Implementation of Iterative Resonance Integral Table and Subgroup Methods in STREAM for High Temperature Reactor Analysis

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1. Introduction

As one of the generation four nuclear reactor designs, a high temperature gas-cooled reactor (HTR) has been in the spotlight with its high safety feature. To analyze HTRs, DeCART uses an iterative resonance integral table (i-RIT) method and subgroup method for resonance treatment [1,2]. The same methods have also been implemented in STREAM [3] in this paer and the accuracy has been evaluated by comparing the solutions with those of Monte Carlo code MCS [4]. Sensitivity tests for energy group structure are also carried out.

2. Methods

2.1 Iterative Resonance Integral Table (i-RIT) Method

The i-RIT method performs an iterative calculation from an initial guess of the macroscopic absorption cross section. It carries out the Method of Characteristics (MOC) calculation to obtain a background cross section and the corresponding resonance integral is calculated from the pre-generated resonance integral table (RIT). Based on the resonance integral, the absorption cross section is updated and the MOC calculation is carried out again with the updated macroscopic absorption cross section until it converges.

Fig. 1 shows the calculation flow of the i-RIT method.

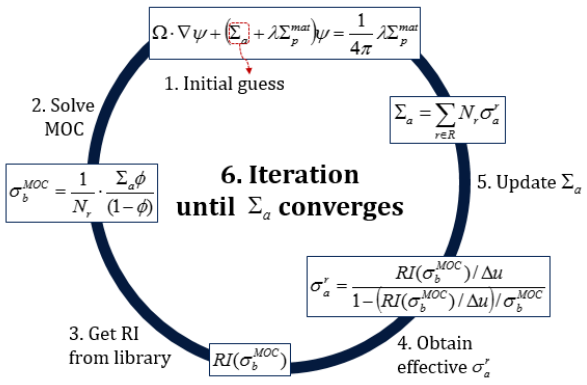

Fig. 1. Calculation flow of i-RIT method

DeCART uses another i-RIT method (i-RIT(a) in [1]) which differs from the Fig. 1 scheme (i-RIT(b) in [1]). The i-RIT(a) method uses the pregenerated absorption cross section tabulated as a function of an equivalence cross section, whereas the i-RIT(b) method directly iterates the absorption cross section.

2.2 Subgroup Method

The subgroup method divides resonance cross section into seven levels typically and obtains background cross sections through the MOC calculation seven times for each level. From the background cross sections, the corresponding absorption cross sections are called from the pregenerated table and summed up with subgroup weights to calculate an effective cross section.

The subgroup levels and weights were generated for STREAM to carry out the subgroup calculation. Eq. (1) describes an effective absorption cross section composed of the subgroup parameters and Eq. (2) describes conditions for the subgroup weights in Eq. (1).

$$\sigma_{ak,eff} \cong \frac{\sum_{n=1}^{7}\left(\omega_n \sigma_{an}\frac{\sigma_{bk}}{\sigma_{an}+\sigma_{bk}}\right)}{1-\sum_{n=1}^{7}\left(\omega_n \frac{\sigma_{bk}}{\sigma_{an}+\sigma_{bk}}\right)} \quad \text{where } 1\le k\le 19, \qquad (1)$$

$$\left[\sigma_{ak,eff}-\sum_{n=1}^{7}\left(\omega_n\sigma_{an}\frac{\sigma_{bk}}{\sigma_{an}+\sigma_{bk}}\right)\right]^2 \cong 0 \text{ and } \sum_{n=1}^{7}\omega_n \cong 1, \qquad (2)$$

where k is the background cross section index (total 19), n is the subgroup level index (total 7), and ω_n is the subgroup weight. Eq. (2) can be expressed in matrix form as shown in Eq. (3).

$$AW \cong \Sigma \Leftrightarrow \begin{bmatrix} \sigma_{a1}\frac{\sigma_{a1,eff}+\sigma_{b1}}{\sigma_{a1}+\sigma_{b1}} & \cdots & \sigma_{a7}\frac{\sigma_{a1,eff}+\sigma_{b1}}{\sigma_{a7}+\sigma_{b1}} \\ \vdots & \ddots & \vdots \\ \sigma_{a1}\frac{\sigma_{a19,eff}+\sigma_{b19}}{\sigma_{a1}+\sigma_{b19}} & \cdots & \sigma_{a7}\frac{\sigma_{a19,eff}+\sigma_{b19}}{\sigma_{a7}+\sigma_{b19}} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \vdots \\ \omega_7 \end{bmatrix} \cong \begin{bmatrix} \sigma_{a1,eff} \\ \sigma_{a2,eff} \\ \vdots \\ \sigma_{a18,eff} \\ \sigma_{a19,eff} \end{bmatrix}. \qquad (3)$$

Fig. 2 shows the flow for the subgroup weight calculation.

