*Transactions of the Korean Nuclear Society Spring Meeting*
*Jeju, Korea, May 17-18, 2018*

# Study of Critical Flow for Supercritical $CO_2$ Turbomachinery Seal Design

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## 1. Introduction

From many previous studies of Supercritical $CO_2$ (S-$CO_2$) Brayton power cycle, it was identified that the S-$CO_2$ Brayton power cycle technology has a big potential to outperform the existing steam cycle and eventually replacing them [1, 2]. The S-$CO_2$ Brayton power cycle is adaptable to a variety of heat sources and has higher cycle efficiency compared to traditional power cycles. As such, it is being considered for multiple energy sources such as nuclear, waste heat recovery, fossil fuel, concentrated solar power, and geothermal energy.

The S-$CO_2$ leakage flow from turbo-machinery via seal becomes one of the important issues since not only it influences the cycle efficiency due to parasitic loss but also it is important for evaluating the system safety under various operating conditions.

In the previous authors' study, the effect of the tooth length on the critical flow and comparing the results to the existing two phase system analysis code calculation were presented. However, since the code is limited to fully reflect the gap effect and the number of tooth effect, a separate numerical model had to be developed. In this paper, the number of tooth effect in a labyrinth seal geometry nozzle are presented by using the same experimental facility described in the previous paper [3].

## 2. $CO_2$ Critical Flow Experiment

Kim et al. constructed a critical flow test facility to validate the S-$CO_2$ critical flow model [3]. Fig. 1 shows the designed experimental facility for the $CO_2$ critical flow. The $CO_2$ flows from a high-pressure tank (left) to a low-pressure tank (right) through the designed nozzle, and pressure and temperature of each position are measured every second. Two tanks are connected by a 1090mm pipe and the designed nozzle that simulates a labyrinth seal geometry is installed between the ball valve and the low-pressure tank. Each tank has 200 mm of inner diameter, 1600 mm in height, and 47 liters of volume. Temperature and pressure limits are 150ºC and 22 MPa, respectively. To control the initial temperature of the high-pressure tank, electrical jacket-type heaters are installed on the external surface of the high-pressure tank. The pressure of the high-pressure tank can be controlled by injecting the $CO_2$ gas through a booster pump.

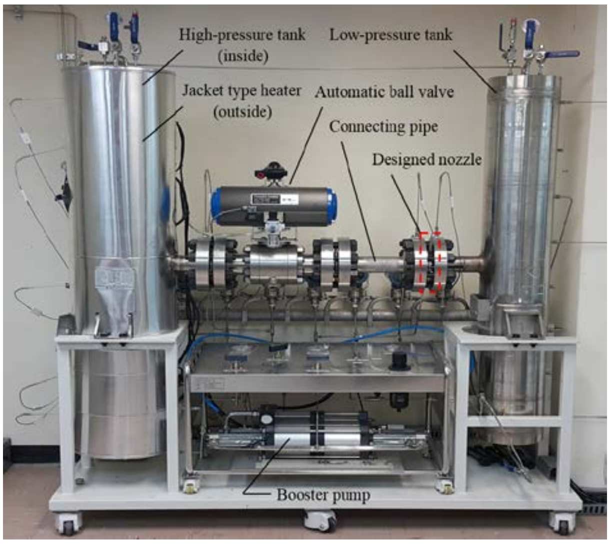


Fig 1. S-$CO_2$ critical flow experimental facility

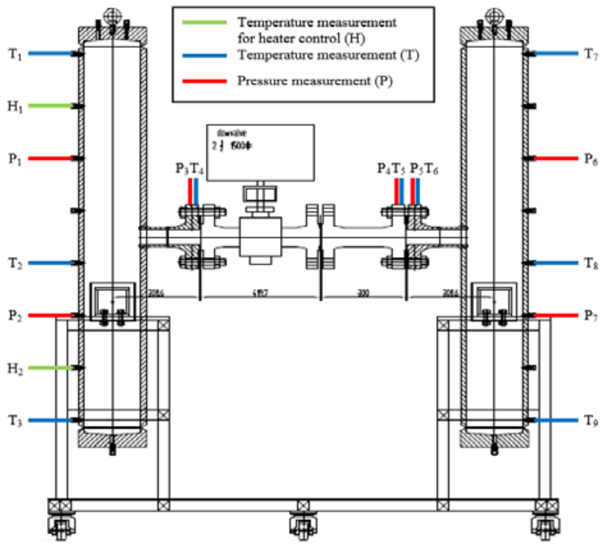


Fig. 2. The location of measurements in the experimental facility

Table I: Known constant values and uncertainties for calculation

| | Known value | Uncertainty | | Uncertainty |
|---|---|---|---|---|
| $D_{nozzle}$ (mm) | 1.5/0.5 | ±0.02 | P (kPa) | ±(0.00025P) |
| $D_{tank}$ (mm) | 200 | ±0.5 | | |
| $H_{tank}$ (mm) | 1600 | ±1.2 | T (ºC) | ±(0.15+0.002T) |
| △Time (sec) | 1 | ±0.03 | | |

