

## Heat transfer correlations for the CO<sub>2</sub> 2-phase flow near the critical point

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### 1. Introduction

A supercritical carbon dioxide cycle is considered as the next-generation power conversion system, and many studies have been conducted for combining it with many heat sources including nuclear reactors. The previous research on supercritical carbon dioxide cycles have mostly focused on the operation in supercritical state only. Recently, however, there has been a growing interest in the trans-critical CO<sub>2</sub> Rankine cycle, which operates under the critical point to perform compression in liquid phase [1, 2]. This will increase the efficiency of the cycle, lower the pressure loss, and ease the limits of turbo machine manufacturing [1]. To analyze the trans-critical CO<sub>2</sub> Rankine cycle involving a phase change near the critical point, it is important to predict the heat transfer and pressure drop in that region. However, limited amount of studies have been conducted in this area. The previous study shows that the heat transfer correlations developed in the single-phase CO<sub>2</sub> region would have a high accuracy in two-phase regions if the two-phases are relatively uniformly distributed in the vicinity of the critical point [1]. However, the applicability of other CO<sub>2</sub> single-phase correlations has not been fully evaluated in the previous study, so it is necessary to examine the applicability of other correlations.

### 2. Correlation review

In order to find the best heat transfer correlation for the two-phase region of CO<sub>2</sub>, this study refers to the research conducted by Bae first [Table1]. In the trans-critical carbon dioxide cycle, CO<sub>2</sub> condensation occurs in the pre-cooler under steady state. In general, a Printed Circuit Heat Exchanger (PCHE) having a wide heat transfer area per unit area is used for this purpose. Since a PCHE channel has a very small flow area, the correlations developed for micro-channels with a diameter of 1.5-2.0 mm have been first identified from several literatures.

In the trans-critical carbon dioxide Rankine cycle, the CO<sub>2</sub> 2-phase region of interest is close to the critical point. The ranges of interest are summarized in Table 1 and drawn the T-s diagram in Figure 1. They show that Bae's test range is on a 2-phase region.

Table 1: Bae's test range

Bae's test range		
Flow parameter Range : $t_m (^{\circ}C) / P_m (MPa)$	Flow geometry range: D(mm)	Re Number range
26.24~29.60/6.62~6.98	1.5-2.0	12000-75000

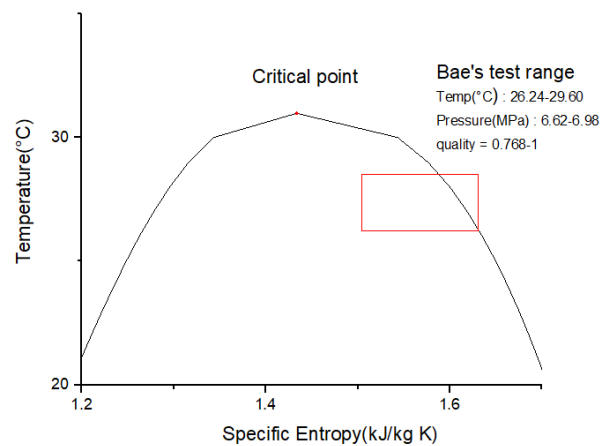


Figure 1. Bae's test range

Fang equation [3], Liao-Zhao [5], Dang-Hibara modification [6], Huai et al [8,9] and Kuang et al [10] were found to be potentially suitable for the conditions. Table 2 summarizes the flow parameters and geometry applicable to those correlations.

Table 2: the flow parameters and geometry of the correlations

Correlations	Flow parameter: $t_m (^{\circ}C) / P_m (MPa)$	Channel Diameter: D(mm)
(1) Fang	25-65/8-12	0.79
(2) Liao-Zhao	20-110/7.4-12	0.5-2.16
(3) Dang-Hibara modification	20-70/8-10	1-6
(4) Huai et al.	22-53/7.4-8.5	1.31
(5) Kuang et al.	45-55/8-10	0.79

Table 2 shows that the correlations were developed in a single phase range. These single-phase correlations were evaluated in the 2-phase range suggested by Bae [Figure 1].

