A Study on Optimization of Albedo Boundary Conditions for the Deterministic Truncation of Monte Carlo Solutions

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- **Computing time**

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Introduction

Monte Carlo (MC) calculation in a reactor criticality analysis

– **A stochastic method** to solve a statistical problem **finding out the average behavior of the unknown parameters**

Pros	Cons
▪ **High accuracy** - Direct simulation of particles' whole behavior - No discretization of variables (energy, angle, space) ▪ No constraints on geometry construction ▪ Simple parallel calculation	▪ **Computationally expensive** - Large memory to describe explicit geometry and to utilize cross section data - Long time to track all particles and to get quantities of interest - Ever after source convergence, it is important to simulate many particles in active cycles to reduce stochastic uncertainty

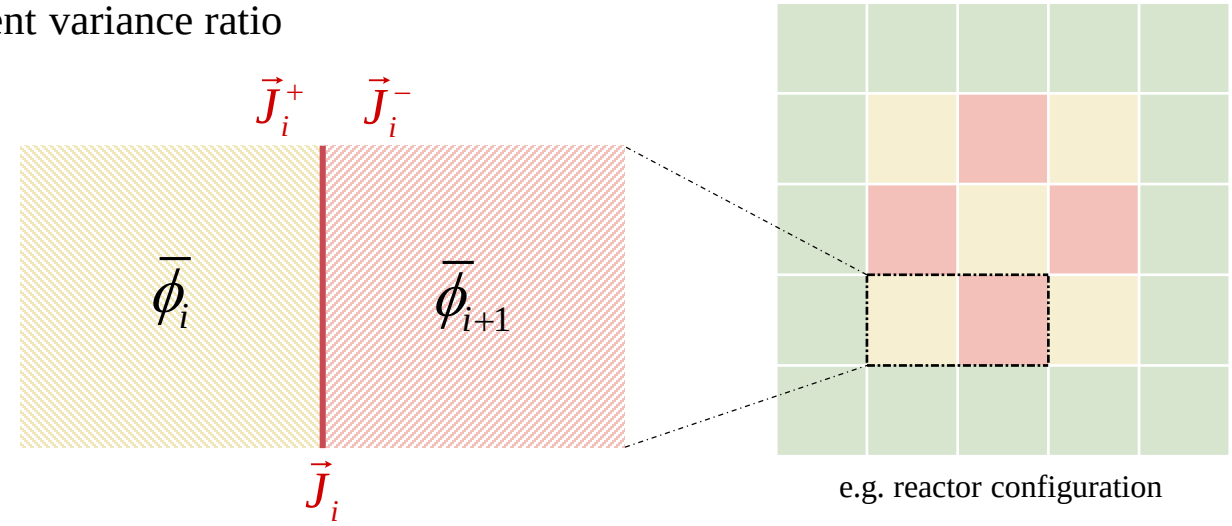
– Several studies have been conducted to accelerate the calculation speed and to reduce stochastic uncertainties more efficiently

- Diffusion based coarse mesh finite difference (CMFD) method
- Modified power method
- Particle ramp-up method
- ...

Introduction

Coarse mesh finite difference (CMFD) method

- **Acceleration scheme** commonly used in a MC eigenvalue problem
- **Preservation of net current and reaction rate** by high fidelity solution (i.e. MC)
- Fast convergence of the fission source distribution (FSD)
  - Shorten the number of inactive cycles and the corresponding computing time
- Weak inter-cycle correlation
  - Reduce the real-to-apparent variance ratio



e.g. reactor configuration

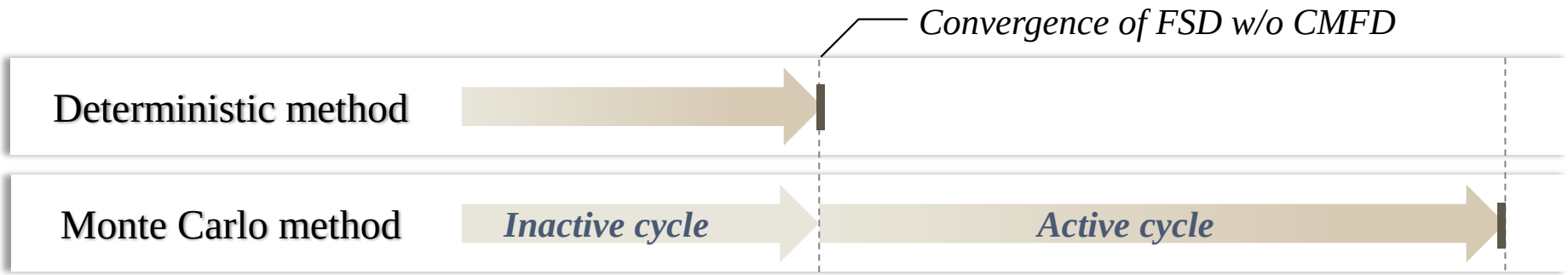
CMFD correction factor : $\hat{D}_{i+1/2}^n = \dfrac{\vec{J}_i^{n+1} + \tilde{D}_{i+1/2}^n (\bar{\phi}_{i+1}^n - \bar{\phi}_i^n)}{\bar{\phi}_{i+1}^n + \bar{\phi}_i^n}$

Difference between MC and deterministic methods

– **When solving the neutron transport eigenvalue problem**

- **Deterministic calculation** : The calculation is finished as soon as FSD converges
- **MC calculation** : The main calculation is activated in active cycle when FSD converges
- MC calculation takes much longer computing time in active cycle than inactive cycle

