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A Study on CMFD Truncation of Monte Carlo Neutron Transport Solution

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1. Introduction

The stochastic Monte Carlo (MC) method is one of many ways to solve the neutron transport equation to analyze nuclear reactors. This method makes it possible to describe the actual environment precisely in any complex geometry and it is considered as the most accurate method. However, the computational time involved in tracking every single particle and recording physical quantities is usually unacceptably long for practical reactor analysis. To reduce the computing time and statistical uncertainty in the MC calculations, a non-linear coarse mesh finite difference (CMFD) acceleration is popularly utilized in the MC calculation [1-3].

The CMFD method including the p-CMFD approach assists the MC calculation by quickly updating the fission source distribution (FSD). Regarding CMFD-related studies, most of researches have mainly focused on the stochastic MC results assisted by the associated CMFD analysis. In the conventional CMFD-coupled MC schemes, the deterministic CMFD results are only supplementary information for improving and accelerating the MC calculation. Taking into account the fact that the CMFD analysis itself provides a subset of solution to the original MC approach and it takes numerical advantages arising from its deterministic attributes, in this paper, we have investigated the feasibility of a CMFD truncation of the MC solution in terms of the solution estimation and variance reduction.

2. Methodologies

In this section, stochastic MC parameters are defined to characterize the numerical performance of each method. In addition, the basic idea of the CMFD truncation of the Monte Carlo solution is introduced.

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2.1 Stochastic MC parameters

In the MC simulation, the accuracy of the calculated results is estimated by evaluating stochastic parameters. The standard deviation obtained from a single MC simulation is usually underestimated due to the inter-cycle correlation, and it is called apparent standard deviation (ASD). Therefore, the real standard deviation (RESD) is calculated by considering a number of independent MC simulations with different random sequences.

Supposing that the MC calculation is performed with N number of batches (i.e., independent simulations), there are N reactor parameters and their associated stochastic uncertainties, respectively:

$$X: x_1, x_2, x_3, \cdots, x_N$$
$$\sigma: \sigma_1, \sigma_2, \sigma_3, \cdots, \sigma_N$$

where σ_i is the ASD of the i^{th} batch. Then ASD and RESD of the quantities in this calculation are obtained as follows [1]:

$$ASD: \sigma_a = \frac{1}{N}\sum_{i=1}^{N}\sigma_i \,, \tag{1.1}$$

$$RESD: \sigma_r = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}. \tag{1.2}$$

2.2 CMFD and p-CMFD truncation of the MC solution

In the conventional CMFD-coupled MC calculations, a CMFD or p-CMFD scheme repetitively solves a one-group neutron diffusion equation in each MC cycle. For the supplementary CMFD analysis in the original reactor problem, a CMFD node system should be predefined as in the standard FDM approximation to the neutron diffusion equation. Depending on the node system considered, the associated CMFD equation can be written in the following standard eigenvalue problem:

$$M\vec{\phi} = \frac{1}{k}F\vec{\phi} \tag{2.1}$$

where M is the migration operator, F is the fission source operator, $\vec{\phi}$ is a one-group neutron flux, and k is the neutron multiplication factor.

In Eq. (2.1), the system matrices M and F are dependent on the one-group cross sections estimated in the MC calculation and the CMFD parameters updated in each MC cycle. In the CMFD method, correction factors are determined such that the reference MC neutron current should be preserved at node interfaces. The neutron current is expressed in terms of the two neighboring node-average fluxes and the associated correction factor.

In the conventional CMFD, the net current is preserved as:

$$\vec{J}^{\,i+1/2}_{CMFD} = -\tilde{D}^{i+1/2}(\phi^{i+1} - \phi^{i}) + \hat{D}^{i+1/2}(\phi^{i+1} + \phi^{i}). \tag{2.2}$$

where $\tilde{D}^{i+1/2} = \dfrac{2d_i d_{i+1}}{d_i + d_{i+1}}$, $d_i = D_i / \Delta x$, D_i is the diffusion coefficient., and $\hat{D}$ is the correction factor determined by

$$\hat{D}_{CMFD} = \frac{\vec{J}^{\,i+1/2}_{MC} + \tilde{D}^{i+1/2}(\phi^{i+1}_{MC} - \phi^{i}_{MC})}{\phi^{i+1}_{MC} + \phi^{i}_{MC}}. \tag{2.3}$$

