Calandria tube sagging model of CAISER based on the elastic-plastic beam theory

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1. Introduction

So far, severe accident progression of the CANDU reactor has been explored by the MAAP-CANDU or MAAP-ISAAC code. Nowadays, necessity on the new mechanistic analysis code for regulatory purpose is arising; CAISER (Candu Advanced Integrated SEveRe accident code) is on development to analyze severe accident progression on the typical CANDU reactors. In the present study, we examined modeling concept for the sagging of calandria tubes in the calandria tank, by using elastic-plastic deflection of clamped-clamped beam theory.

2. Problem define

2.1 Elastic deflection of clamped-clamped beam

In the calandria tank, 380 calandria tubes (pressure tubes) are horizontally laid. The length of calandria tubes was about 6 meters, and it was supported at the both ends with clamping. So, the deflection of calandria tube by its own weight is modeled by elastic deflection of a beam at the both ends with clamping. Figure 1 shows problem definition of elastic deflection. Uniform distributed load q(x) is assigned along the length of L. Then the bending moment along the beam is expressed by the simple equation as follows (equation 1).

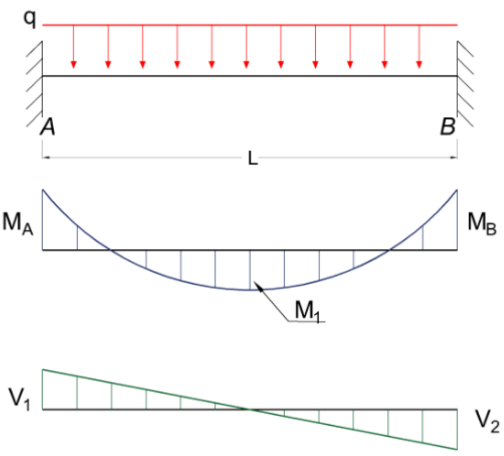

Figure 1 Schematic of the elastic deflection of clamped-clamped beam [1].

$$
\begin{aligned}
&\frac{d^2y}{dx^2} = \frac{M(x)}{EI} \\
&EI\frac{d^4y}{dx^4} = -q(x) \\
&EI\frac{d^3y}{dx^3} = V(x) = -q(x)x + C_1 \\
&EI\frac{d^2y}{dx^2} = M(x) = -\frac{1}{2}q(x)x^2 + C_1x + C_2 \\
&EI\frac{dy}{dx} = EI\theta(x) = -\frac{1}{6}q(x)x^3 + \frac{1}{2}C_1x^2 + C_2x + C_3 \\
&EIy(x) = -\frac{1}{24}q(x)x^4 + \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + C_3x + C_4
\end{aligned} \qquad (1)
$$

Here, M(x) is bending moment, E is elastic modulus, I is area moment of inertia, V(x) is force, θ(x) is curvature, and y(x) is deflection length of the beam. Above equations with 4 unknown coefficients are determined by appropriate boundary conditions as follows

$$\theta(x) = 0,\ y(x) = 0 \qquad \textit{for } x = 0, L \qquad (2)$$

Using this, resultant deflection of the beam is expressed as

$$EIy(x) = -\frac{1}{24}q(x)x^4 + \frac{1}{12}q(x)Lx^3 - \frac{1}{24}q(x)L^2x^2 \qquad (3)$$

2.2 Plastic deflection of clamped-clamped beam

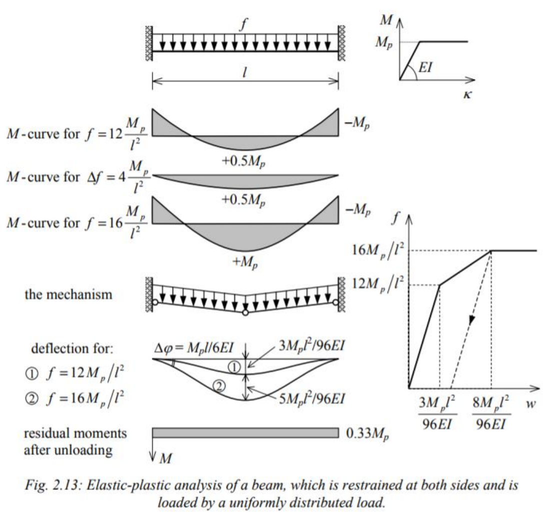

Figure 2 Schematic of the elastic-plastic deflection of clamped-clamped beam [2].