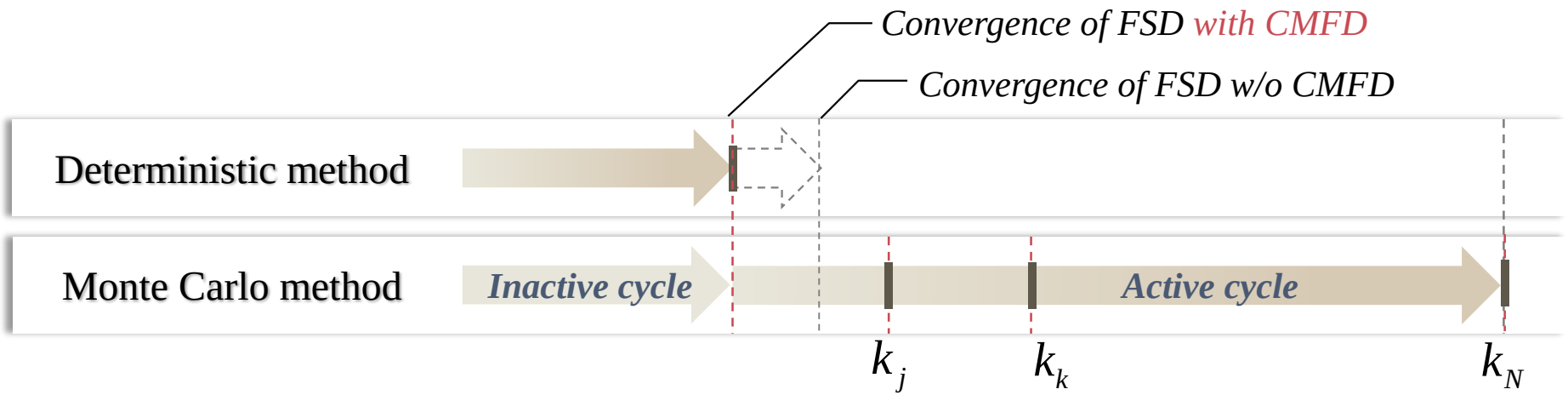
– **Calculation process**



- If the FSD truly converges in the inactive MC cycle,
  the reactor parameters should be already determined at the end of the inactive cycle

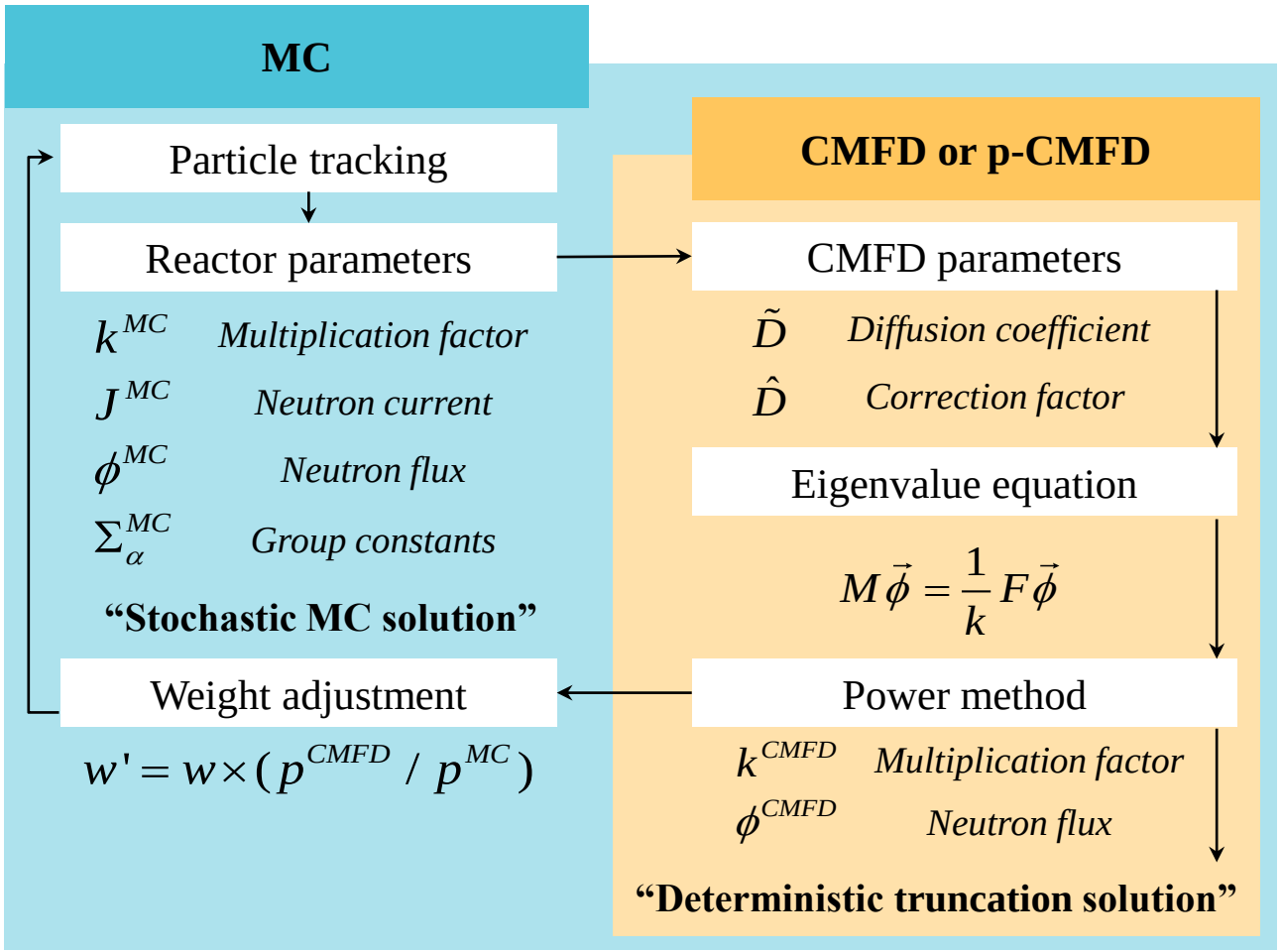
CMFD solution in MC simulation

- **The CMFD method enables a fast convergence of the FSD**
- **Solution can be truncated from the original MC solution in a systematic / deterministic way**
- Combination of the flexible and versatile MC method and the efficient deterministic analysis
  - No limitation in geometry modeling
  - Continuous energy
  - Numerically cheap computation



Methods

Concept of deterministic truncation of MC solution (DTMC) method



- The next FSD in MC simulation is corrected by the CMFD solution
- **The deterministic result is a subset of the solution to the original MC approach**

Methods

Deterministic truncation of MC solution (DTMC) method

– **Solving a standard eigenvalue problem**

- **Eigenvalue** : k_{eff}
- **Eigenvector** : power distribution
- Generalized equivalence theory confirms that the CMFD solution is equivalent to MC ones

