Generation of Exact Discontinuity Factors for Nodal Expansion Method Using 2 by 2 Nodes per Assembly

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1. Introduction

Modern reactor physics calculations for PWR are largely based on two-step analysis to save computational cost and time. The key component of this two-step method is homogenization of various cross sections into coarse-mesh level to calculate with simple diffusion solver. As homogenization of a fuel assembly is done based on nodal equivalence theory, a flux discontinuity between the neighboring nodes occurs from different flux shapes in heterogeneous and homogeneous models. In order to set up continuity equations for the homogenized assemblies, the discontinuity factor (DF) concept was adopted [1]. The DF was defined on assembly interfaces, and nodal calculations with homogenized cross sections were done in assembly-wise coarse-mesh basis.

However, for the case that we need to perform post processing for detailed information about the assembly, such as pin power reconstruction, more information is required. Also, assembly-wise nodal calculations often result in poor accuracy, particularly with the PWR fuel assemblies which are 20cm wide in width. For this reason, subdivision of an assembly for the nodal expansion method (NEM) calculation with transverse leakage integration and generation of corresponding DFs are suggested. In the following sections, possible methods to generate DFs for 2 by 2 division of a fuel assembly are suggested and tested.

2. NEM-based 2 by 2 DF Generation Method

In this section, DF generation for 2 by 2 division of a single assembly based on NEM method is described in detail. The reference heterogeneous solution and homogenized cross sections were provided by the 2-dimensional method of characteristics (MOC) lattice code, DeCARD2D [2].

2.1 Two-kernel NEM calculation

Based on reference net currents on the assembly boundaries, four two-kernel NEM calculations were done per an assembly. At each NEM calculation, two surface fluxes for DFs and two node-wise fluxes were determined. In order to define DFs only on assembly perimeter surfaces, DFs on inter-assembly interfaces were set as unity as illustrated in Fig. 1. Here, two DFs are defined per an assembly surface.



Fig. 1. 2 by 2 division of an assembly and DFs

As an assembly was divided into 2 by 2 nodes, four two-kernel NEM calculations were able to be done for four sets of neighboring nodes. For these NEM calculations, 1-dimensional fluxes were expanded by the 4^{th} order polynomial basis functions. The 2^{nd} order transverse leakage terms were calculated using the node-averaged transverse leakage values in three sequential nodes. Approximation of 1-dimensional transverse leakage into 2^{nd} order polynomial is as follows:

$$L_{u}(u) = \sum_{i=0}^{2} L_{i}P_{i}(u) = L_{0}P_{0}(u) + L_{1}P_{1}(u) + L_{2}P_{2}(u).$$
(1)

Here, the coefficients are defined as follows with respect to a centered cell indexed as *m*:

$$L_{0u} = \int_{0}^{L} L_{u}(u) du = \overline{L_{u}}, \qquad (1-a)$$

$$L_{1u} = L_{u,right} - L_{u,left}, \qquad (1-b)$$

$$L_{2u} = 3 \times (L_{u,right} + L_{u,left} - 2 \times L_{0u}),$$
(1-c)

with,

$$L_{gu,left}^{m} = \frac{D_{g}^{m} \overline{L}_{gu}^{m} + D_{g}^{m-1} \overline{L}_{gu}^{m-1}}{D_{g}^{m} + D_{g}^{m}},$$
 (1-d)

$$L_{gu,right}^{m} = \frac{D_{g}^{m} \overline{L}_{gu}^{m} + D_{g}^{m+1} \overline{L}_{gu}^{m+1}}{D_{g}^{m} + D_{g}^{m+1}}.$$
 (1-e)

Since the NEM calculations were done based on net current boundary conditions from DeCART2D, we obtained different node-wise fluxes from two NEM calculations sharing a single node. These were resulted from different net current values defined from MOC transport and NEM diffusion method on inter-assembly node boundaries. DFs on assembly boundaries were able to be obtained despite of this unphysical result, but expected to show a large error when reproducing the reference solution.

2.2 Convergence of node-wise flux

As discussed in previous section, the reference net currents on inter-assembly interfaces are not identical to the NEM-calculated net currents. From the nodal equivalence theory, net currents on the assembly's perimeter must be same for the heterogeneous and the homogeneous problems to preserve reaction rates. However, the flux distribution and interstitial net currents of adjacent nodes are different between the two cases. In order to get consistent node-wise fluxes from the four two-kernel NEM sweeps, an iterative method to update the interfacial net current was needed. Starting from transport net current boundary conditions given by DeCART2D, we performed a sweep, or four two-kernel NEM calculations, inside an assembly. Then, using the updated net currents, re-do the NEM sweep. As the iterative sweeps were proceeded, the net currents were converged from transport net current to diffusion net current, and node-wise fluxes as well. Fig. 2 illustrates how the NEM sweep is done to update net currents.



