# Probabilistic Reasoning with Multilevel Flow Modeling based on Time-related Concepts

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## 1. Introduction

As modern industrial systems become larger and more complex, the importance of system modeling has been consistently emphasized and accordingly various system modeling methods have been suggested. Particularly, since existing systems are not always sufficiently understood to apply quantitative modeling methods, not only studies on quantitative modeling methods but also qualitative modeling methods have been conducted actively.

Among various kinds of qualitative modeling methods, multilevel flow modeling (MFM) which represents a system's goals and functions with flows of mass, energy and their interactions [1] can be considered as one of the major qualitative modeling methods. MFM can model various industrial systems without the reliance on detailed knowledge or domain specific assumptions. Moreover, by accompanying reasoning algorithm with MFM, it is able to conduct qualitative reasoning regarding event causes and consequences not only fast but also precisely.

With these advantages, MFM has been applied for various purposes [2-5]. However, current MFM method has a limitation that it is not able to consider the dynamic characteristics of the modelled system, and accordingly, MFM's applicability has been restricted since the concept of time is important in explaining the characteristics of many systems.

Therefore, in this paper, the time-related concepts including time-to-detect (TTD) and time-to-effect (TTE) were adopted from the system failure model, in order to enhance MFM to be capable of considering dynamic characteristics. Additionally, the method for probabilistic reasoning based on these time-related concepts is also introduced.

## 2. Preliminaries

## 2.1 Multilevel Flow Modeling

MFM is a qualitative modeling method for general industrial processes, which represents the goals and functions of the system with mass, energy flows and their interactions; and represents the system's structure hierarchically with means-end and part-whole abstractions [1]. Although MFM models are simple, it is intuitive and capable of consisting many fundamental characteristics of the system. Fig. 1 represents the various symbols which are used during MFM modeling.

Functions								
Mass and Energy Flow						Control		
source ti	ransport 令	stora	ge conv	ersion	separation	steer	trip d	
sink	barrier	balan	ce distr	ibution		regulate	suppress	
Targets	Targets Relations							
objective	Influ	Influence		Means-end			Control	
function structure	influencer	participant	maintain produce	suppress destroy I€ ●	producer- product mediate	enable 	actuate	

Fig. 1. Various symbols which are used during MFM modeling [1]

As MFM based models are established with considering energy conservation and mass conservation laws, not only the whole system can be easily and precisely modeled but also it is able to conduct qualitative reasoning which is a process that revealing the causes and consequences of the observed events [6].

The main characteristics of MFM can be summarized as follows.

(1) System representation with flows and interactions: as MFM models the functions of target system with elementary flows, MFM can be applied easily and accurately without detailed knowledge and excessive domain-specific assumptions.

(2) Qualitativeness: a system can be modeled with MFM without detailed quantitative relations. Accordingly, the application of MFM would be inappropriate if quantitative information is required.

(3) Model-based reasoning: once a model is established, the model is not able to consider additionally acquired information on system configurations unless the model is updated.

(4) Snap-shot evidence and results: it is not able to consider the dynamic characteristics of the systems since current MFM methods do not involve time-related concepts.

Among these characteristics of MFM, the fourth characteristic (i.e. snap-shot evidence and results) is regarded as one of the main disadvantages of MFM, since time-related evidences are often utilized importantly during the cause and consequence reasoning processes. If MFM becomes able to consider dynamic characteristics of system, it is expected that more detailed and delicate cause and consequence reasoning would be possible.

## 2.2 System Failure Model

A System failure model (tentative name) was introduced as one of the main concepts of functional fault analysis (FFA), which is a systematic design methodology for the integration of a system health management (SHM) to the early design stage of complex systems such as spacecraft. With system failure model, it is able to consider the effect propagation due to various failure modes and its timing along the modeled physical paths [7]. The system failure model is represented in Fig. 2.



Fig. 2. Schematic of the system failure model and its timing definitions [7]

However, the fore-mentioned system failure model was introduced only for spacecraft cases, and therefore the modified system failure model was suggested by applying several modifications to make the model more general. The modified system failure model is represented in Fig. 3.