– **Fine mesh CMFD grid** to obtain a detailed pin power profile

- Radial node size : pin (1-2 cm)
- Axial node size : 5-10 cm

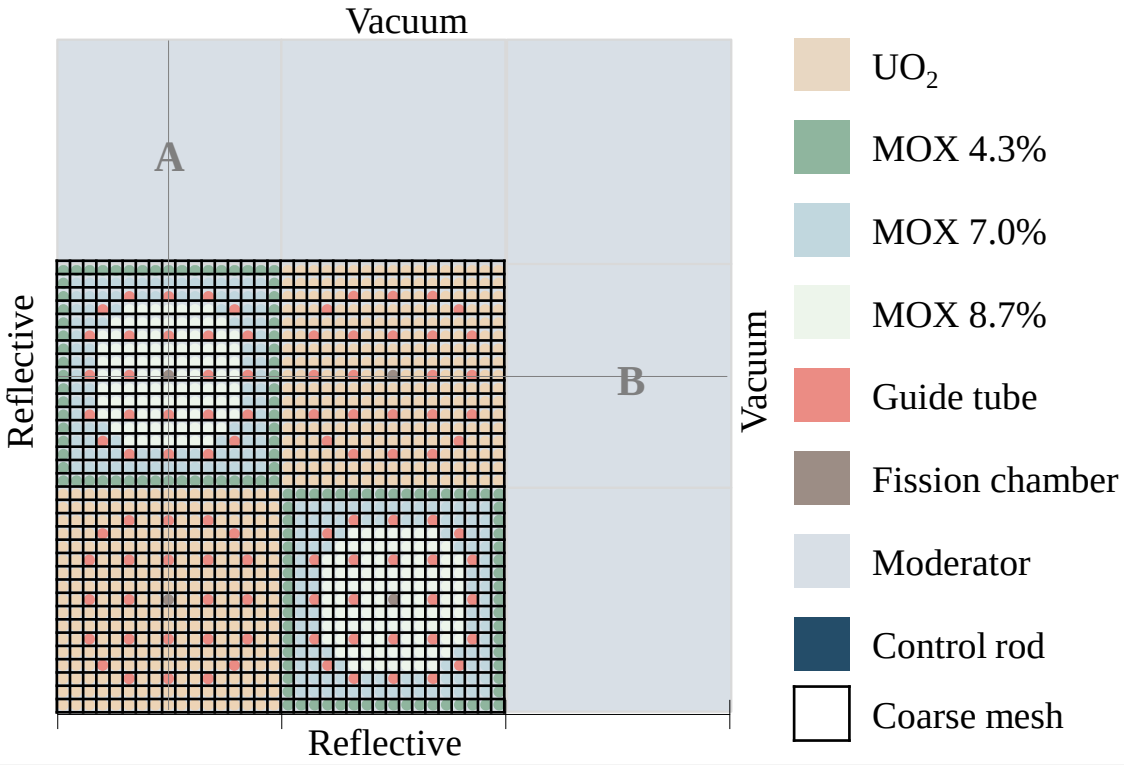
– **Characteristics**

- Boundary for CMFD domain is confined to the active core
  where the particle flux is high to get reliable CMFD parameters in pin-sized grid system
- Inter-cycle correlation can be higher in the DTMC method;
  thus, a minimum of 2 cycles is accumulated to generate the CMFD parameters
- The CMFD solution only retains the uncertainties originating from the CMFD parameters,
  and is free from the stochastic random process

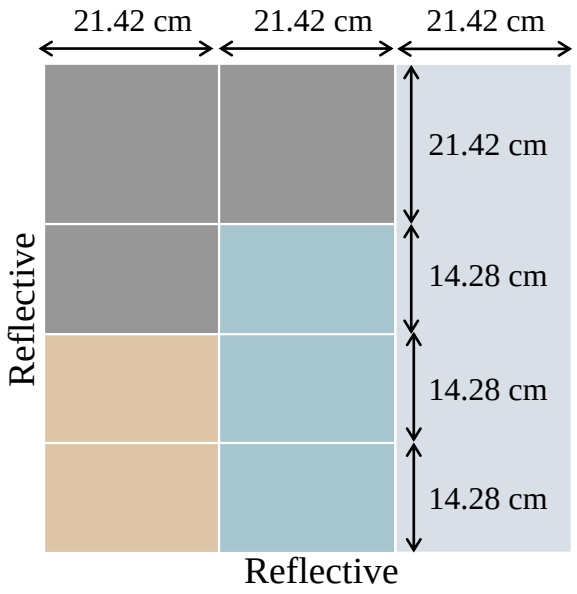
Numerical Results

Problem description

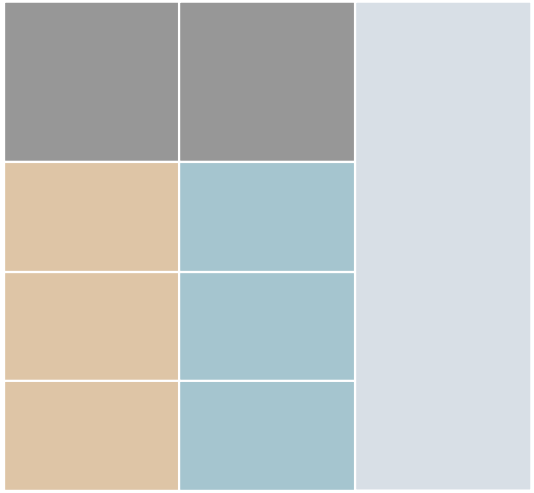
- Solved by in-house MC code
- C5G7 rodded A type benchmark problem
- 10-30 inactive cycles, 100 active cycles, 1.0M histories, 60 batches
- Pin-size CMFD mesh
- Reference solution (100 active cycles & 14.0M histories)