Table II: Design specifications of the experimental facility

| | Design Parameters | |
|---|---|---|
| High/Low-pressure tank | Pressure (MPa) | 22 |
| | Temperature (ºC) | 150 |
| | Volume (L) | 47 (I.D.:200 mm, H: 1600mm) |
| Pipe connecting two tanks | Internal diameter (mm) | 57 |
| | Length (mm) | 1090 |
| Heater (Jacket-type) | Electric capacity (kW) | 5 |
| Valve type | Ball valve | |

The ball valve is automatically opened and driven by hydraulic pressure (1MPa) from an air compressor to minimize the valve opening time.

## 3. Numerical Model

To model the $CO_2$ leak flow mechanism realistically, several factors should be considered and a few assumptions are necessary for a simplified model. The base calculation algorithm of the $CO_2$ critical flow model for calculating the leak rate of $CO_2$ in a turbo-machinery was referred from the $CO_2$ leak model of Na-$CO_2$ heat exchanger study in the S-$CO_2$ power cycle [4]. It was assumed that $CO_2$ flows through a nozzle from a high pressure $CO_2$ tank to a low pressure $CO_2$ tank, and the nozzle diameter plays the same role as the seal gap size.

For an isentropic flow, the frictional pressure loss and heat transfer are neglected thus the flow state can be easily calculated with the following governing equations (i.e. continuity equation, isentropic flow equations and corrected mass flux equation) [5]:

$$G = \rho V_{elocity} = constant \tag{1}$$

$$\frac{P_0}{P_{critical}} = (1+\frac{\gamma-1}{2})^{\gamma/(\gamma-1)} \tag{2}$$

$$M = \sqrt{\frac{2}{\gamma-1}[(\frac{P_0}{P})^{(\gamma-1)/\gamma}-1]} \tag{3}$$

$$G = \frac{P_0}{\sqrt{RT_0}}\sqrt{\gamma}M(1+\frac{\gamma-1}{2}M^2)^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{4}$$

$$G_{max} = \frac{P_0}{\sqrt{RT_0}}\sqrt{\gamma}(\frac{\gamma+1}{2})^{-\frac{\gamma+1}{2(\gamma-1)}}(M_{exit}=1.0) \tag{5}$$

where, $G$ is mass flux, $\rho$ is density, $V_{elocity}$ is velocity of flow, $P_0$ is stagnation pressure, $P_{critical}$ is critical pressure, $\gamma$ is specific heat ratio, $M$ is Mach number, $P$ is static pressure, and $R$ is ideal gas constant, $T_0$ is stagnation temperature, and $M_{exit}$ is Mach number at nozzle exit.

Based on the above governing equations, the critical pressure obtained from equation (2) is compared to the low pressure side at every time step. The choked condition is then checked. If the flow is not choked, the Mach number is calculated from equation (3) and it is applied to equation (4). On the other hand, equation (5) with the Mach number of unity is used to calculate the choked mass flux. The temperature and pressure of the two tanks were measured during the experiment and the $CO_2$ density can be obtained from the NIST reference fluid thermodynamic and transport properties database (REFPROP) after the experiment [6]. Since the tank volume was an already known value, which is 47 liters, $CO_2$ mass change can be calculated for every second. In other words, mass flow rate can be obtained by calculating the mass difference of the tank for every second. Finally, the mass flux can be obtained from the known nozzle area.

