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Xenon Stability Analysis of SMART

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1. Introduction

Xenon (Xe) induced spatial oscillation is one of the significant safety issues in reactor core design. Thus Xe stability evaluation is required by the regulatory body before constructing and operating nuclear power plants. Although it is known that small reactors are relatively free from instability issues caused by Xe oscillation compared to large PWRs [1][2], Xe stability analysis is also required for SMART.

One of the useful strategies for Xe stability analysis is to evaluate the stability index of some representative parameters which are oscillated in perturbed Xe dynamics. The stability index is defined as a natural exponent which describes the growing or decaying amplitude of the oscillation, so it can be measured by introducing an exponential-sinusoidal function to the time-dependent oscillatory parameters. The stability index is the coefficient of the exponent in the function [3].

Three types of Xe oscillations, namely radial, azimuthal, and axial oscillations are simulated for SMART using the MASTER code [4]. And three oscillatory parameters such as radial shape index (RSI), azimuthal shape index (AZI), and axial offset (AO) are evaluated for each oscillation type. These parameters are then plotted to the exponential-sinusoidal function to evaluate the stability index.

In the next section, background and methodologies applied to Xe stability analysis of SMART in this paper are described. The third section presents analysis results. Conclusion of this paper is discussed in the last section.

2. Background & Analysis Methodology

2.1. Stability index

Xe stability can be analyzed by evaluating a factor called stability index. This index can be estimated by expressing time-dependent oscillatory parameters to an exponential-sinusoidal function represented as Eq. (1) [3].

$$f(t;f_0,b,T,t_0,f_{eq}) = f_0 e^{bt}\sin(2\pi t/T+t_0)+f_{eq} \qquad (1)$$

where

$f(t)$: Time-dependent oscillatory functions
f_0 : Amplitude of the parameters
b : Stability index
T : Period
t_0 : Phase shift
f_{eq} : Parameter at the equilibrium state

The stability of a reactor under Xe-induced spatial oscillation can be characterized by the stability index '*b*' of Eq. (1). A positive stability index indicates an unstable core, and a negative value describes that the core is stable as the oscillation continues.

2.2. Time-dependent oscillatory parameters

Xe oscillations are classified into three types according to radial, azimuthal, and axial directions in this study. Radial, azimuthal, and axial Xe oscillations occur between center and peripheral regions, one side and the other side on the X-Y plane, and top and bottom halves of the reactor core, respectively. RSI, AZI, and AO are selected as the parameters to represent the spatial oscillation to evaluate the stability index on each direction. Their definitions are as follows.

$$RSI(t) = \left(P_{in}(t) - P_{out}(t)\right)/P_{tot}(t) \qquad (2)$$

$$AZI(t) = \left(P_L(t) - P_R(t)\right)/P_{tot}(t) \qquad (3)$$

$$AO(t) = \left(P_T(t) - P_B(t)\right)/P_{tot}(t) \qquad (4)$$

where the subscripts '*in*', '*out*', '*L*', '*R*', '*T*', and '*B*' of region-average power *P* mean center region, peripheral region, left side, right side, top half, and bottom half of a core, respectively. Center and periphery regions, and right and left sides of the SMART core are defined as shown in Fig. 1 for radial and azimuthal stability analyses.

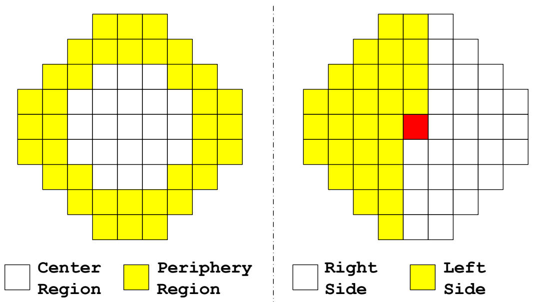

Fig. 1. Center and periphery regions, and right and left sides of the SMART core for radial and azimuthal stability analyses

Behavior of node-wise power distributions in Xe dynamics is obtained by MASTER [4]. The time-dependent parameters are then obtained using the node-wise power distributions.