(1) Fang [3] equation

$$Nu_w = \frac{(f_w/8)(Re_w-1000)Pr_w}{A+12.7(f_w/8)^{1/2}(Pr_w^{2/3}-1)} \left(1+0.001\frac{q}{G}\right) \left(\frac{\bar{c}_p}{c_{p,w}}\right)^n$$

$$A = \begin{cases} 1+7 \times 10^{-8} Re_w & \text{if } Re_w < 10^6 \\ 1.07 & \text{if } Re_w \geq 10^6 \end{cases}$$

$$n = \begin{cases} 0.66+4 \times 10^{-4}(q/G) & \text{if } \bar{c}_p/c_{p,w} \leq 1 \\ 0.9+4 \times 10^{-4}(q/G) & \text{if } \bar{c}_p/c_{p,w} > 1 \end{cases}$$

where  $f_w$  is the friction factor evaluated at  $T_w$  by the Churchill [4] equation.

$$f_w = 8 \left[ \left( \frac{8}{Re} \right)^{12} + B^{-3/2} \right]^{1/12}$$

$$B = \left[ 2.457 \ln \frac{1}{(7/Re)^{0.9} + 0.27Rr} \right]^{16} + \left( \frac{37,530}{Re} \right)^{16}$$

where  $Rr$  is the channel relative roughness.  $Rr = \varepsilon / D$ , and  $\varepsilon$  is the channel roughness. This equation is effective in the range of  $3000 \leq Re_w < 10^6$  and  $0 \leq q/G < 350$  J/kg.

Because the heat transfer coefficient is determined by the inner wall temperature rather than the bulk temperature of the fluid, the applicability of the correlation can be determined according to the condition of the wall temperature in the experiment.

(2) Liao-Zhao [5]

$$Nu_w = 0.128 Re_w^{0.8} Pr_w^{0.3} \left( \frac{Gr}{Re_b^2} \right)^{0.205} \left( \frac{\rho_b}{\rho_w} \right)^{0.437} \left( \frac{\bar{C}_p}{C_{p,w}} \right)^{0.411}$$

where  $Gr$  is the Grashof number, defined as

$$Gr = \frac{g(\rho_w - \rho_b)\rho_b D^3}{\mu_b^2}$$

In the Liao-Zhao correlation, the Reynolds number  $Re_b$  and Prandtl number  $Pr_b$  ranged from  $10^4$  to  $2 \times 10^5$  and from 0.9 to 10, respectively [5], which meet the condition of Bae' test.

(3) Dang-Hibara modification [6]

$$Nu_w = \frac{(f_f/8)(Re_b-1000)Pr}{1.07+12.7(f_f/8)^{1/2}(Pr^{2/3}-1)}$$

$$Pr = \begin{cases} C_{p,b}\mu_b/k_b & \text{for } C_{p,b} \geq \bar{C}_p \\ \bar{C}_p\mu_b/k_b & \text{for } C_{p,b} < \bar{C}_p \text{ and } \mu_b/k_b \geq \mu_f/k_f \\ C_p\mu_f/k_f & \text{for } C_{p,b} < \bar{C}_p \text{ and } \mu_b/k_b < \mu_f/k_f \end{cases}$$

where the friction factor  $f$  is calculated with the Filonenko [7] equation:

$$f_f = (1.82 \log Re - 1.64)^{-2}$$

For  $(10^4 \leq Re < 5 \times 10^6)$

(4) Huai et al [8,9]

$$Nu_w = 0.022186 Re_w^{0.8} Pr_w^{0.3} \left( \frac{\rho_b}{\rho_w} \right)^{-1.4652} \left( \frac{\bar{C}_p}{C_{p,w}} \right)^{0.0832}$$

Comparison of the Liao-Zhao equation at the Nusselt number with respect to the average fluid temperature graph [9] suggests that Reynolds number range of Huai correlation is similar to that of Liao-Zhao. It means that Huai correlation covers Bae' Reynold number range.

(5) Kuang et al [10]

$$Nu = 0.001546 Re^{1.054} Pr^{0.653} \left( \frac{\rho_w}{\rho} \right)^{0.367} \left( \frac{\bar{C}_p}{C_p} \right)^{0.4}$$

The Kuang equation is developed with the modification of mean specific heat in the Ghajar and Asadi's equation [11]. Comparison of the Huai equation at the experimental Nusselt number with respect to calculated Nusselt number graph [10] means that Reynolds number range of Kuang correlation is similar to that of Huai.