Fig. 3. Schematic of the modified system failure model and its timing definitions

Timing definitions included in the modified system failure model are as follows.

(1) Time-to-effect (TTE): the time from the 'onset of failure' to the point when its effects are potentially detectable.

(2) Time-to-detect (TTD): the time from the 'onset of failure' to the confirmation of fault existence.

(3) Time-to-diagnosis: the time from the 'onset of failure' to the identification of the fault (e.g., fault location, fault type, etc.).

(4) Time-to-mitigation: the time from the 'onset of failure' to the complete prevention of the critical system failure.

(5) Time-to-criticality: the time from the 'onset of failure' to the critical system failure.

## 3. Application of the Time-related Concepts to MFM

# 3.1 Modified definitions of the TTD and TTE concepts from an MFM perspective

Since the reasoning processes are conducted when one or more functions are not in normal states, which include failed states, it is necessary to modify the timing definitions from an MFM perspective.

Firstly, among the upper five timing definitions, only TTD and TTE are associated to the MFM, since the others are related to the diagnosis and mitigation processes which are out of the MFM's scope.

To redefine TTD and TTE in MFM perspective, it is convenient to consider the simple system with only two connected functions and the corresponding instrumentation systems.

If a state change in function A occurs and is detected by its corresponding instrumentation system, TTD for function A ( $t_A$ ) can be defined as the time from the 'state change in function A' to the 'detection of the state change in function A'. If a state change in function A occurs and it induces a state change in function B, TTE between function A and function B ( $t_{AB}$ ) can be defined as the time from the 'state change in function A' to the 'state change in function B'. These are represented in Fig. 4.



Fig. 4. Diagram of the simple two-function system with corresponding TTDs and TTE

These concepts can be easily expanded to multifunction systems with the assumption that the effect propagates one-by-one from the antecedent function to the subsequent function.

Substantially, due to the uncertainties and many kinds of factors, it is natural to represent the TTD and TTE values as distributions instead of fixed constants. In this case, it is now able to conduct enhanced probabilistic reasoning, rather than conventional deterministic reasoning.

#### 3.2 Estimation of TTD and TTE as distribution

In order to utilize TTD and TTE profiles for enhanced reasoning processes, it is necessary to estimate these values with proper level of uncertainties. If TTD and TTE are represented as distributions, TTD and TTE estimation problems can be considered as distribution estimation problems.

In the case of TTD, it is expected that the estimation of TTD distribution is relatively easy, since most of the applied instrumentation systems are both theoretically and empirically well-defined. However, in the case of TTE, it is expected that the estimation of TTE distribution is much harder since it may vary due to many complicated factors such as input conditions or state definitions. Furthermore, analytical methods are inappropriate to be considered since MFM is not likely to be applied for the well-understood systems.

Instead, if the time of the event occurrences can be measured, it is able to consider the empirical approaches for the TTE distribution estimation (e.g. estimation of the likelihood from the samples). Representatively, two kinds of approaches can be utilized for the estimation.

3.2.1 Estimation of the TTE distributions based on Bayesian update

Bayesian update (i.e. Bayesian inference) is the statistical inference method based on Bayes' theorem, which can be used to update the probability for a hypothesis with observed evidences.

Equation for the Bayesian update can be represented as follows.

$$P(\theta \mid Data) = \frac{P(Data \mid \theta)}{P(Data)} \cdot P(\theta)$$
(1)

Where  $\boldsymbol{\theta}$  is the parameter of the data point's distribution.

To apply Bayesian update, it is essential to define the forms of the prior distribution and likelihood, and it highly affects the update results. The beta distribution is one of the widely used distributions since it is able to approximate many other distributions. However, since most of the commonly used distributions are inappropriate for approximating multimodal distributions (i.e. distributions with multiple peaks), Bayesian update is also difficult to deal with multimodal hypotheses. Many studies have been conducted to solve the problem of multimodality within the Bayesian update, but they are still ongoing [8].