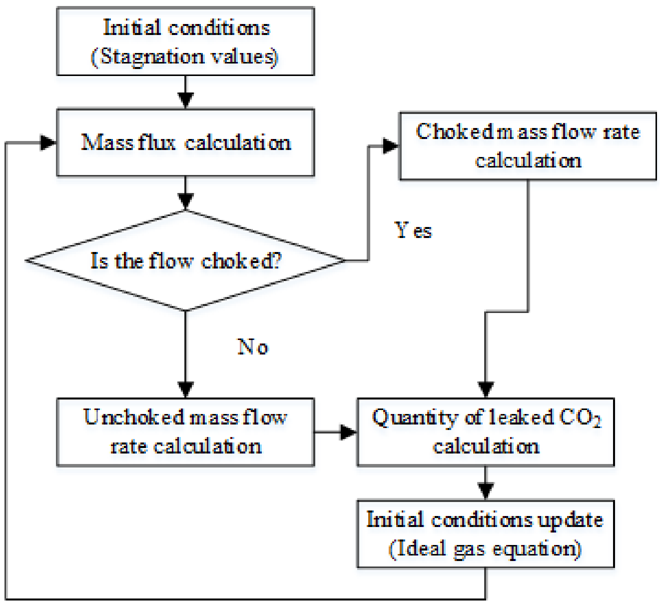


Fig. 3. Flow chart of $CO_2$ critical flow model

To simplify the model, it was assumed that temperature and pressure of $CO_2$ at the seal exit are at equilibrium with $CO_2$ in the low pressure tank. This assumption actually neglects expansion process of $CO_2$ at the nozzle exit although the $CO_2$ pressure at the exit is higher than that of $CO_2$ in the low pressure tank when the flow is choked. Fig. 3 shows a flowchart of the $CO_2$ critical flow model. Based on the algorithm of the $CO_2$ critical flow model, the sensitivity study of the transient response during the leak process was performed while varying nozzle diameter and initial conditions (temperature and pressure).

## 4. Experimental Results

To observe the number of tooth effect in the labyrinth seal and to validate the isentropic $CO_2$ critical flow model further, the experiments with various labyrinth seal geometry nozzles were performed. The detail internal geometry of a labyrinth seal simulating orifice is shown in Fig. 4. The information of three experimental cases is summarized in Table III. It is noted that the total length and tooth length of nozzle geometry is kept constant while changing the tooth number and the cavity length. This means that when the tooth number is increased, the cavity length is reduced accordingly. The initial conditions of the high pressure tank were set to 10.6MPa and 133ºC to maintain the gaseous state after the expansion.

The experimental results are shown in Fig. 5. This figure shows that as the number of tooth increases, the time required for reaching equilibrium is delayed. Fig. 5 shows that as the number increase from one to three, the equilibrium reaching time is delayed about 291s. Thus, the experimental data confirms that the leak rate is reduced as the number increases even though the total nozzle length is the same.

To model the labyrinth seal geometry, Hodkinson's equation was adopted in this study [7]. Hodkinson modified Egli's approach [8] to provide a semi empirical relation that was based on assumptions of a gas jet geometry. Hodkinson's assumption was as follows. The fluid jet expands conically from the tip of an upstream tooth at a small angle, $\beta$. A part of the jet impinges on the downstream tooth to recirculate in the cavity, dissipating the kinetic energy associated with it, while a portion of the jet travels under the downstream tooth and carries over the kinetic energy to the next cavity. He assumed the angle $\beta$ to be only a function of seal geometry. The Hodkinson's equation is shown in equations (6) and (7).

$$G = \mu_i \frac{AP_0}{\sqrt{RT_0}} \sqrt{\frac{1-(\frac{P}{P_0})^2}{n-ln(\frac{P}{P_0})}} \tag{6}$$

$$\mu_i = \sqrt{\frac{1}{1-(\frac{n-1}{n})(\frac{c_i/s_i}{c_i/s_i+0.02})}} \tag{7}$$

where, $G$ is mass flux, $A$ is cross section, and $P$ and $P_0$ is pressure of low- and high-pressure tanks, $T_0$ is temperature of high-pressure tank, $n$ is tooth number, $\mu_i$ is carry-over coefficient, $c_i$ is clearance at $i$ tooth, and $s_i$ is length between $i$ and $i+1$ tooth.