- Cross section A



- Cross section B



Numerical Results

Multiplication factor

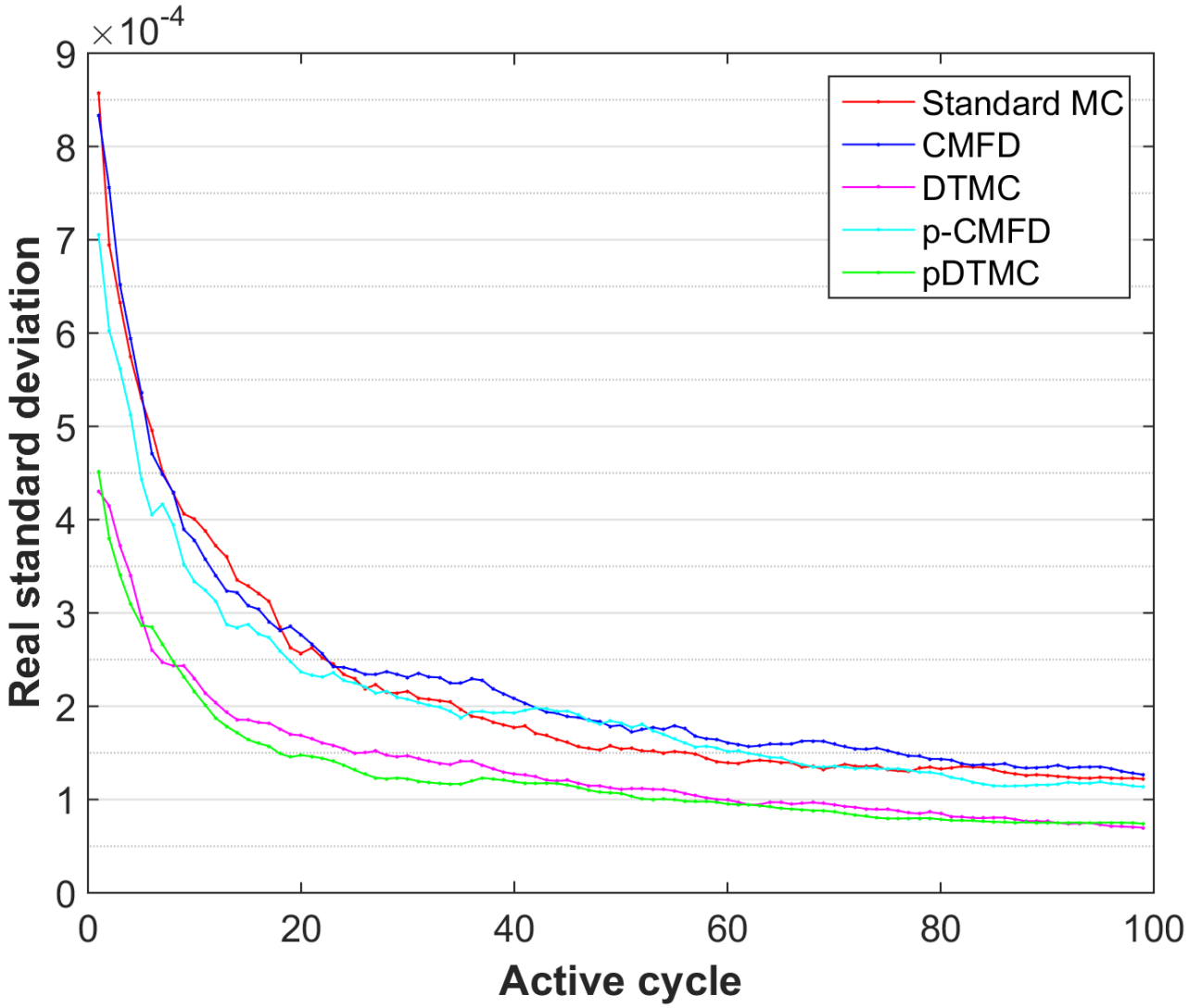
Active cycle	Parameter	MC	CMFD	DTMC	p-CMFD	pDTMC
1	k_{eff}	1.12867	1.12845	1.12821	1.12830	1.12816
	σ_a (pcm)	-	-	-	-	-
	σ_r (pcm)	85.7	83.3	43.1	70.5	45.1
5	k_{eff}	1.12783	1.12834	1.12826	1.12766	1.12776
	σ_a (pcm)	39.2	39.9	22.1	40.6	17.2
	σ_r (pcm)	53.0	53.6	29.5	44.3	28.7
10	k_{eff}	1.12787	1.12826	1.12842	1.12811	1.12792
	σ_a (pcm)	30.2	31.3	17.1	30.3	13.7
	σ_r (pcm)	40.1	37.8	23.0	33.4	21.6
20	k_{eff}	1.12810	1.12807	1.12824	1.12803	1.12807
	σ_a (pcm)	**21.9**	**23.3**	**12.4**	**22.4**	**10.2**
	σ_r (pcm)	**25.7**	**27.7**	**16.9**	**23.7**	**14.7**
100	k_{eff}	1.12819	1.12811	1.12818	1.12817	1.12812
	σ_a (pcm)	**10.2**	**11.0**	**5.9**	**10.4**	**4.7**
	σ_r (pcm)	**12.2**	**12.7**	**7.0**	**11.4**	**7.4**

***MC** : standard MC results*
***CMFD & p-CMFD** : MC results with CMFD & p-CMFD*
***DTMC & p-CMFD** : Deterministic results with CMFD & p-CMFD*

k_{ref} *(reference)* = 1.12808 ± 1.7 pcm
σ_a : *apparent standard deviation*
σ_r : *real standard deviation*

Numerical Results

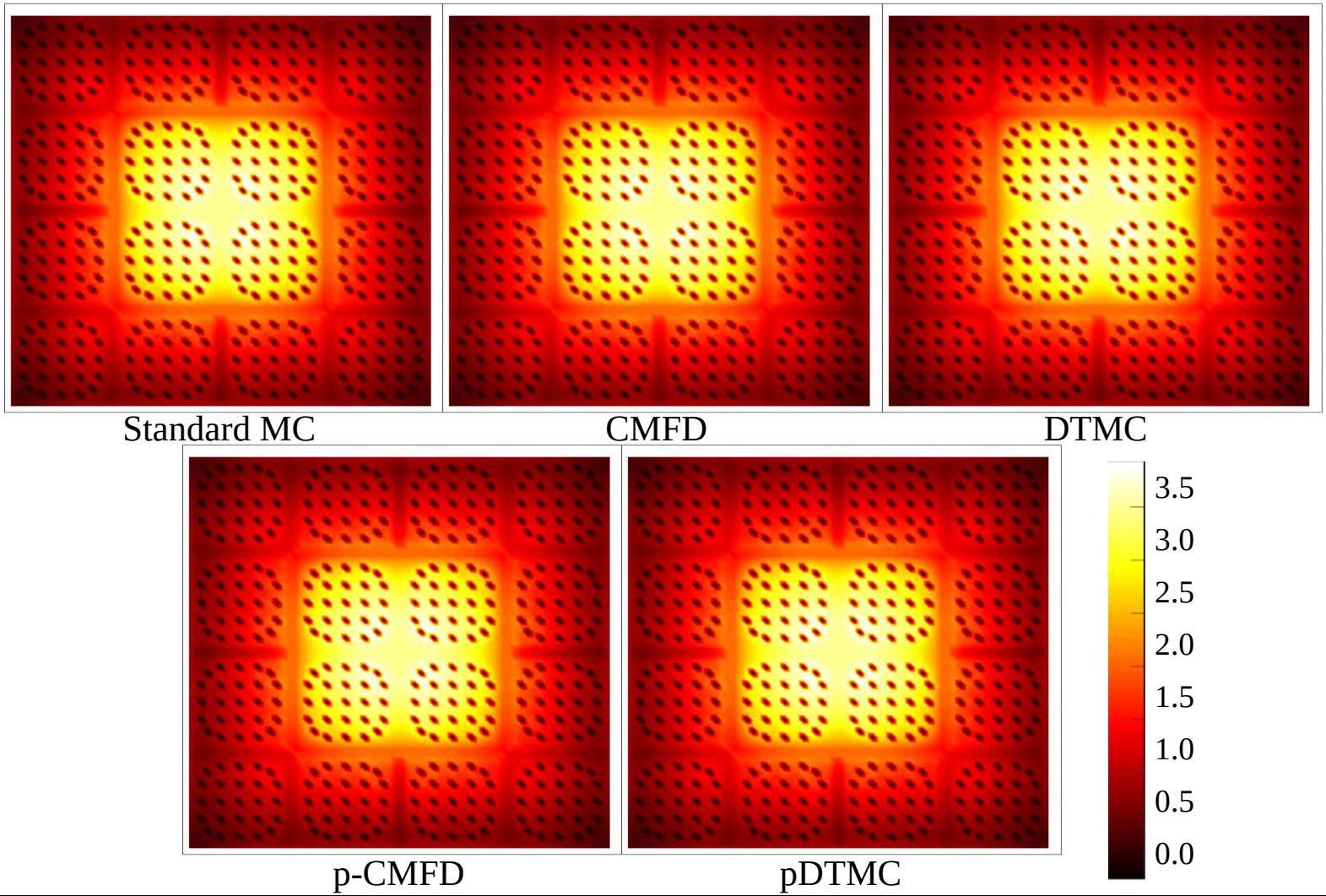
Cumulative real standard deviation of k_{eff}