2.3. Perturbations for Xe Dynamics

Xe dynamics in this paper is simulated by perturbing core condition as shown in Fig. 2. Each state is defined as the function of the state of control rod position (CR), core power (P), various operating conditions (C), and Xe distribution (Xe). State A is the initial state. For the simulation of Xe dynamics, State A is shifted to B by changing the position of some control rods as the perturbation. In consequence, Xe distributions are changed over time. After time t_0 that is applied for Xe buildup under perturbations, the position of the moved control rods returns to the initial position. Xe dynamics calculation is then started from the time point t_0 to record the oscillatory parameters.

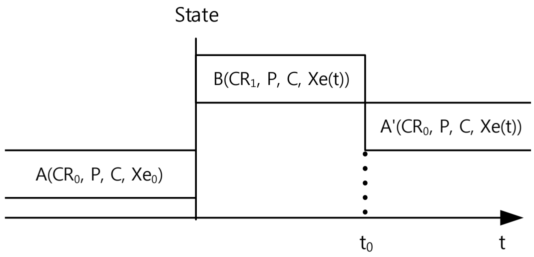

Fig. 2. Scenario for SMART Xe dynamics calculation

Behavior of Xe-induced spatial oscillation depends on various core design and operating parameters such as core size, inserted reactivity, reactivity feedback, power level. Since Xe dynamics simulation should be performed under sufficiently conservative conditions to verify that SMART is stable under Xe oscillation transients, proper conditions are selected and applied in Xe dynamics calculations. Four parameters, namely inserted control rods, core power, T/H feedback, and Xe buildup time (t_0) are considered to set the initial and perturbed states for Xe dynamics. And the previous study [2] had performed parametric studies that

- The larger the inserted reactivity and core power, the larger the amplitude of oscillation. Magnitude of inserted reactivity and core power has almost no effect on the stability index.
- T/H feedback has an effect to make the core stable in Xe dynamics.

In order to confirm the effect of Xe buildup time on the behavior of oscillatory parameters, behaviors of AO with two different buildup times of 1- and 4-hour are compared, while keeping other conditions are same. 4-hour is selected as the expected time to maximize Xe number density at a local position in SMART. Fig. 3 shows AO behaviors.

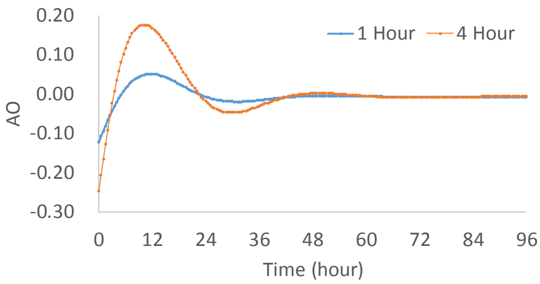

Fig. 3. Behavior of AO with different Xe buildup time

Stability indices for the oscillation shown in Fig. 3 are estimated as -0.080 and -0.079, respectively. These results show that maximization of Xe buildup increases the magnitude of oscillation whereas stability properties are not affected by Xe buildup.

In this regard, the following perturbations are considered to enlarge the amplitude as well as to obtain severe stability index.

1) Inserted control rods
   ⇨ All rods are initially fully out and
   A. All rods located in the center region are fully inserted for radial stability analysis.
   B. All rods located in the right side are fully inserted for azimuthal stability analysis.
   C. All rods are inserted to the top half of the core for axial stability analysis.
2) Core power : HFP
3) T/H feedback : Not considered.
4) Xe buildup time : 4 hour

Note that inactivation of T/H feedback is adopted for more conservative analyses although it is unrealistic. Also, the maximization of reactivity insertion and Xe buildup are considered to increase the amplitude of oscillation in order to clarify the behavior of oscillatory parameters.

2.4. Non-linear curve fitting

In order to evaluate Xe stability, the time-dependent oscillatory parameters such as RSI, AZI, and AO are approximated to an exponential-sinusoidal function represented as Eq. (1). Since this function is non-linear, any numerical methods are required to obtain the solution. Also, this function has 5 unknowns whereas lots of data points are available for curve fitting, a numerical approach such as the least square method is required to get a unique optimum solution.