After the bending moment on the beam exceeds its own yield stress of material (corresponds to the M_p), the beam is going to the plastic deflection. Maximum bending moment is applied at the both ends of the beam, so the additional deflection acts like the both ends are no longer clamped. Figure 2 shows schematic diagram of elastic-plastic deflection of the beam. After the yield bending moment, the maximum deflection at the middle of the beam is increased.

3. Estimation by the model

3.1 Material properties

As expressed in the equation (3), to estimate deflection of the beam, Young's modulus (E) and area moment of inertia (I) should be determined. Young's modulus is estimated by MATPRO as follows [3]

$$E = \begin{cases} 1.088\times10^{11} - 5.475\times10^{7}T & \text{for } T < 1090K \\ 4.912\times10^{10} - 4.827\times10^{7}(T-1090) & \text{for } 1090\text{K} \le T \le 1240K \\ \max\left(1\times10^{10}, 9.21\times10^{10} - 4.05\times10^{7}T\right) & \text{for } T > 1240K \end{cases} \quad (4)$$

Area moment of inertia is calculate for cylindrical pipe shell as follows

$$I = \frac{\pi\left(D^4 - d^4\right)}{64} = \frac{\pi\left(R^4 - r^4\right)}{4} \quad (5)$$

Here, D and R is outer diameter and radius of pipe, d and r is inner diameter and radius.

3.2 Resultant deflection length

Above mentioned model can estimate deflection of calandria tube in normal operation. Young's modulus at temperature of 500K is about 81.4E9 GPa. Area moment of inertia for calandria tube is about 1.1347E-6 m^4. Weight q(x) is about 490.5N/m, is estimated by the total mass of fuel channel (300 kg). Therefore, in the elastic deflection range, maximum displacement is at the center location of calandria tube and the amount is about -0.0179m (~1.8cm) at 500K. It is similar value from previous study [3].

In the case of plastic deflection, we found yield moment of bending acting on the calandria tube. From [4], yield stress of zirconium alloy is about 85 MPa at 1000K. The estimated yield bending moment is estimated as follows.

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

$$M(x) = -\frac{1}{2}w(x)x^2 + \frac{1}{2}w(x)Lx - \frac{1}{12}w(x)L^2$$

$$\sigma_{\max} = \frac{M(x)y_n}{I} \quad (6)$$

y_n is distance from neutral axis to the edge of surface

$$\sigma_{\max} = \frac{M(0)y_n}{I} = \frac{-\frac{1}{12}*490.5*36*0.0659}{1.1347E-6} = 85.46E6$$

$$\sigma_{\max} = \frac{M(\frac{L}{2})y_n}{I} = \frac{\frac{1}{24}*490.5*36*0.0659}{1.1347E-6} = 42.73E6$$

At 973K, maximum yield stress is about 85.46E6Pa, is nearly the same in the literature. Therefore, the local temperature of calandria tube reaches to about 1000K, it starts to plastic deflection.

4. Summary

Calandria tube deflection by elastic-plastic beam theory was performed. The elastic deformation was modeled by the clamped-clamped beam theory with the boundary conditions. Plastic deflection was modeled by yield stress (maximum bending moment) of the elastic beam theory. It was about 85MPa, showed good agreement with literature. Based on this approach, CAISER can calculate sagging phenomena of calandria tube in the calandria tank. We are now implementing present conceptual model to the CAISER, and will validate with experimental results in the literature.

ACKNOWLEDGMENTS

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