Meanwhile, in the p-CMFD scheme, both incoming and outgoing partial currents are preserved by introducing two correction factors on each interface:

$$\vec{J}^{\pm,i+1/2}_{pCMFD} = -\frac{1}{2}\tilde{D}^{i+1/2}(\phi^{i+1}-\phi^{i}) + \hat{D}^{\pm,i+1/2}\phi^{i+1/2\mp1/2}, \quad (2.4)$$

where $\hat{D}^{\pm}$ is the correction factor determined by

$$\hat{D}^{\pm}_{pCMFD} = \frac{\vec{J}^{\pm}_{MC} \pm 0.5\tilde{D}^{i+1/2}(\phi^{i+1}_{MC}-\phi^{i}_{MC})}{\phi^{i+1/2\mp1/2}_{MC}}. \quad (2.5)$$

Once the CMFD parameters are generated from the MC simulation, the deterministic eigenvalue problem in Eq. (3) is solved so as to calculate the eigenvalue (multiplication factor) and eigenvector (flux distribution) at every cycle. In this work, this CMFD solution is proposed as a deterministic truncation of the MC (DTMC) solution.

In Fig. 1, a flow chart for the CMFD-coupled MC calculation is described and the basic idea of the DTMC method is given again. In the conventional method, the deterministic CMFD results are utilized only for updating the fission source distribution (FSD) in the original stochastic MC analysis, not for the solution itself.

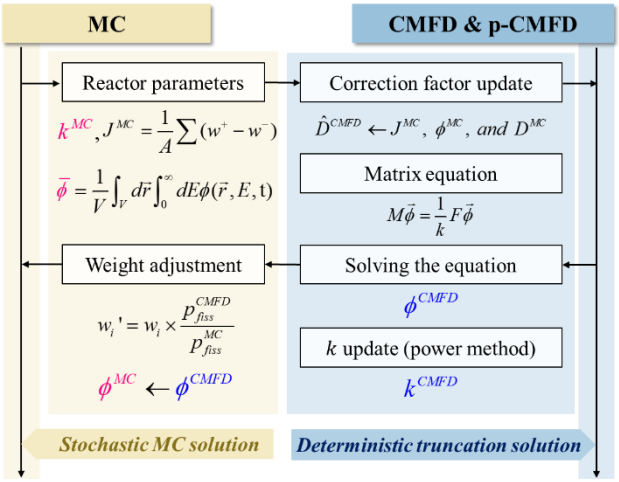

Fig. 1 Algorithm for CMFD in MC method

It is worthwhile to note that the deterministic CMFD results are also statistical samples capable of predicting the solution. The generalized equivalence theory confirms that the CMFD solutions should be equivalent to the MC ones in view of the eigenvalue and reaction rates in each CMFD node. One big advantage of the DTMC solution is that it provides both the reactor eigenvalue and detailed power profile in each MC cycle in the deterministic way and it is very cheap in terms of the computational costs.

The deterministic solution can be essentially calculated based on the MC reference parameters. As shown in Fig. 1, once the deterministic results are obtained, they are used to update the MC FSD in the conventional CMFD-coupled MC analysis. It is important to note that the DTMC solution can be obtained without adjusting the FSD using the CMFD solution, which is a one-way coupling for the DTMC solution and this approach is evaluated in the current paper. It is obvious that the DTMC solution is cycle-wise and subject to the stochastic uncertainties. The statistical quantities associated with the DTMC estimation are also evaluated in the standard way as in the MC calculation.

3. Numerical Results

Numerical tests were carried out to compare the performance of the stochastic MC and deterministic CMFD results, and assess the feasibility of the CMFD truncation method, the DTMC solution, with regard to the solution estimation and variance reduction. In this numerical tests, the unrodded C5G7 [4] benchmark problem was enlarged for a higher dominance ratio, as shown in Figs. 2 and 3. In this work, a multi-group in-house MC code has been used for the MC and CMFD calculations.