Numerical Results

Pin-wise power profile

– Axially integrated 2D distribution



Numerical Results

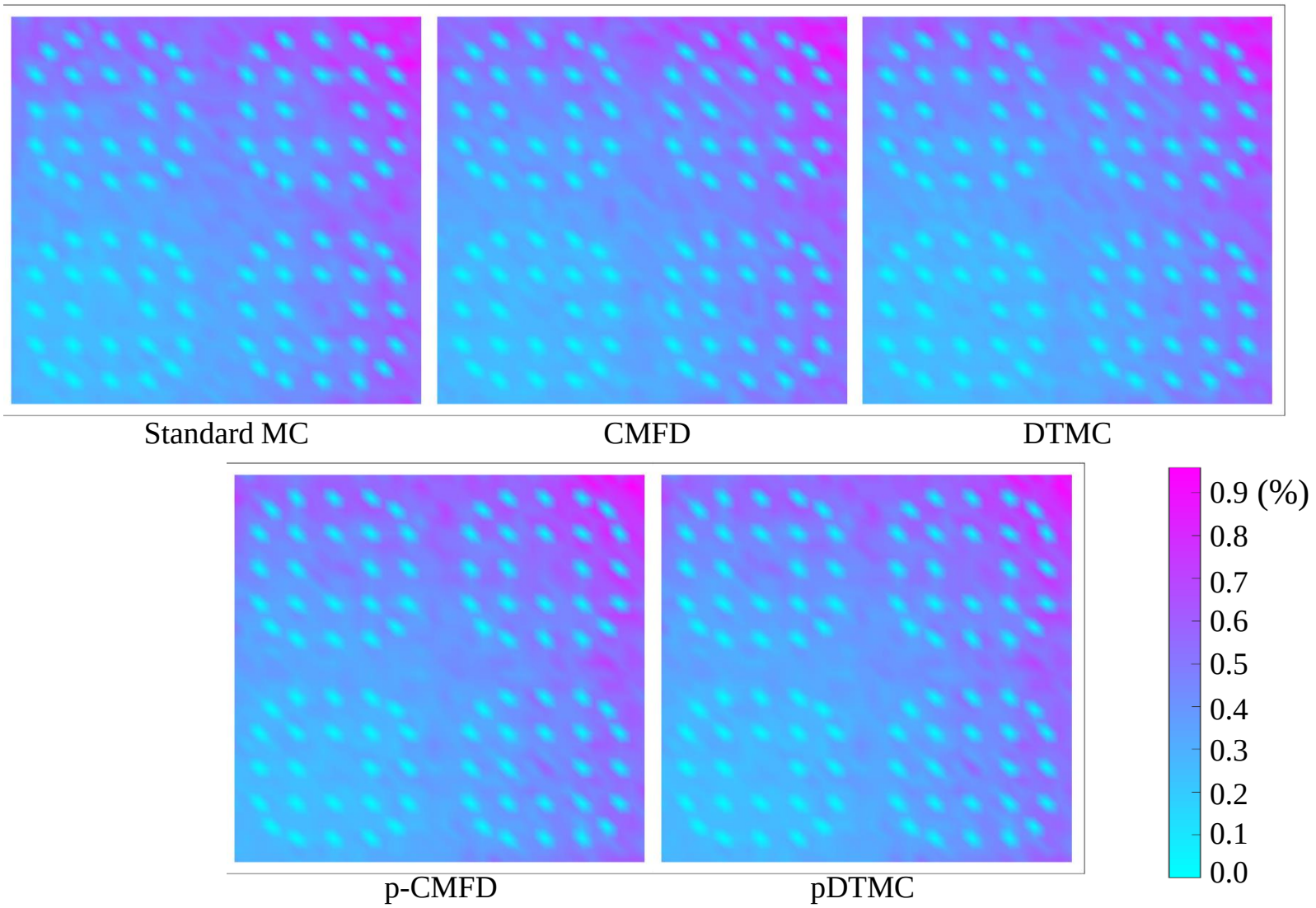
Errors of 3D pin-wise power distribution