$\omega_n^{(0)}$ initialization

Start loop (i)

$$\mathrm{A}^{\mathrm{T}}\mathrm{A}\mathrm{W}^{(i)} = \mathrm{A}^{\mathrm{T}}\lambda^{(i)}\Sigma \quad \text{where} \quad \lambda^{(i)} = \sum_{n=1}^{N}\omega_n^{(i)}$$

Least square fitting using *fminsearch* function

$$\mathrm{W}^{(i+1)} = \mathrm{W}^{(i)} / sum(\mathrm{W}^{(i)})$$

$$\lambda^{(i+1)} = \sum_{n=1}^{N}\omega_n^{(i+1)}$$

$$\textbf{if} \quad \frac{\sum_{n=1}^{N}\left(\omega_n^{(i+1)} - \omega_n^{(i)}\right)^2}{N} < eps \quad \Rightarrow \textbf{break}$$

End loop (i)

Fig. 2. Flow for subgroup weight calculation

As shown in Fig. 2, from an initial guess $\lambda^{(0)}$, a set of weights are found to satisfy Eq. (2). These weight sets minimize an effective cross section error, which are used to the STREAM subgroup calculation.

3. Numerical Results

3.1 Problem Description

An HTR compact problem as shown in Fig. 3 was solved by DeCART and STREAM.

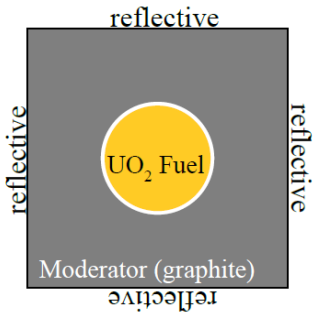

Fig. 3. HTR homogenized compact problem

The geometry and materials of the problem are listed in Table I. The compact is homogenized for simplicity.

Table I: Problem Description

Region	Material	Radius or Pitch (cm)
Fuel	UO_2 + Graphite	0.6225 [Radius]
Gap	Helium	0.6350 [Radius]
Moderator	Graphite	1.749165 [Pitch]

To check the consistency of the two codes, the same 190 group DeCART library was used to both codes. The results are shown in Fig. 4.

3.2 Results

In Fig. 4, it is observed that the STREAM i-RIT results (pink pentagons) show similar tendencies as the DeCART i-RIT results (black triangles). Also, the STREAM subgroup results (sky-blue stars) show the same tendencies as the DeCART subgroup results (red-brown triangles). Thus, the consistency has been confirmed between STREAM and DeCART.

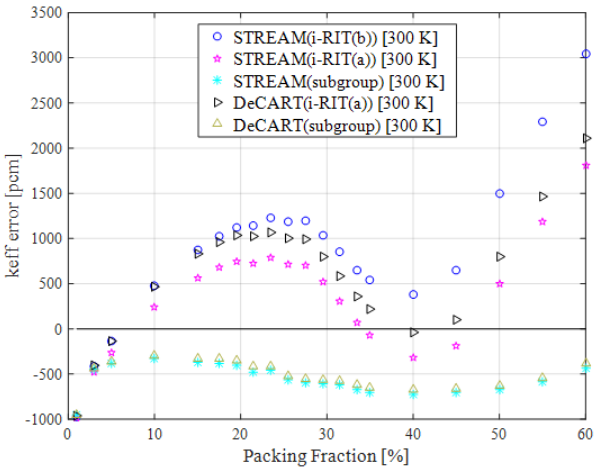

Fig. 4. Comparison between STREAM and DeCART

Now, a newly generated 190 group STREAM library (same energy group structure as the DeCART library) was applied to STREAM. Table II and Fig. 5 show the analysis results of the i-RIT and subgroup methods for the 190 group STREAM library at a fuel temperature of 300 K.

Table II: STREAM k-eff using 190 group library

Packing Fraction (%)	*MCS Reference	STREAM i-RIT	STREAM Subgroup
1	1.50988	1.53254	1.51079
3	1.63471	1.69131	1.59898
5	1.60550	1.68741	1.53638
10	1.50458	1.62109	1.42773
15	1.42293	1.55719	1.36357
17.5	1.38857	1.52727	1.34041
19.5	1.36376	1.50563	1.31065
21.5	1.34140	1.48435	1.28642
23.5	1.32004	1.46440	1.26461
25.5	1.30125	1.44595	1.25134
27.5	1.28312	1.42884	1.23995
29.5	1.26611	1.41158	1.22780
31.5	1.25023	1.39551	1.21179
33.5	1.23578	1.38050	1.19678
35	1.22554	1.36989	1.21199
40	1.19420	1.33693	1.18124
45	1.16738	1.30731	1.14978
50	1.14460	1.28189	1.12694
55	1.12494	1.24078	1.10648
60	1.10764	1.23784	1.09081

*MCS solutions have 15~25 pcm of standard deviations.

