A Study on in-situ Two-group SPH Factor Correction in GET-Based Pin-by-Pin Core Analysis

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1. Introduction

Advanced reactor designs need accurate neutronic calculations. In addition, high-fidelity multi-physics and multi-dimensional reactor analyses also require accurate prediction of detailed power profile. The direct whole-core transport calculation is quite promising, however it needs a lot of computational burden and memory. Another possible solution is the pin-by-pin core calculation with a low-order operator such as the diffusion one.

As in the standard two-step nodal core analysis, two-step pin-wise analysis is based on pin-wise spatial homogenization in the conventional single lattice transport analysis with the reflective boundary condition. Consequently, resulting pin-wise two-group cross sections are always subject to unavoidable errors due to the unphysical boundary condition. There are two well-known homogenization techniques to reduce the error of key quantities; generalized equivalence theory (GET) [1,2] and super-homogenization method (SPH) [3,4]

Recently, W. Kim and Y. Kim [5] introduced the albedo-corrected parameterized equivalence constants (APEC) method to reduce error of fuel assembly homogenization by parameterizing assembly two-group constants as a function of neutron leakage. Similarly, the APECp (APEC for Pin-cell) method [6] was investigated to reduce error of pin-wise spatial homogenization. With GET-based APECp method, two-group cross-section corrections can be rather noticeable, while it is very difficult to adjust the surface-wise discontinuity factors. In this paper, we introduce the SPH concept into the APECp method to improve the accuracy of the pin-wise core analysis.

Pin-wise two-group constants are produced using a 2-D method of characteristics (MOC) based lattice code DeCART2D [7]. Pin-by-pin core analyses are performed using the HCMFD method [8] in this work.

2. Leakage correction with SPH parameters

In the standard two-step procedure, the flux-volume-weighted group constants (FWCs) and surface-wise discontinuity factors are generated from a single fuel assembly transport analysis. The key idea of APEC (or APECp) method is correcting the initial group constants by taking into account neutron leakage. Previously, we tried to correct the group constants directly as a function of an effective neutron leakage. In this work, the SPH concept is considered to reduce the error of the initial FWCs. A XS-dependent SPH factor is introduced and is functionalized with a pin-wise leakage information

2.1 XS-dependent SPH factor and Functionalization

In the SPH method, the average reaction rate is to be preserved by introducing the group-dependent SPH factor μ_g:

$$\tau_g = \mu_g \Sigma_g^{ref} \phi_g = \Sigma_g^{ref} \phi_g^{ref}, \quad (1)$$

where Σ_g^{ref} is reference cross-section, ϕ_g^{ref} is reference heterogeneous flux, ϕ_g is homogenous flux.

From the definition of the SPH factor, the reference reaction rate can be preserved if reference cross-sections and SPH factors are available. For the initial core calculation, however, the available information is only initial single fuel assembly group constants such as cross-sections and discontinuity factors. Therefore, we introduce a XS-dependent SPH factor:

$$SPH_{\Sigma g\alpha} = SPH_g \times \frac{\Sigma_{\alpha g}^{ref}}{\Sigma_{\alpha g}^{SA}}, \quad (2)$$

where $\Sigma_{\alpha g}^{SA}$ is cross-sections from the single fuel assembly calculation and SPH_g is the standard SPH factor.

Previous studies [6] showed that two-group pin-wise group constants have a relationship with the node-average CFR, defined as below;

$$CFR_g^m = \frac{\sum_s J_g^s}{\phi_g^m}, \quad (3)$$

where J_g^s is the net outward current ϕ_g^m is the node-average flux.

Similarly, the XS-dependent SPH factor in Eq. (2) is parameterized in this work. For the functionalization of the SPH correction factor in terms of the CFR, we consider three different regions since they have different characteristics; 1) FA of inner core region, 2) FA near the baffle-reflector, 3) baffle-reflector regions.

The pin-wise changes in XS-dependent SPH factor from their initial values are functionalized by change of node-average CFR as follows:

1) FA of inner core region

$$\Delta SPH_{1st\_\Sigma\alpha} = a_{1,F}\Delta CFR_{1st} + a_{2,F}\Delta CFR_{2nd} \quad (4)$$

$$\Delta SPH_{2nd\_\Sigma\alpha} = a_{1,T}\Delta CFR_{1st} + a_{2,T}\Delta CFR_{2nd} \quad (5)$$