Cycle	Value	MC	CMFD	DTMC	p-CMFD	pDTMC
1	σ_a	-	-	-	-	-
	σ_r	0.038	0.038	0.034	0.038	0.034
	RMS (%)	4.20	4.22	3.77	4.23	3.74
	ε_{avg} (%)	3.44	3.46	3.09	3.47	3.07
5	σ_a	0.021	0.021	0.014	0.021	0.014
	σ_r	0.022	0.022	0.021	0.022	0.021
	RMS (%)	2.46	2.47	2.37	2.47	2.37
	ε_{avg} (%)	2.01	2.03	1.95	2.03	1.94
10	σ_a	0.016	0.016	0.011	0.016	0.011
	σ_r	0.016	0.016	0.016	0.016	0.016
	RMS (%)	1.828	1.828	1.781	1.831	1.785
	ε_{avg} (%)	1.497	1.501	1.463	1.504	1.466
20	σ_a	0.0114	0.0115	0.0084	0.0115	0.0081
	σ_r	**0.0119**	**0.0119**	**0.0117**	**0.0119**	**0.0117**
	RMS (%)	**1.327**	**1.327**	**1.303**	**1.329**	**1.308**
	ε_{avg} (%)	**1.085**	**1.089**	**1.069**	**1.094**	**1.078**
100	σ_a	0.0053	0.0053	0.0040	0.0053	0.0039
	σ_r	**0.0055**	**0.0055**	**0.0054**	**0.0055**	**0.0054**
	RMS (%)	**0.615**	**0.616**	**0.609**	**0.615**	**0.609**
	ε_{avg} (%)	**0.503**	**0.505**	**0.500**	**0.505**	**0.501**

σ_a : apparent standard deviation; σ_r : real standard deviation; RMS : root mean square error; ε : relative error

Numerical Results

RMS error distribution of pin power



Summary

Findings

- The DTMC results showed **good agreement with the reference** solution
- The DTMC solution has **lower stochastic uncertainties** than the MC solutions
- The DTMC method can estimate **the accurate and reliable solution at the early active cycle**
- The DTMC solution of **power distribution** is **not improved as much as k_{eff}**
- The **maximum errors appear in the boundary region** for each method

Next work

Optimization of the boundary condition for solution improvement

Boundary condition in DTMC method

- **The CMFD domain is confined to the active core region**
- **The CMFD parameters near the boundary surface are more unreliable**
  - Less particles
  - Simpler correction
  - More sensitive to parameters

Correction factor in the inner region

$$\hat{D} = \frac{J^{i+1/2} + \tilde{D}^{i+1/2}(\phi^{i+1} - \phi^{i})}{\phi^{i+1} + \phi^{i}}$$

Correction factor at the boundary surface

Albedo boundary condition : $\hat{D} = J^{i+1/2} / \phi^{i}$

Optimization of the boundary condition

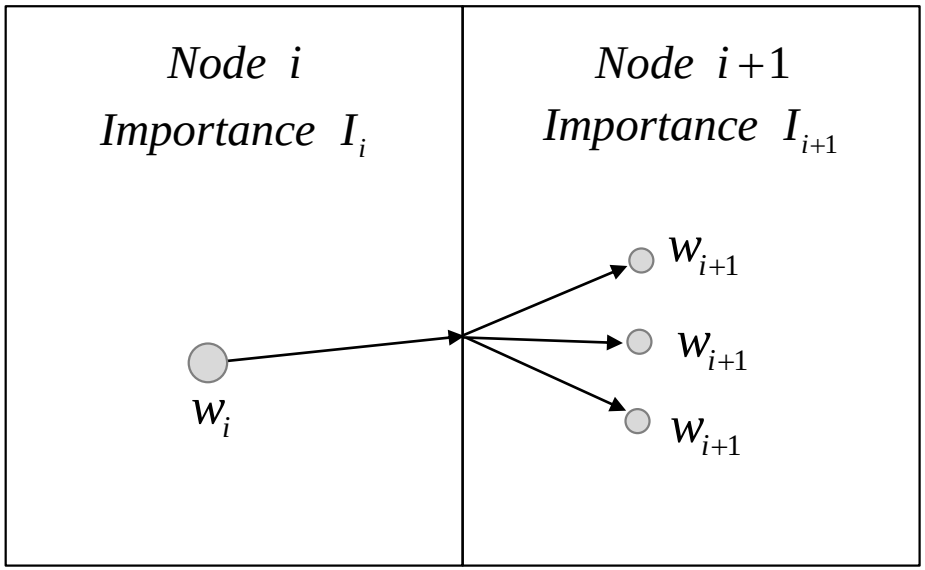
1. Weight adjustment method at the boundary region

2. Vacuum boundary condition with irregular nodes

Methods (1)

Weight adjustment method at the boundary region

– **Particle splitting with cell importance**



– $I_{i+1} > I_i$: $w_{i+1} = w_i / n$

– $I_{i+1} < I_i$: kill with probability $1 - p = 1 - I_{i+1} / I_i$

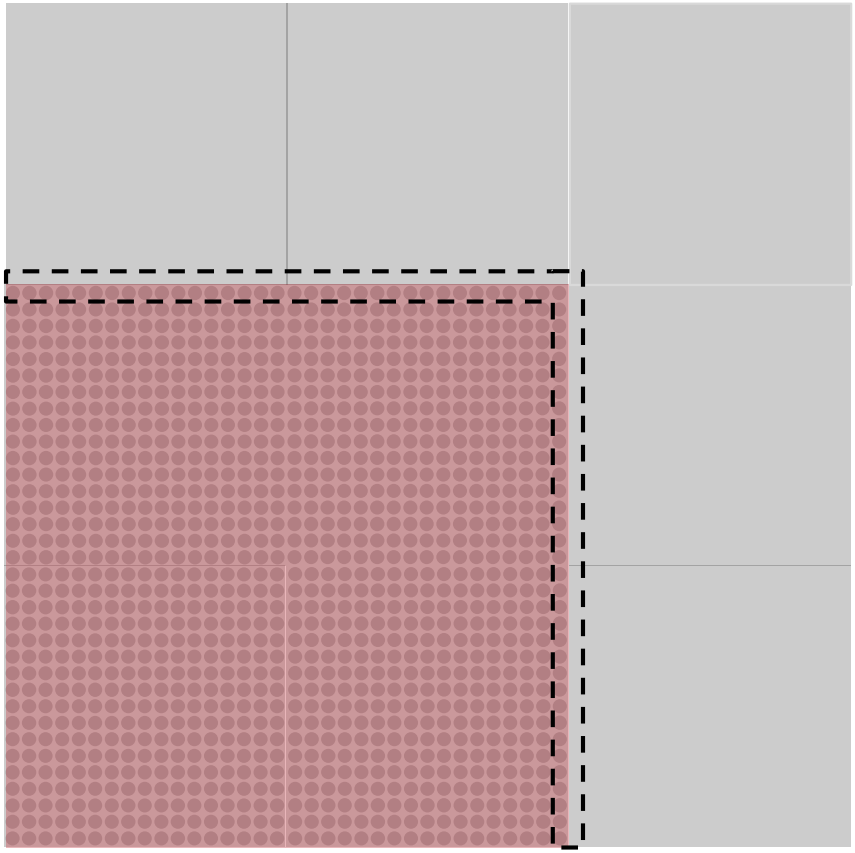
Track with $w_{i+1} = w_i \times I_i / I_{i+1}$ with probability $p = I_{i+1} / I_i$