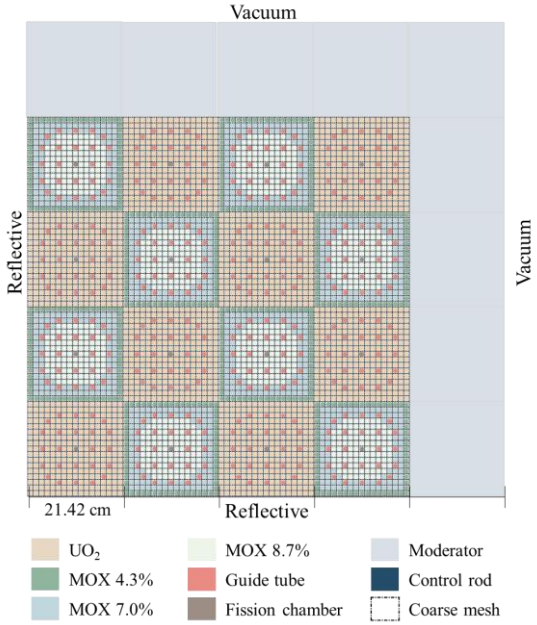

Fig. 2 Radial core configuration

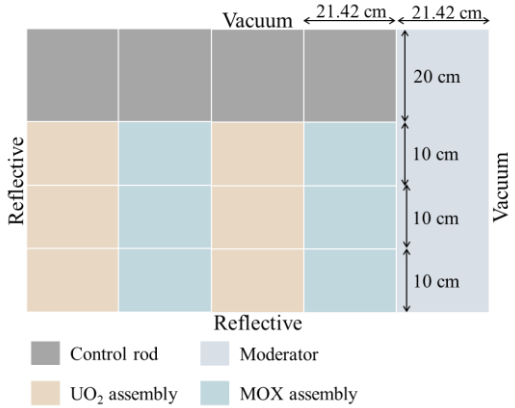

Fig. 3 Axial core configuration

The CMFD node is set to be equivalent to the fuel pin size because detailed local information should be obtained from the DTMC calculation for practical applications. Each simulation for comparison was simulated with 100 inactive cycles, 500 active cycles,

and 1,000,000 histories per cycle, while the reference solution was obtained with 1,000 active cycles. For evaluation of the RESD, 30 independent MC runs were simulated with different random seeds. In the CMFD calculation, ten successive cycles are accumulated for the estimation of the CMFD parameters.

In Table 1 and Figs 4-6, numerical results for the eigenvalue are compared in terms of mean value, ASD, and RESD for each case: standard MC, and DTMC (CMFD/p-CMFD). Note that the standard MC indicates stand-alone MC calculation without CMFD coupling in the current work. The DTMC means the deterministic solutions with CMFD or p-CMFD scheme applied in the MC simulation.

It should be noted that the DTMC multiplication factors show a good agreement to the reference within the stochastic uncertainties even from the 1st active MC cycle. It is important to note that the ASD of the DTMC is about 3~4 times lower than that of MC results. The RESD of the DTMC solutions are not decreased as low as in the ASD case, but it is still quite lower than that of MC results throughout the simulation (Figs. 4 and 5). This indicates that accurate solution can be obtained by the deterministic results with less number of active cycle compared to the MC results; thus, the corresponding computational cost can be decreased a lot.

Table 1. Comparison of ASD and RESD for k_{eff}

Method	Cycle	k_{eff}	ASD (pcm)	RESD (pcm)
Reference		1.152157	3.01	-
Standard MC	1	1.151972	-	87.8079
	10	1.152215	22.06	39.14
	500	1.152162	3.84	5.87
DTMC (CMFD)	1	1.152120	-	27.6861
	10	1.152144	4.53	17.50
	500	1.152152	1.01	3.43
DTMC (p-CMFD)	1	1.152178	-	20.3433
	10	1.152155	3.60	18.75
	500	1.152159	0.94	3.03

