-n*, denoted by $\phi_{g,i}^{\dagger,n}$, can be estimated as

$$\phi_{g,i}^{\dagger,n} = \frac{1}{M_g^{i-n}}\sum_{j=i}^{M^i}\sum_{k=1}^{K^{ij}}\frac{w^{ijk}}{v^{ijk}}\frac{1}{\Sigma_t^{ijk}}, \quad (24)$$

where *j* and *k* are history and collision indices, respectively.

Then $\left\langle \phi_0^\dagger, \beta_i \mathbf{F}_i \psi\right\rangle$, $\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle$, and $\left\langle \phi_0^\dagger, \mathbf{F}\psi\right\rangle$ in Eqs. (17) and (18) can be calculated at iteration *i* by

$$\left\langle \phi_0^\dagger, \beta_i F_i \psi\right\rangle = \frac{1}{M_i}\sum_{j=1}^{M^i}\sum_{k=1}^{K^{ij}} w^{ijk}\frac{\nu_d \Sigma_f^{ijk}}{\Sigma_t^{ijk}}\phi_{g,i-1}^{\dagger,n}, \quad (25)$$

$$\left\langle \phi_0^\dagger, \frac{\psi}{v}\right\rangle = \frac{1}{M_i}\sum_{j=1}^{M^i}\sum_{k=1}^{K^{ij}}\frac{w^{ijk}}{v^{ijk}}\frac{1}{\Sigma_t^{ijk}}\phi_{g'',i-1}^{\dagger,n}, \quad (26)$$

$$\left\langle \phi_0^\dagger, \mathbf{F}\psi\right\rangle = \frac{1}{M_i}\sum_{j=1}^{M^i}\sum_{k=1}^{K^{ij}} w^{ijk}\frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}}\phi_{g',i-1}^{\dagger,n}. \quad (27)$$

The index *g* in Eq.(25) and the index g' in Eq.(27) are energy groups of delayed fission neutron and fission neutron, which is generated from *k*-collision, respectively. The index g'' in Eq.(26) indicates the energy group which the neutron has before the *k*-th collision.

3. Numerical Results

The proposed method to calculate the α-adjoint weighted kinetics parameters is verified for an infinite homogeneous problem characterized by two-group cross sections given in Table 1.

The MC a iteration calculation is conducted with 1000 active iterations on 100,000 sources per iteration. Table 2 shows the comparison of MC estimates of the α-adjoint weighted kinetic parameters and α-eigenvalue with their analytic solutions. From the table, one can see that the MC estimates agree well with the references within their 95% confidence intervals.

Table 1. 2-group cross-section for the infinite homogeneous problem

Cross section	First group (g=1)	Second group (g=2)
Σ_{tg}	0.500	0.500
Σ_{fg}	0.025	0.175
ν_{pg}	2.000	2.000
Σ_{sgg}	0.100	0.200
$\Sigma_{sg'g}(g\neq g')$	0.024372	0.000
χ_1	0.800	0.500
χ_2	0.200	0.500
$\chi_{d,1}$	0.800	0.800
$\chi_{d,2}$	0.200	0.200
β_0	0.006	0.006
$1/v_g$[sec/cm]	2.28626×10^{-10}	1.29329×10^{-6}

Table 2. α adjoint -weighted kinetics parameters for the infinite homogeneous problem

Parameter	Ref	mean	RSD [%]	Error [%]
α	83448.3	83438.7	0.008	-0.011
β_{eff}	3.16578E-03	3.17000E-03	0.079	0.133
Λ	6.52997E-06	6.52687E-06	0.038	-0.048

4. Conclusions

A new point kinetic equation using the α-adjoint weighted kinetics parameters is derived by making the best of a fact that the subcritical system can be accurately represented by the α-mode eigenvalue equation, rather than the *k*-mode eigenvalue equation. Algorithms to calculate the required kinetics parameters are suggested for the MC α iteration calculations. Application results of the proposed PKE for a subcritical system will be presented.

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