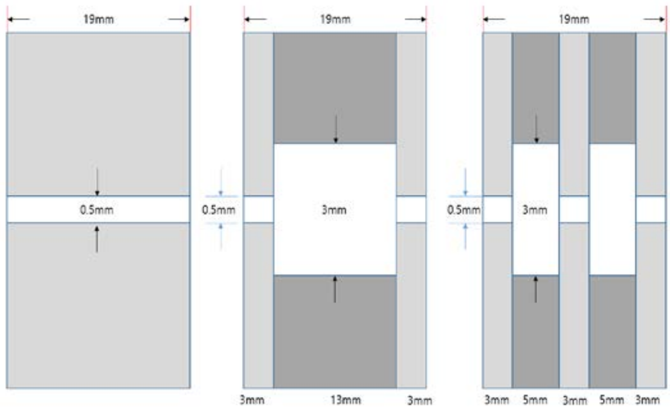


Fig. 4. Internal geometry of labyrinth seal geometry nozzle

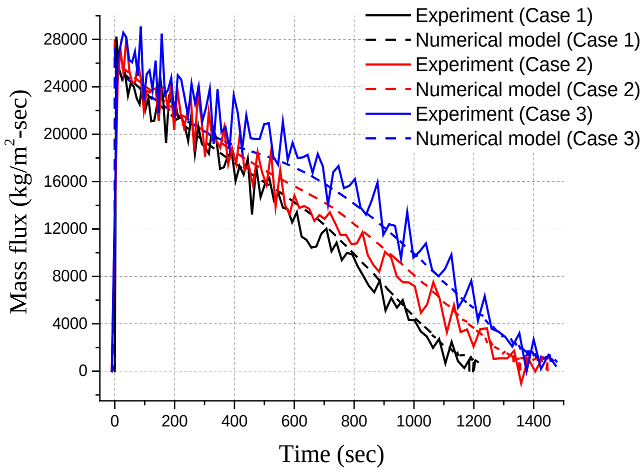


Fig. 5. Comparison of mass flux between the experimental and numerical results

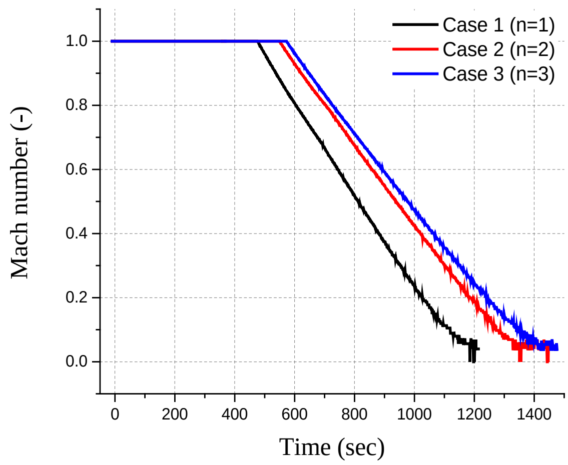


Fig. 6. Mach number of experimental result

Table III: Summary of experimental cases

| | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| D (mm) | 0.5 | 0.5 | 0.5 |
| $L_{tooth}$ (mm) | 19 | 3 | 3 |
| $L_{cavity}$ (mm) | - | 13 | 5 |
| n (-) | 1 | 2 | 3 |
| Diameter ratio (-) | 1 | 6 | 6 |
| Pressure ratio (-) | 105.0 | 105.0 | 105.0 |

As shown in Fig. 5, the mass flux of the $CO_2$ experiment and the model agrees with each other quite well in critical and sub-critical flow regimes. An isentropic $CO_2$ critical flow model of Hodkinson has a good accuracy for predicting the real $CO_2$ leak flow in single phase condition regardless of the upstream condition at the supercritical state or the gaseous state.

## 3. Conclusions

Predicting the leak flow rate in turbo-machinery seals is imperative to secure high performance of an S-$CO_2$ power cycle. Thus, an isentropic $CO_2$ critical flow model was selected from the literature and compared to the experimental results to identify the mass flow rate of $CO_2$ leakage in turbo-machinery. This paper includes the experimental data obtained under various conditions. Although this study does not immediately contribute for reducing the leakage flow, it provides useful information for further understanding of labyrinth seal leakage performance and eventually leading to the design optimization.

To validate the isentropic $CO_2$ critical flow model with experimental results, experiments with nozzles that simulate the condition in the labyrinth seal were performed. As the number increases, the equilibrium reaching time is delayed further. The model suggested by Hodkinson for predicting the critical flow in the seal geometry gave a good agreement with the experimental data.

The newly developed model should reflect the gap effect, the number of tooth effect as well as the two phase effect to capture all important physical processes in the seal and showing a good prediction capability for the seal design. Therefore, more complex assumptions will apply to the $CO_2$ critical flow model to reflect the real gas effect. Furthermore, Henry-Fauske model will be studied to accurately estimate the dynamic behavior for $CO_2$ critical flow under two-phase conditions in the near future.

## ACKNOWLEDGEMENTS

This study was conducted with the support of Korea Evaluation Institute of Industrial Technology (Project No.: 10063187, Engineering Technique for Power Generation System Design using Industry Waste Heat) as part of the core industrial technology development project supported by the Ministry of Trade, Industry and Energy, which is gracious acknowledged.

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