2) FA near the baffle-reflector

$$\Delta SPH_{1st\_\Sigma\alpha} = a_{1,F}\Delta CFR_{1st} + a_{2,F}\Delta CFR_{2nd} + a_{3,F} \quad (6)$$

$$\Delta SPH_{2nd\_\Sigma\alpha} = a_{1,T}\Delta CFR_{1st} + a_{2,T}\Delta CFR_{2nd} + a_{3,T} \quad (7)$$

3) Baffle-reflector region

$$\Delta SPH_{1st\_\Sigma\alpha} = a_{1,F}\Delta CFR_{1st} + a_{2,F}\Delta CFR_{2nd} + a_{3,F} \quad (8)$$

$$\Delta SPH_{2nd\_\Sigma\alpha} = a_{1,T}\Delta CFR_{1st} + a_{2,T}\Delta CFR_{2nd} + a_{3,T} \quad (9)$$

where $a_{1,g}$, $a_{2,g}$, and $a_{3,g}$ are constants and the initial XS-dependent SPH is unity and the ΔCFR is defined as

$$\Delta CFR_g = CFR_g - CFR_g^{initial} \quad (10)$$

In this preliminary work, one-point pre-correction of the group constants is considered since the flux is not preserved after application of SPH factor. Therefore, corresponding node-average CFR of SA color-set calculation are used for functionalization. For determination of the coefficient in Eqs. (4) to (9), the standard least square method is used.

2.2 Color-set Analyses for SPH Functionalization

In order to determine proper coefficient of the fitting functions, a few color-set calculations are needed. In this work, only six different simple color-set calculations were considered to determine three different APECp functions. Figure 1 shows color-set problem to generator fitting data for the interior FAs.

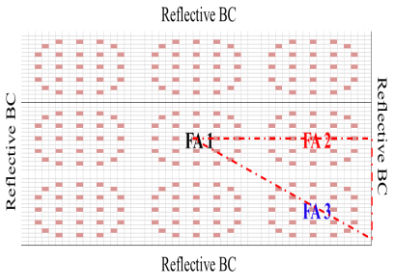
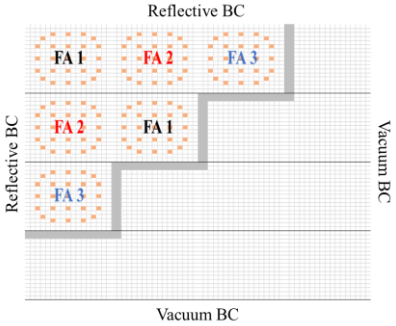
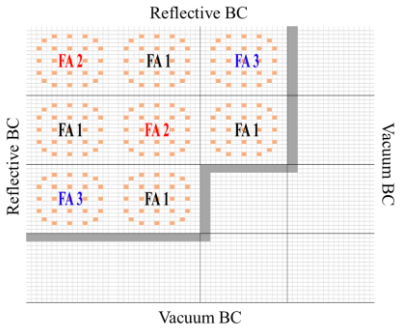


Fig. 1. Simple color-set problem for interior FAs

Two different baffle-color-set problems are considered as shown in Fig. 2. In addition, the initial reference group constants for each different type of baffle-reflector are determined from these baffle color-set calculations. In this study, 6~9 baffle-color-set problems were used for fitting both FAs near baffle-reflector and baffle-reflector itself.



a) Type 1 baffle color-set



b) Type 2 baffle color-set

Fig. 2. Color-set problem for FAs near baffle and reflector.

3. Results and Discussions

To demonstrate the feasibility of the new APECp method using the SPH correction factor, a small UOX core in Fig. 3 was chosen as a test problem. The UOX core is modified from the KAIST 1A benchmark [9]. In the UOX core, there are three typical 17x17 fuel assembles (UOX-1: 2.0 w/o, UOX-2: 3.3 w/o UOX-2 with 16 BA fuel pins). For the consistency, the baffle-reflector regions are also treated with pin-wise group constants.

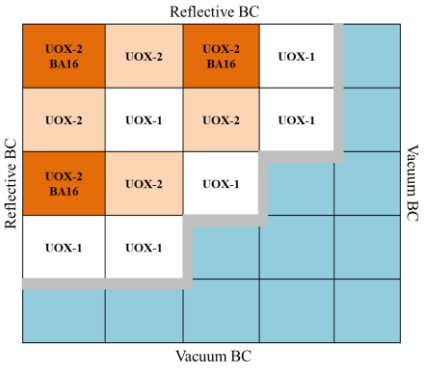


Fig. 3. Quarter core configuration of UOX core

In this APECp-SPH method, the pin-by-pin core calculation initially starts with pin-wise cross-sections and discontinuity factors from the single fuel assembly calculation. For the baffle-reflector region, the initial two-group constants of each type such as flat baffle, L-shape baffle and corner baffle are determined from corresponding region of the baffle color-set type 1 and 2 as shown in Fig 2.