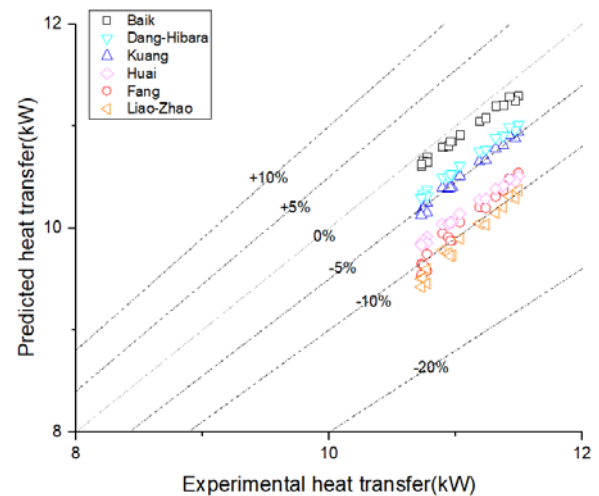


Figure 2. Comparison results of the measured and predicted heat transfer

Figure 2 shows the comparison results between the measured data and the predictions from the six heat transfer correlations. As a result, the correlations developed in the single phase region near the critical point showed accuracy within 12%. The correlations produce similar values to the experimental values. This suggests that the two-phase condition of CO<sub>2</sub> close to the critical point are close to the CO<sub>2</sub> single-phase condition in terms of heat transfer.

### 3. Conclusions

In this study, the heat transfer correlations developed in the single-phase region near the critical point of CO<sub>2</sub> were investigated, since the preceding research showed that the correlations developed in the single-phase region can be applied to the two-phase region near the critical point of CO<sub>2</sub>. As a result, Liao-Zhao [5], Dang-Hibara modification [6], Huai et al [8,9], Kuang et al [10] etc. are newly identified as potentially well fit correlations that can successfully predict the heat transfer coefficient in the two-phase region near the critical point. The six correlations are compared with the measured values and producing similar values to the experimental values as shown in Figure 2. This means that the two-phase condition of CO<sub>2</sub> close to the critical point are close to the CO<sub>2</sub> single-phase condition from the perspective of heat transfer and shows the validity of the previous study corresponding to the applicability of other CO<sub>2</sub> single-phase correlations as well [1].

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### REFERENCES

- [1] Bae, Seong Jun. A Study of Trans-critical CO<sub>2</sub> Power Cycle for Nuclear Marine Application. Department of Nuclear and Quantum Engineering. 2018. 147+vi pages. Advisor: Lee, Jeong Ik.
- [2] <https://www.echogen.com/>
- [3] X.D. Fang, Modeling and Analysis of Gas Coolers, ACRC CR-16. Department of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, USA, 1999.
- [4] S.W. Churchill, Friction-factor equation spans all fluid-flow regimes, Chemical Engineering 84 (7) (1977) 91e92.
- [5] S.M. Liao, T.S. Zhao, Measurement of heat transfer coefficient from supercritical carbon dioxide flowing in horizontal mini/micro channels, Journal of Heat Transfer 124 (2002) 413e420.
- [6] C. Dang, E. Hihara, In-tube cooling heat transfer of supercritical carbon dioxide. Part 1. Experimental measurement, International Journal of Refrigeration 27 (2004) 736e747.
- [7] Detlev G. Kröger, Air-cooled Heat Exchangers and Cooling Towers, p.70
- [8] X.L. Huai, S. Koyama, T.S. Zhao, An experimental study

of flow and heat transfer of supercritical carbon dioxide in multi-port mini channels under cooling conditions, Chemical Engineering Science 60 (2005) 3337e3345.

[9] X. Huai, S. Koyama, Heat transfer characteristics of supercritical CO<sub>2</sub> flow in small-channeled structures, Experimental Heat Transfer 20 (2007) 19e33.

[10] G. Kuang, M. Ohadi, S. Dessiatoun, Semi-empirical correlation of gas cooling heat transfer of supercritical carbon dioxide in microchannels, HVAC & RRResearch 14 (6) (2008) 861e871.

[11] Ghajar, A.T., and A. Asadi. 1986. Improved forced convective heat transfer correlation for liquids in the near critical region. *AIAA Journal* 24(12):2030–37.