## 3.2.2 Estimation of the TTE distributions based on non-Bayesian probability distribution approximation algorithm

As alternatives of Bayesian update, studies on non-Bayesian probability distribution approximation algorithms also have been actively conducted to consider the multimodal distributions [9]. Although none of them show the matchless performance, many non-Bayesian probability distribution approximation algorithms show better performance on considering multimodal distributions.

Since non-Bayesian probability distribution approximation algorithms are relatively data-inefficient, it would be proper to consider the mixed approach rather than choosing only one method for the TTE distribution estimation (e.g. application of Bayesian update when the amount of data is small, and then application of another method when the amount of data is sufficient).

# 3.3 Probabilistic reasoning based on the TTD and TTE distributions

To conduct probabilistic cause and consequence reasoning based on TTD and TTE distributions, it is necessary to consider the summation of the distributions. If the distributions are independent to each other, the summation of the distributions can be solved through convolution operation.

For the continuously distributed random variables X and Y with probability density functions f and g respectively, convolution operation to get the distribution of the sum Z=X+Y is as follows.

$$h(z) = (f * g)(z) = \int_{-\infty}^{\infty} f(z - t)g(t)dt$$
 (2)

In section 3.3.1 and 3.3.2, probabilistic cause reasoning and consequence reasoning processes are described.

### 3.3.1 Probabilistic cause reasoning

To simplify the problem, assume that there are two event paths which can affect both function A and function B. Since these event paths include different functions, the results of serial convolution operations would also be different. Through the serial convolution operations of TTD and TTE distributions, it is able to obtain the probability distributions that represents when will the state of A, and state of B changes for both event paths. Accordingly, it is able to calculate the time gap distributions between function A and function B for both event paths.

When the state changes in function A and function B are both observed and the time gap between these changes is measured, then it is able to calculate the probabilities of event occurrence due to two event paths. If the observed time gap is denoted as  $t_m$ , and time-gap distribution for event path 1 and 2 are denoted as  $pd_1$  and  $pd_2$  respectively, then the probabilities of event occurrence due to each event path ( $P_1$  and  $P_2$ ) can be obtained as follows.

$$P_{1}(t = t_{m}) = \frac{pd_{1}(t_{m})}{pd_{1}(t_{m}) + pd_{2}(t_{m})}$$
(3)

$$P_{2}(t = t_{m}) = \frac{pd_{2}(t_{m})}{pd_{1}(t_{m}) + pd_{2}(t_{m})}$$
(4)



Fig. 5. Schematic of probabilistic cause reasoning

For example, if the actual time gap is measured and corresponding probability values of  $pd_1$  and  $pd_2$  are 0.2 and 0.05 respectively, it is able to infer that the probability of the event occurrence due to event path 1 is 80% (0.2/0.2+0.05), and the probability of the event occurrence due to event path 2 is 20% (0.05/0.2+0.05).

For the cases with **n** event paths, the upper equations can be generalized as follows to get the event occurrence probability due to event path  $\mathbf{x}$ .

$$P_x(t=t_m) = \frac{pd_x(t_m)}{\sum_{k=1}^n pd_k(t_m)}$$
(5)

### 3.2.1 Probabilistic consequence reasoning

To simplify the problem, assume that the state change in function A is observed and it eventually induces the state change in function B. Through the serial convolution operations of TTD and TTE distributions, it is able to obtain the probability distributions that represents when will the state of B changes, and when will the state change in B detected. If new observations on state changes in functions between function A and function B are obtained, then the prediction about when will the state of B changes can be conducted with reduced uncertainty.

#### 4. Conclusion

In this paper, the time-related concepts including TTD and TTE were adopted from the modified system failure model, which is a generalized version of the system failure model. Moreover, enhanced probabilistic reasoning processes based on the estimated TTD and TTE distributions are briefly introduced.

Based on the concepts introduced in this paper, it is expected that the MFM's applicability will be increased, especially for the systems with sparse instrumentation systems.

For future studies, additional case studies should be conducted in order to further examine the applicability to real-world systems. In addition, since the accurate and precise estimation of TTD and TTE distributions are extremely important for the probabilistic reasoning processes, continuous monitoring on probability distribution estimation methods is necessary.

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