In order to obtain the parameters numerically, the non-linear least square method is applied. Consider a set of m data points, (t_i, y_i) where i is from 1 to m. Then square sum of residual of each data point represented in Eq. (5) is minimized when optimum parameters are found.

$$S(\mathbf{x}) = \sum_{i=1}^{m}\left(y_i - f(t_i;\mathbf{x})\right)^2 = \sum_{i=1}^{m} r_i(\mathbf{x})^2 \tag{5}$$

where

$\mathbf{x} = \left(f_0, b, T, t_0, f_{eq}\right)^T$: Solution vector

The Gauss-Newton method can be used to find the optimum solution iteratively. Using the solution vector obtained at the k^{th} iteration step, the residual of the i^{th} data point for the $k+1^{th}$ iteration step can be guessed by applying the first order linear approximation as follows.

$$r_i(\mathbf{x}^{k+1}) \simeq r_i(\mathbf{x}^k) + \sum_{j=1}^{5} \left.\frac{\partial r_i(\mathbf{x})}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}^k} \Delta x_j^{k+1} \tag{6}$$

where

$\Delta x_j^{k+1} = x_j^{k+1} - x_j^k$

Also, we know that partial derivatives of the target function S is zero when S is minimized.

$$\left.\frac{\partial S(\mathbf{x})}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}_{opt}} = \sum_{i=1}^{m} \left.\frac{\partial \left(r_i(\mathbf{x})^2\right)}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}_{opt}} = 2\sum_{i=1}^{m} r_i(\mathbf{x}) \left.\frac{\partial r_i(\mathbf{x})}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}_{opt}} = 0 \quad (7)$$

where x is the solution vector and x_{opt} is the optimum solution vector to minimize S. If the solution vector converges at the k+1[th] step ($x_{k+1}= x_{opt}$) and the difference of the solution vectors at the k^{th} and k+1[th] step is marginal ($x_{k+1} \approx x_k$), inserting Eq. (6) into Eq. (7) leads to

$$\sum_{i=1}^{m} \left.\frac{\partial r_i(\mathbf{x})}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}^k} \sum_{j'=1}^{5} \left.\frac{\partial r_i(\mathbf{x})}{\partial x_{j'}}\right|_{\mathbf{x}=\mathbf{x}^k} \Delta x_{j'}^{\,k+1} = -\sum_{i=1}^{m} \left.\frac{\partial r_i(\mathbf{x})}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}^k} r_i(\mathbf{x}^k) \quad (8)$$

Eq. (8) can be converted to a matrix formulation as follows.

$$\left(\mathbf{J}^{\mathbf{T},k}\mathbf{J}^k\right)\Delta\mathbf{x}^{k+1} = \mathbf{J}^{\mathbf{T},k}\mathbf{r}^k \quad (9)$$

where $\boldsymbol{J}^k$ and r^k are Jacobian matrix and residual vector, respectively.

$$\mathbf{J}^k = -\left.\begin{pmatrix} \frac{\partial r_1(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial r_1(\mathbf{x})}{\partial x_5} \\ \vdots & & \vdots \\ \frac{\partial r_m(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial r_m(\mathbf{x})}{\partial x_5} \end{pmatrix}\right|_{\mathbf{x}=\mathbf{x}^k} = \left.\begin{pmatrix} \frac{\partial f_1(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_1(\mathbf{x})}{\partial x_5} \\ \vdots & & \vdots \\ \frac{\partial f_m(\mathbf{x})}{\partial x_1} & \cdots & \frac{\partial f_m(\mathbf{x})}{\partial x_5} \end{pmatrix}\right|_{\mathbf{x}=\mathbf{x}^k}$$

$$\mathbf{r}^k = \left(r_1(\mathbf{x}^k) \quad \cdots \quad r_m(\mathbf{x}^k)\right)^T$$

Then the solution vector for the k+1[th] step is estimated by solving the following equation derived from Eq. (9).