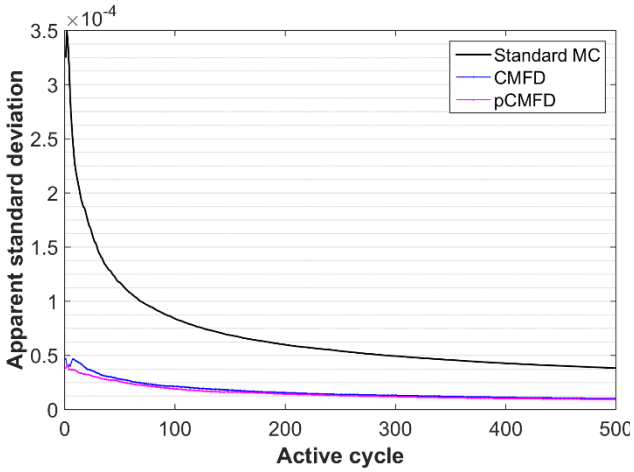

Fig. 4 Cumulative ASD of the multiplication factor

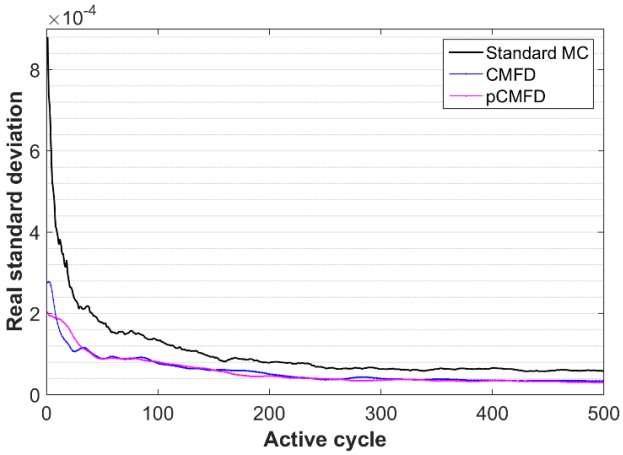

Fig. 5 Cumulative RESD of k_{eff}

As shown in Figs. 6.1 and 6.2, the DTMC eigenvalues are well within the acceptance range (1σ) of the reference solution even from the very initial cycle. It certainly demonstrates that the DTMC solution truncated at the early stage of the MC simulation could have a high accuracy and reliability to precisely estimate the DTMC solution.

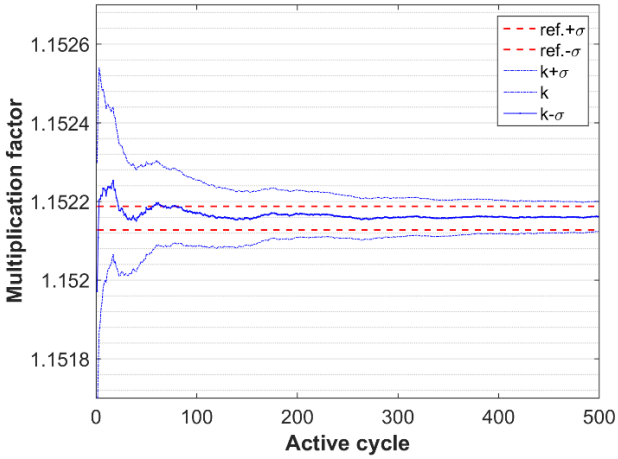

Fig. 6.1 Cumulative k_{eff} (MC) with the reference

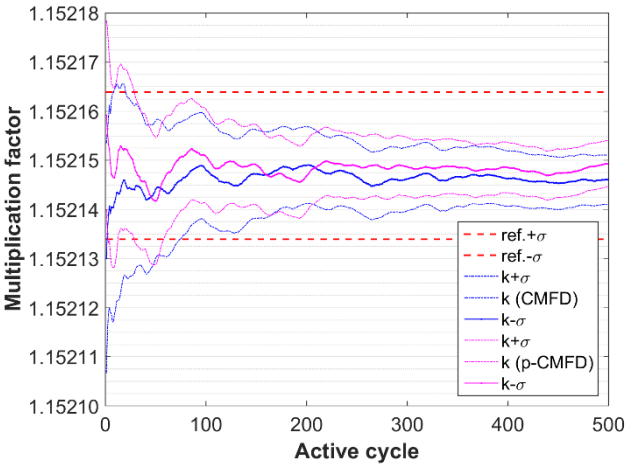

Fig. 6.2 Cumulative k_{eff} (DTMC) with the reference

Next, the pin-level power distribution was compared for each method. The pin-wise detailed stochastic error distributions are illustrated in Fig. 7, and the maximum and average values of both ASD and RESD are provided in Table 2. Similarly to the multiplication factor results, the ASD and RESD of the DTMC are smaller than or similar to those of the MC results.

In Figs. 8.1 and 8.2, a relative pin power at the specific position is compared to the 68% acceptance range of the reference solution. The specific pin is randomly chosen to be five pins to the right and five pins up from the center. The DTMC solutions quickly converge to the reference solution.