Fig. 2. Flowchart of the net current update algorithm

2.3 Generation of assembly-wise DFs

In order to apply the homogenized cross sections and DFs from a color-set calculation for large whole-core problems with a better accuracy, the Albedo-corrected Parameterized Equivalence Constants (APEC) method was proposed by Kim and Kim [3,4]. The APEC method along with functionalization of DFs was able to achieve significant improvement accuracy in nodal calculations [5]. However, this DF correction method was defined on "a single DF per an assembly surface" basis. If a single DF can be defined from 2 by 2 nodal

color-set calculation with marginal accuracy defect, it can be a better option with the functionalization of DFs method.

From the aforementioned motivation, an assemblywise DF, which is based on surface flux average was defined as follows:

$$DF_{avg} = \frac{\frac{1}{2} \left(\phi_{s,1}^{\text{het}} + \phi_{s,2}^{\text{het}} \right)}{\frac{1}{2} \left(\phi_{s,1}^{\text{hom}} + \phi_{s,2}^{\text{hom}} \right)},$$
 (2)

where, $\phi_{s,1}$ and $\phi_{s,2}$ are surface fluxes of two adjacent nodes for the NEM calculation.

3. Results and Discussions

In previous sections, several methods to generate DFs for 2 by 2 assembly division were suggested. In order to test the accuracy of those methods, a 3 by 2 assembly color-set problem was considered to verify the accuracy of the evaluated DFs. The problem was consisted of a baffle-reflector layer along with L-shape fuel assemblies. Detailed illustration of the problem is shown in Fig. 3 with corresponding boundary conditions.

The L-shape color-set problem with heterogeneous geometry was solved by DeCART2D code to provide reference multiplication factor and assembly-wise power distribution. Detailed information of DeCART2D input and resulted multiplication factor is described in Table I.



Fig. 3. L-shape baffle included color-set problem

With homogenized cross sections, DFs were evaluated from the suggested methods. In addition, 1 by 1 NEM based DFs were evaluated as well for comparison.

Table I: Model description and the reference keff

Fuel Assembly 1	16 by 16 fuel pin (2.82 w/o)
Fuel Assembly 2	16 by 16 fuel pin (4.88 w/o)
Pin pitch	1.2658 cm
Assembly pitch	20.2528 cm
$\overline{k_{eff}}$	1.181939

Homogenized cross sections and the evaluated DFs were then imported to conventional NEM calculation

algorithm to confirm if they can reproduce the reference solution. Table II lists the suggested methods and the corresponding results. Here, DFs from 1 by 1 and 2 by 2 divisions were used to reproduce the reference solution based on the corresponding assembly divisions. Meanwhile, DFs from 1 by 1 NEM resulted 140pcm of reactivity error, showed the necessity of this research. DFs from 1 by 1 NEM were able to reproduce the reference solution correctly, while 2 by 2 NEM calculation without iterative net current correction failed as expected. With the iterative NEM sweep method, node-wise fluxes converged in 4~5 iterations. Only with this simple correction, the reactivity error from the reference decreased to less than 1pcm. DF distributions for the two cases are shown in Figs. 4 and 5.

Table II: Reproductions of the reference solution from the evaluated DFs with the suggested methods

DF generation method	$\Delta \rho$ (pcm)	Assembly power Max. error (%)			
NEM 1x1	0.10	0.002			
NEM 2x2 (No iteration)	-3.51	0.277			
NEM 2x2 (Iteration)	0.93	0.009			
NEM 2x2 (Assembly-wise DF)	6.01	0.108			

The last row of Table II is showing a result from the assembly-wise DFs from 2 by 2 DFs with iterative correction. The approximation from Eq. (2) failed to closely estimate DFs which is shared by neighboring nodes in 2 by 2 NEM.

0.982		0.998		1.043			1.057				
0 <mark>.992</mark>		1.00	1.00		0.990	1.053		1.00	1.00		1.031
	1.00			1.00			1.00			1.00	
	1.00			1.00			1.00			1.00	
0.990		1.00	1.00		<mark>0.990</mark>	1.050		1.00	1.00		1.048
	0.979			1.002			1.042			1.081	

Fig. 4. Fast (left) and thermal (right) group 2 by 2 DFs obtained from the single-sweep (no iteration) method for the assembly located at upper left side.

	0.991			0.988			1.052			1.046	
0.992		1.00	1.00		0.989	1.052		1.00	1.00		1.030
	1.00		-	1.00 1.00			1.00 1.00			1.00	
<mark>0.99</mark> 1	1.00	1.00	1.00	1.00	0.991	1.051	1.00	1.00	1.00	1.00	1.050
	0.989			0.990			1.052			1.067	

Fig. 5. Fast (left) and thermal (right) group 2 by 2 DFs obtained from the iterative net current update method for the assembly located at upper left side.

4. Conclusions

New DF generation methods for 2 by 2 NEM calculation were proposed and tested for a color-set problem. Cross section homogenization and net currents calculations were done using the DeCART2D transport code. Since the nodal method is based on diffusion method, two-kernel NEM analyses were done iteratively to force net currents to approach to the reference values. From the test on a color-set problem, it was demonstrated that the iterative method was able to reproduce the reference solution. For a potential application of the method to practical design and analysis, further studies are necessary for a simpler and efficient formulation.

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