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \left(\mathbf{J}^{\mathbf{T},k}\mathbf{J}^k\right)^{-1}\mathbf{J}^{\mathbf{T},k}\mathbf{r}^k \quad (10)$$

Convergence of the solution can be checked by observing S. In this calculation, iteration procedure is ended when the derivative of the target value S is smaller than a specified criterion.

$$\max\left(\left|\left.\frac{\partial S(\mathbf{x})}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}^{k+1}}\right|\right)_j < \varepsilon_{crit} \quad (11)$$

In the Gauss-Newton method, initial-guessed solution affects the convergence of the problem since the matrix is dependent on the solution vector. In other words, improper initial solution can make this matrix become close to a singular matrix and the solution diverges. Thus it is required to guess the initial solution properly. The initial solutions can be guessed as Eq. (12).

$$\mathbf{x}_0 = \left\{\begin{array}{l} f_{eq} = \frac{1}{m}\sum_{i=1}^{m} f_i \\ T = 2\times\left(f_p(N_p) - f_p(1)\right)/\left(N_p - 1\right) \\ b = \frac{1}{N_p - 1}\sum_{i=1}^{N_p-1} \ln\left|\frac{f_p(i+1) - f_{eq}}{f_p(i) - f_{eq}}\right| / \left(t_p(i+1) - t_p(i)\right) \\ t_0 = \begin{cases} -2\pi t_{f_{eq}}/T & \left(f'(t_{f_{eq}}) \geq 0\right) \\ -2\pi t_{f_{eq}}/T + \pi & \left(f'(t_{f_{eq}}) < 0\right) \end{cases} \\ f_0 = \frac{1}{m}\sum_{i=1}^{m}\left|f_i - f_{eq}\right| e^{-bt_i} / \sin\left(\frac{2\pi t_i / T}{T} + t_0\right) \end{array}\right\} \quad (12)$$

where

- N_p : # of local peaks in time-dependent parameters
- $f_p(i)$: The i^{th} local peak value of the time-dependent parameters
- $t_p(i)$: Time point of $f_p(i)$
- $t_{f_{eq}}$: The first time point when the solution is f_{eq}

And Fig. 4 demonstrates the parameters used in Eq. (12).

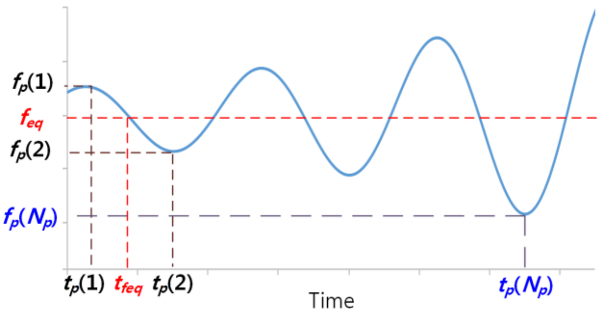

Fig. 4. Definition of parameters to define initial-guessed solution

Although proper initial solution is guessed, solution can diverge during iteration. Unless converged, iteration process should be started from a different initial solution. A new initial solution can be guessed applying small variation on the initial solution determined by Eq. (12). Amount of the variation can be selected using random number.

The algorithm to find the solution vector using non-linear least square curve fitting with the Gauss-Newton method is shown in Fig. 5. A computer code realizing Fig. 5 is developed in this study for non-linear least square fitting of the oscillatory parameters.

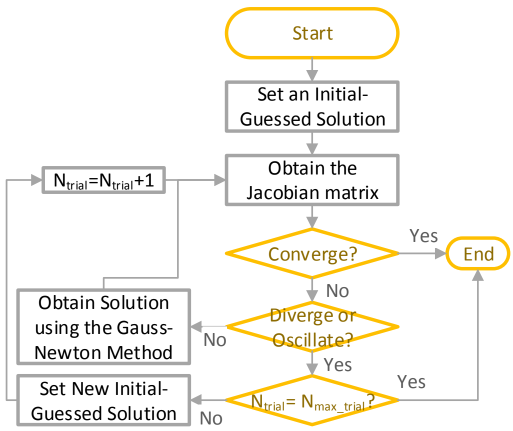

Fig. 5. Flow chart to find the solution vector

3. Analysis Results

Xe stability of SMART is analyzed for the initial and equilibrium cores. BOC, MOC, and EOC for each cycle are selected as the representative burnup for the analysis.