Methods (1)

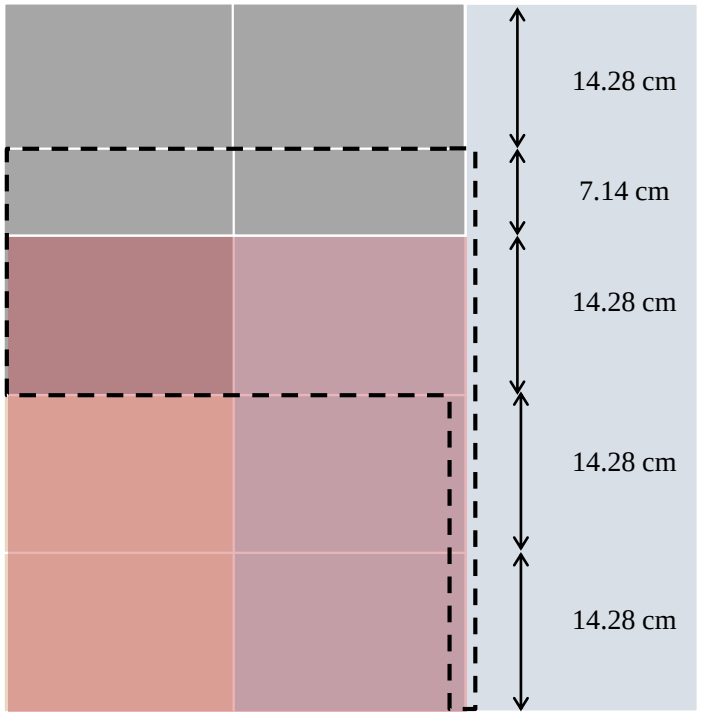
Weight adjustment method at the boundary region

– **Higher importance at the boundary regions**

❖ **Radial configuration**



❖ **Axial configuration**



Active core

High importance region

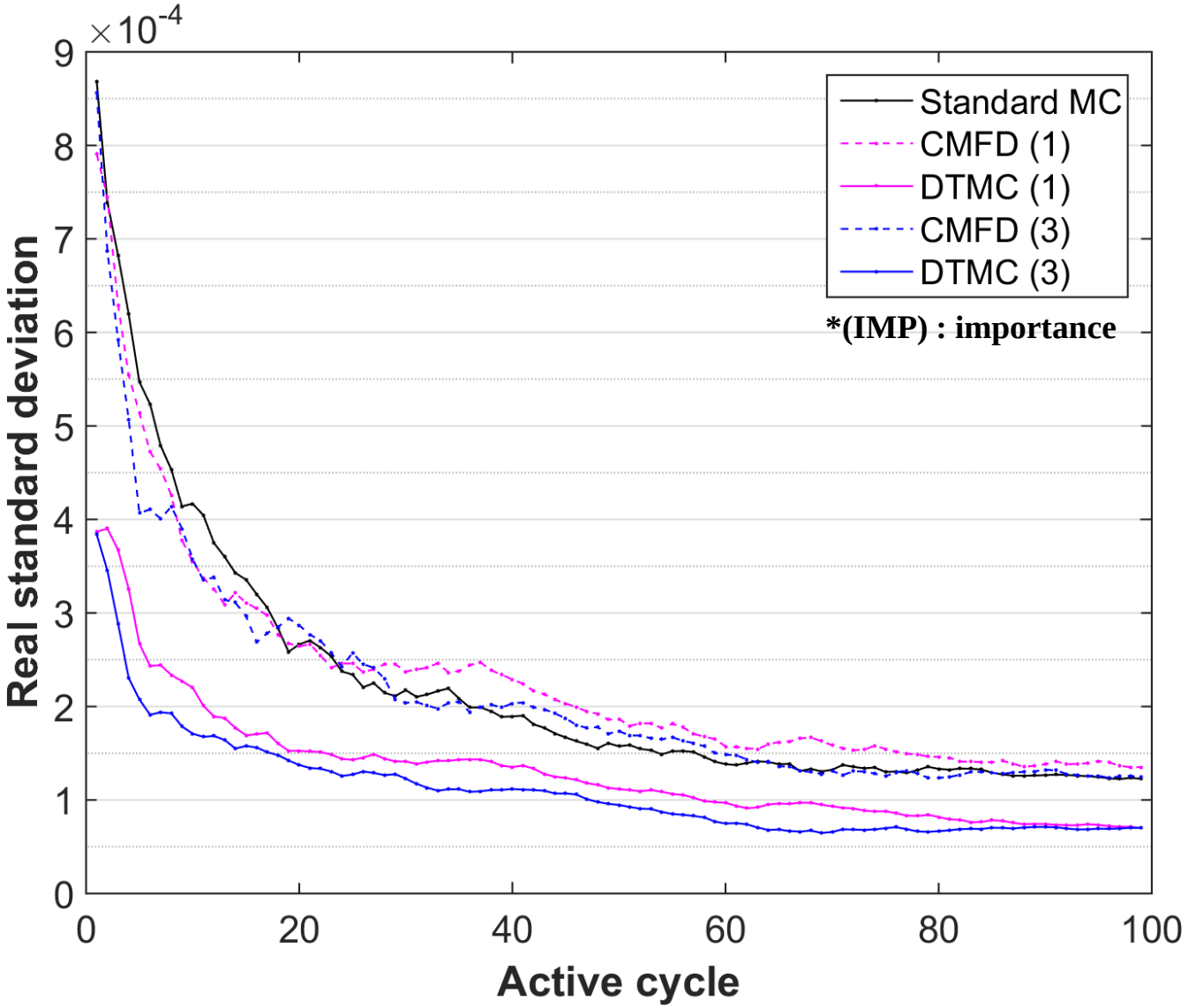
Numerical Results (1)