Table 2. Comparison of pin power profile

Method	Cycle	Avg. ASD	Max. ASD	Avg. RESD	Max. RESD
Standard MC	1	-	-	0.176	0.503
	10	0.071	0.156	0.077	0.216
	500	0.011	0.023	0.012	0.036
DTMC (CMFD)	1	-	-	0.079	0.221
	10	0.015	0.039	0.065	0.183
	500	0.004	0.009	0.012	0.035
DTMC (p-CMFD)	1	-	-	0.078	0.244
	10	0.014	0.038	0.064	0.180
	500	0.004	0.008	0.011	0.033

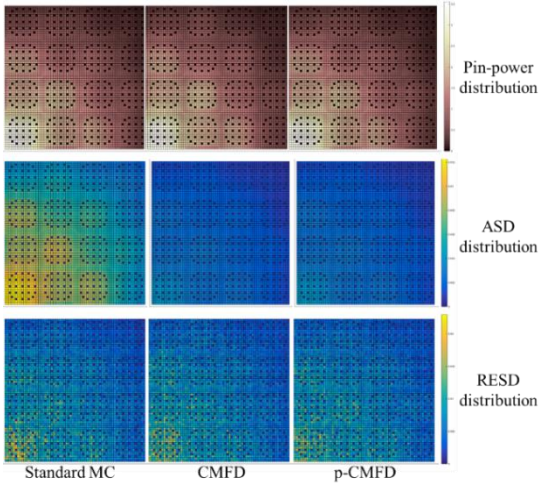

Fig. 7 Distribution of ASD and RESD for each case

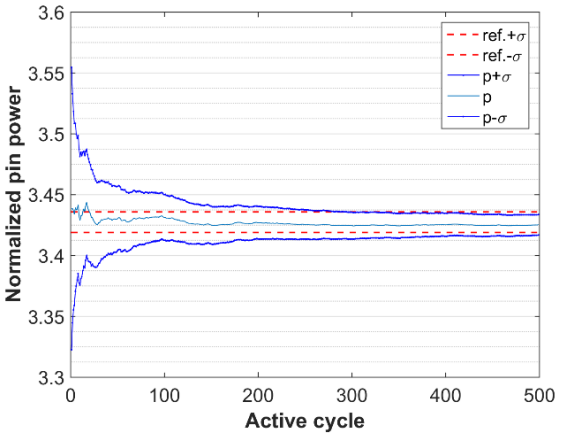

Fig. 8.1 Relative pin power (MC) at UO_2 pin in UO_2 fuel assembly; (x,y)=(5,5)

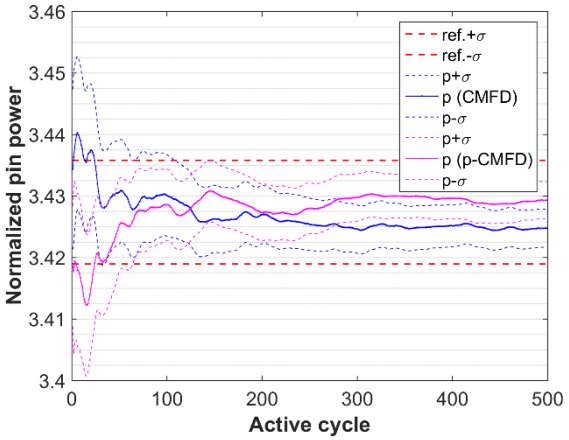

Fig. 8.2 Relative pin power (CMFD) at UO_2 pin in UO_2 fuel assembly; (x,y)=(5,5)

3. Conclusions

The feasibility of a deterministic truncation to the MC solution (DTMC) has been proposed and evaluated in terms of the solution estimation and variance reduction. Since the DTMC method is based on the MC simulation, it solves the reactor system problem without any approximations. We have demonstrated that the DTMC solution quickly approaches to very accurate solution in the 3D pin-wise eigenvalue problem. It is found that both ASD and RESD for the DTMC solutions are much smaller for the reactor eigenvalue and they are rather comparable or smaller for the detailed pin-power profile. We also found that the DTMC approximation is very accurate even for the very first active cycle of the MC analysis. This implies that computing cost can be saved dramatically by using the DTMC solution for standard reactor problems and it deserves a lot more studies.

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