Thus total 6 core states are evaluated for Xe stability analysis for each oscillation type. As a calculation option, 30-minute time step size is used in Xe dynamics.

Table I shows the severest stability index of each oscillation type and its core state. Fig. 6 through Fig. 8 present the behavior of raw and fitted RSI, AZI, and AO of the core state where the severest stability index occurs. These figures prove that the developed code for fitting of the oscillatory parameters can predict the original data with sufficient accuracy. Since the stability indices in Table I are estimated by the code, it is assured that they are also sufficiently accurate. Table I shows that all the severest stability indices have negative value regardless of the oscillation type. Thus it is confirmed that SMART is stable in any transients caused by Xe-induced spatial oscillation.

Table I: Severest Stability Index of Each Oscillation Type and its Core State

Type	Stability Index (1/hr)	Core State
Radial	-0.133	Eq. Cycle, BOC
Azimuthal	-0.090	Init. Cycle, BOC
Axial	-0.079	Init. Cycle, EOC

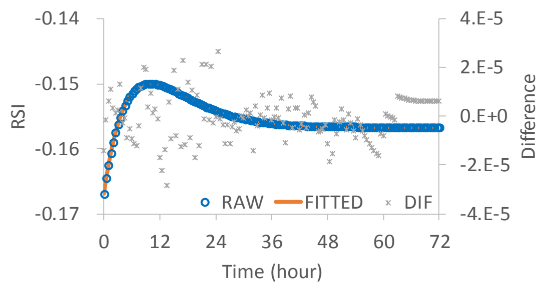

Fig. 6. Raw and fitted RSIs for radial Xe stability analysis

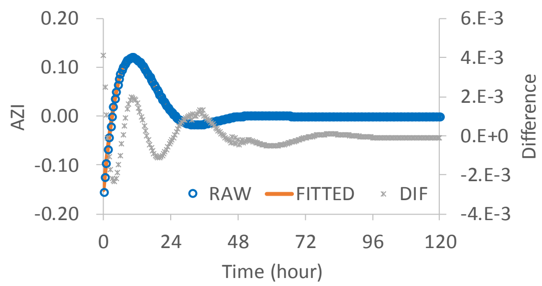

Fig. 7. Raw and fitted AZIs for azimuthal Xe stability analysis

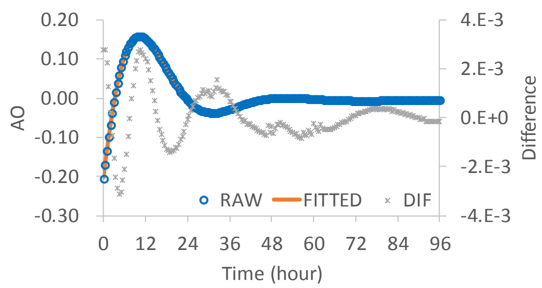

Fig. 8. Raw and fitted AOs for axial Xe stability analysis

4. Conclusions

Xe stability of the SMART core is analyzed for radial, azimuthal, and axial spatial oscillations by evaluating the stability indices which describe the growing or decaying amplitude of the oscillation on each direction. In order to evaluate them, Xe dynamics calculations using MASTER are performed at various core states with proper perturbations and then three oscillatory parameters of RSI, AZI, and AO are estimated for each oscillation type. Their stability indices are obtained by using a computer code developed in this work, which is based on the non-linear least square curve fitting and the Gauss-Newton iterative method.

Although unrealistic perturbations such as no consideration of T/H feedback are adopted for conservative analyses of Xe stability, all the evaluated stability indices are negative in any core states. Thus it can be concluded that SMART is stable for any transients caused by Xe-induced spatial oscillations.

ACKNOWLEDGMENT

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