For the preliminary study, the pin-wise XS-dependent SPH factor are updated only once after the problem is solved with the initial group constants. In the current study, the SPH factors are not continuously updated in a non-linear way to guarantee numerical stability. In the baffle-reflector region, pin-level XS-dependent SPH factors are also updated since they are closely coupled with the fuel pins near baffle.

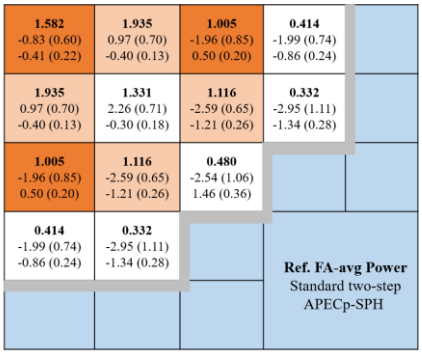
Table 1 shows several sets of HCMFD solutions which satisfy different condition. ‘Standard SPH’ means standard SPH iteration to preserve reference reaction

rate. Nevertheless, one can see that the 'standard SPH' solution has noticeable discrepancy for the eigenvalue. This is one of known shortcomings of the SPH method. It is mentioned that the standard SPH method cannot produce an exact balance in geometry with reflectors or void boundary condition [10] due to a lack in degrees of freedom. In Table I, 'SPH with DF' means SPH iteration with initial DF information. It is interesting to note that the standard SPH method can be improved noticeably with GET-based initial discontinuity factor. One can clearly note that the newly proposed APECp-SPH pre-correction provides clearly more accurate eigenvalue than the conventional two-step pin-wise analysis ('standard two-step').

Figure 4 shows the FA-average power profiles for both standard two-step and APECp-SPH schemes. It is noteworthy that the new SPH correction substantially reduces both maximum and RMS pin power error.

Table I: Results of Test UOX core

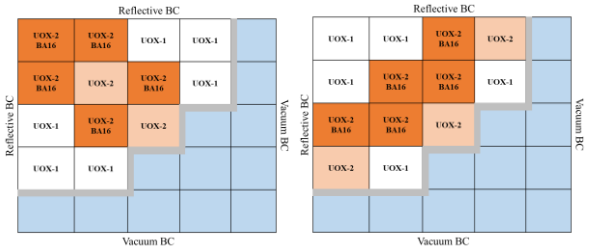
UOX test core	k-eff	k-eff difference [pcm]	pin-power %error Max (RMS)
Ref. DeCARD2D	1.112455	Ref	Ref
Standard SPH	1.112773	31.83	0.00 (0.00)
SPH with DF	1.112470	1.53	0.00 (0.00)
Standard two-step	1.113307	85.22	-2.95 (0.82)
APECp-SPH	1.112692	23.65	1.46 (0.24)



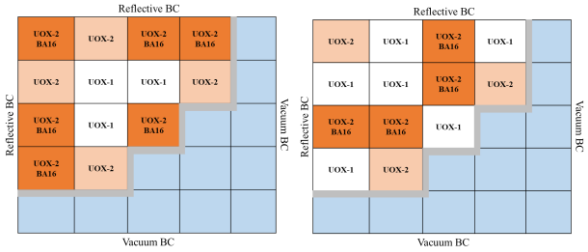
*Format :: max Pin power %error (RMS error)

Fig. 4. Assembly-wise maximum and RMS pin power %error distribution of UOX core

For further test of the APECp-SPH method with the small PWR core, four different variants of the small benchmark are also considered by rather randomly changing the FA loading pattern, as shown in Fig 5. Actually, the variants have very different power distributions from the original core.



a) UOX variant 1 (left) 2(right)



b) UOX variant 3 (left) 4(right)

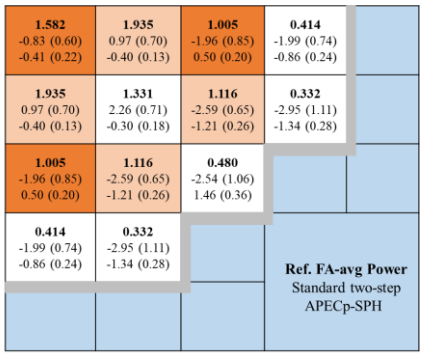
Fig. 5. Quarter core configuration of UOX variants

Table 2 shows summary of the HCMFD pin-wise analyses for each variant. With the initial discontinuity factors, neutron balance for each core is enhanced. For each variant, it is clear that APECp-SPH pre-correction results in a noticeably improved eigenvalue than the standard two-step method, as in the original problem.