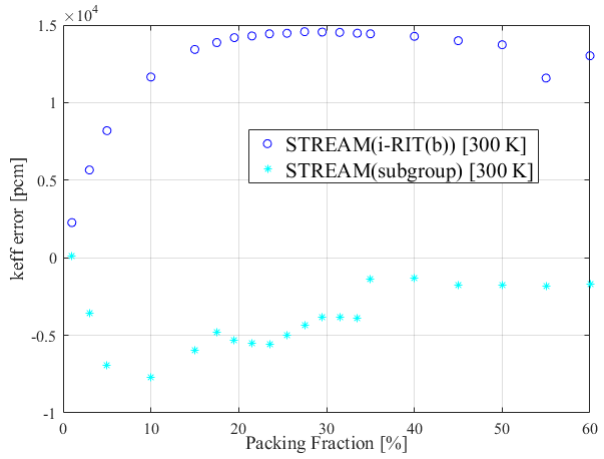

Fig. 5. STREAM k-eff error using 190 group library

In Fig. 5, the scale of the y-axis is 10^4. Since both i-RIT and subgroup methods showed large k-eff errors, the reactivity errors in resonance energy groups at a packing fraction of 23.5 % were investigated as shown in Fig. 6.

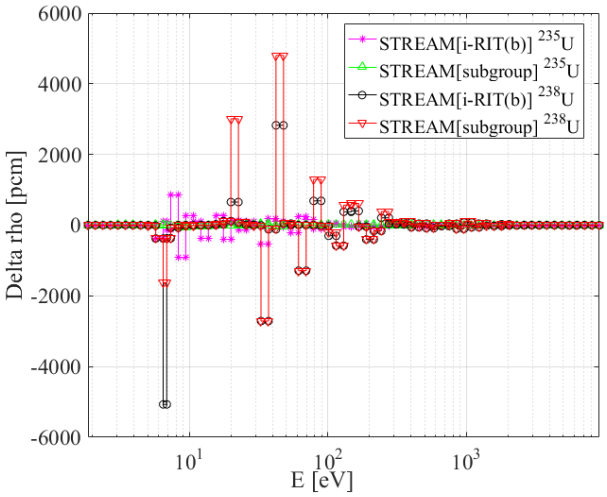

Fig. 6. Reactivity errors in resonance energy group

In Fig. 6, it is observed that the largest errors occur at the energies of 6.67, 20.87, 35, and 45 eV by ^{238}U.

3.3 Sensitivity Test for Energy Group Structure

To decrease the errors in Fig. 5, the energy group located at 6.67 eV (one of the largest resonance peaks of ^{238}U) has been divided into 10 intervals with energy width of approximately 0.04 eV. Likewise, the energy group located at 20.87 eV has been divided into 22 intervals with energy width of approximately 0.12 eV. Thus, the 190 energy group structure became a 220 energy group structure.

Tables III and Fig. 7 show the analysis results of the i-RIT and subgroup methods using the 220 group STREAM library at a fuel temperature of 300 K.

Table III: STREAM k-eff using 220 group library

Packing Fraction (%)	*MCS Reference	STREAM i-RIT	STREAM Subgroup
1	1.50988	1.51294	1.51214
3	1.63471	1.64152	1.63618
5	1.60550	1.61468	1.60666
10	1.50458	1.51724	1.50441
15	1.42293	1.43725	1.42225
17.5	1.38857	1.40347	1.38765
19.5	1.36376	1.37907	1.36272
21.5	1.34140	1.35647	1.33995
23.5	1.32004	1.33564	1.31848
25.5	1.30125	1.31640	1.29931
27.5	1.28312	1.29853	1.28124
29.5	1.26611	1.28165	1.26425
31.5	1.25023	1.26602	1.24849
33.5	1.23578	1.25144	1.23391
35	1.22554	1.24116	1.22364
40	1.19420	1.21004	1.19258
45	1.16738	1.18333	1.16590
50	1.14460	1.16059	1.14344
55	1.12494	1.13530	1.12199
60	1.10764	1.12348	1.10691

*MCS solutions have 15~25 pcm of standard deviations.

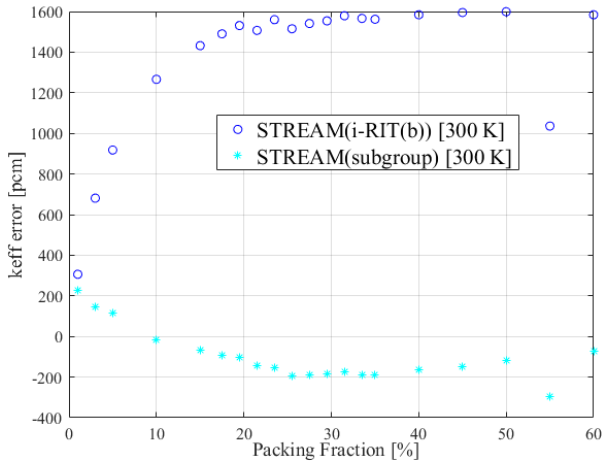

Fig. 7. STREAM k-eff error using 220 group library

As shown in Fig. 7, the i-RIT and subgroup methods show improved accuracy when compared to Fig. 5. Especially, the subgroup method shows very accurate k-eff values with errors below 300 pcm compared with the MCS reference solutions.

4. Conclusions

The i-RIT and subgroup methods were implemented in STREAM. For the 190 group STREAM library, both methods showed large k-eff errors. For the 220 group library dividing the ^{238}U resonance peaks into several intervals, both methods showed improved accuracy. Especially, the subgroup method with the 220 energy group structure showed very high accuracy with k-eff errors below 300 pcm.

Acknowlegment

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MSIP) (No. 2017M2A8A1014757).

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