Figures 6-9 show the FA-wise power distribution and maximum and RMS errors in the pin-wise power profile of the APECp-SPH and standard two-step approaches for the variants. It should be noted that, for all the variants calculation, both maximum and RMS pin power error are clearly reduced by application of the APECp-SPH pre-correction.

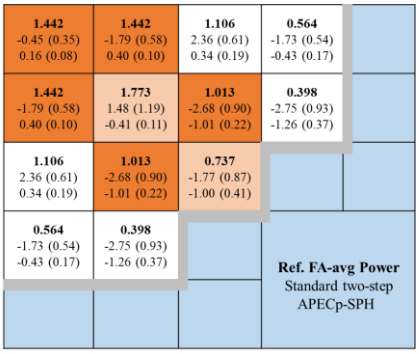
Table 2: Results of Test UOX variant cores

	k_{eff}	Δk_{eff} [pcm]	pin-power %error Max (RMS)
UOX variant 1			
Ref. DeCARD2D	1.054424	Ref	Ref
Standard SPH	1.054791	36.68	0.00 (0.00)
SPH with GET	1.054430	0.57	0.00 (0.00)
Standard two-step	1.054993	56.88	-2.75 (0.77)
APECp-SPH	1.054663	23.09	-1.26 (0.23)
UOX variant 2			
Ref. DeCARD2D	1.055231	Ref	Ref
Standard SPH	1.055635	40.41	0.00 (0.00)
SPH with GET	1.055215	-1.59	0.00 (0.00)
Standard two-step	1.055511	27.97	2.76 (0.82)
APECp-SPH	1.055362	13.14	-1.12 (0.25)
UOX variant 3			
Ref. DeCARD2D	1.098787	Ref	Ref
Standard SPH	1.099088	30.14	0.00 (0.00)
SPH with GET	1.098786	-0.06	0.00 (0.00)
Standard two-step	1.099636	84.93	-4.31 (1.03)
APECp-SPH	1.099031	24.35	1.33 (0.26)
UOX variant 4			
Ref. DeCARD2D	1.105228	Ref	Ref
Standard SPH	1.105484	25.64	0.00 (0.00)
SPH with GET	1.105231	0.30	0.00 (0.00)
Standard two-step	1.105852	62.38	-3.24 (0.90)
APECp-SPH	1.105236	0.75	-1.45 (0.35)



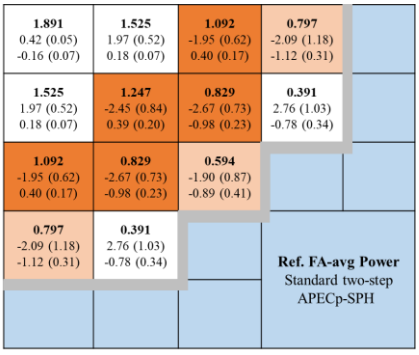
*Format :: max Pin power %error (RMS error)

Fig. 6. Assembly-wise maximum and RMS pin power %error distribution of UOX variant 1



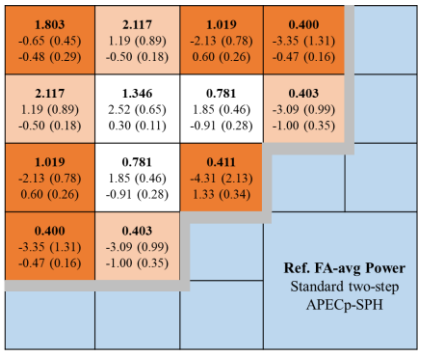
*Format :: max Pin power %error (RMS error)

Fig. 7. Assembly-wise maximum and RMS pin power %error distribution of UOX variant 2



*Format :: max Pin power %error (RMS error)

Fig. 8. Assembly-wise maximum and RMS pin power %error distribution of UOX variant 3



*Format :: max Pin power %error (RMS error)

Fig. 9. Assembly-wise maximum and RMS pin power %error distribution of UOX variant 4

4. Conclusions

This paper presents a new way to combine the conventional equivalence theory with the SPH method to correct two-group constant obtained from the standard lattice analysis. Pin-wise XS-dependent SPH factors are functionalized as a function of pin-wise leakage and then initial two-group group constants are corrected only once in the core analysis. We found that the newly proposed APECp-SPH correction works quite well and improves the accuracy of pin-wise core analysis, relative to the standard two-step calculation. It was also found that the APECp functions determined with several color-set problems can be applied to various modified cores without any modifications, indicating generality of the current leakage correction method.

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