Multiplication factor

Cycle	Parameter	MC	IMP = 1		IMP = 2		IMP = 3	
			CMFD	DTMC	CMFD	DTMC	CMFD	DTMC
1	k_{eff}	1.12867	1.12845	1.12821	1.12799	1.12790	1.12734	1.12801
	σ_a (pcm)	64.2	68.5	29.3	78.0	25.7	69.6	30.5
	σ_r (pcm)	85.7	83.3	43.1	80.8	47.1	83.7	43.8
5	k_{eff}	1.12783	1.12834	1.12826	1.12793	1.12828	1.12817	1.12802
	σ_a (pcm)	39.2	39.9	22.1	45.8	22.6	43.3	21.9
	σ_r (pcm)	53.0	53.6	29.5	48.9	28.6	45.1	23.8
10	k_{eff}	1.12787	1.12826	1.12842	1.12771	1.12813	1.12811	1.12784
	σ_a (pcm)	30.2	31.3	17.1	35.0	17.0	33.0	17.0
	σ_r (pcm)	40.1	37.8	23.0	36.1	21.4	33.0	16.2
20	k_{eff}	1.12810	1.12807	1.12824	1.12798	1.12799	1.12792	1.12788
	σ_a (pcm)	21.9	23.3	12.4	25.0	12.7	24.8	12.6
	σ_r (pcm)	25.7	27.7	16.9	25.9	15.7	26.2	13.5
100	k_{eff}	1.12819	1.12811	1.12818	1.12793	1.12802	1.12790	1.12796
	σ_a (pcm)	10.2	11.0	5.9	11.4	5.9	11.3	5.7
	σ_r (pcm)	12.2	12.7	7.0	12.4	7.1	11.8	6.9

Numerical Results (1)

Cumulative real standard deviation for k_{eff}



Numerical Results (1)

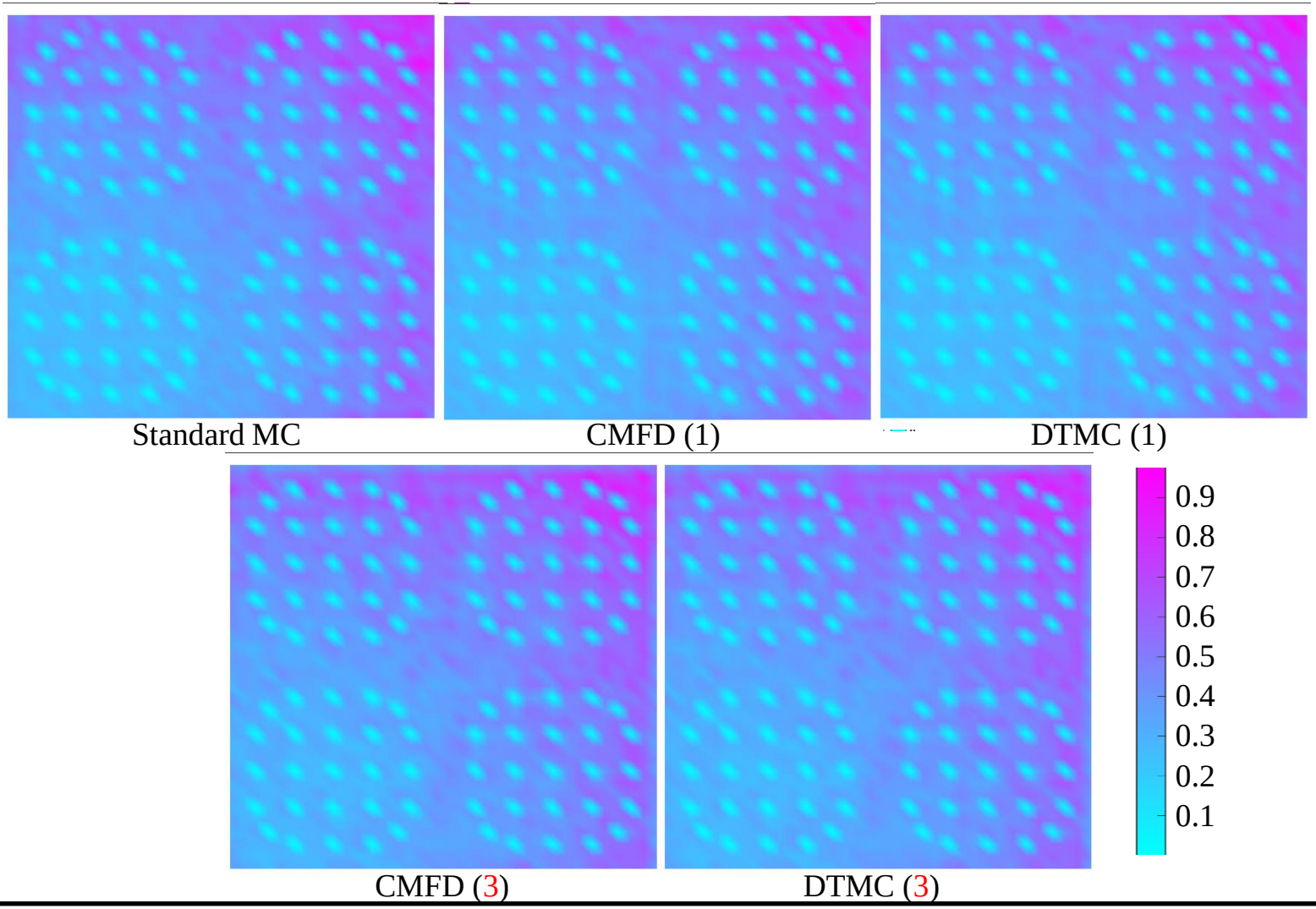
Error of power distribution

Cycle	Parameter	MC	IMP = 1		IMP = 2		IMP = 3	
			CMFD	DTMC	CMFD	DTMC	CMFD	DTMC
1	RMS (%)	4.20	4.22	3.77	4.07	3.64	3.99	3.57
	ε_{avg} (%)	3.44	3.46	3.09	3.02	2.70	2.82	2.53
5	RMS	2.46	2.47	2.37	2.38	2.28	2.34	2.24
	ε_{avg}	2.01	2.03	1.95	1.78	1.71	1.66	1.60
10	RMS	1.83	1.83	1.78	1.78	1.73	1.75	1.70
	ε_{avg}	1.50	1.50	1.46	1.33	1.30	1.25	1.21
20	RMS	1.33	1.33	1.30	1.30	1.27	1.28	1.25
	ε_{avg}	1.09	1.09	1.07	0.98	0.97	0.92	0.91
100	RMS	0.62	0.62	0.61	0.63	0.63	0.63	0.62
	ε_{avg}	0.50	0.51	0.50	0.51	0.50	0.49	0.49

Numerical Results (1)

Relative error distribution (%)

– **1st layer, IMP = 3**



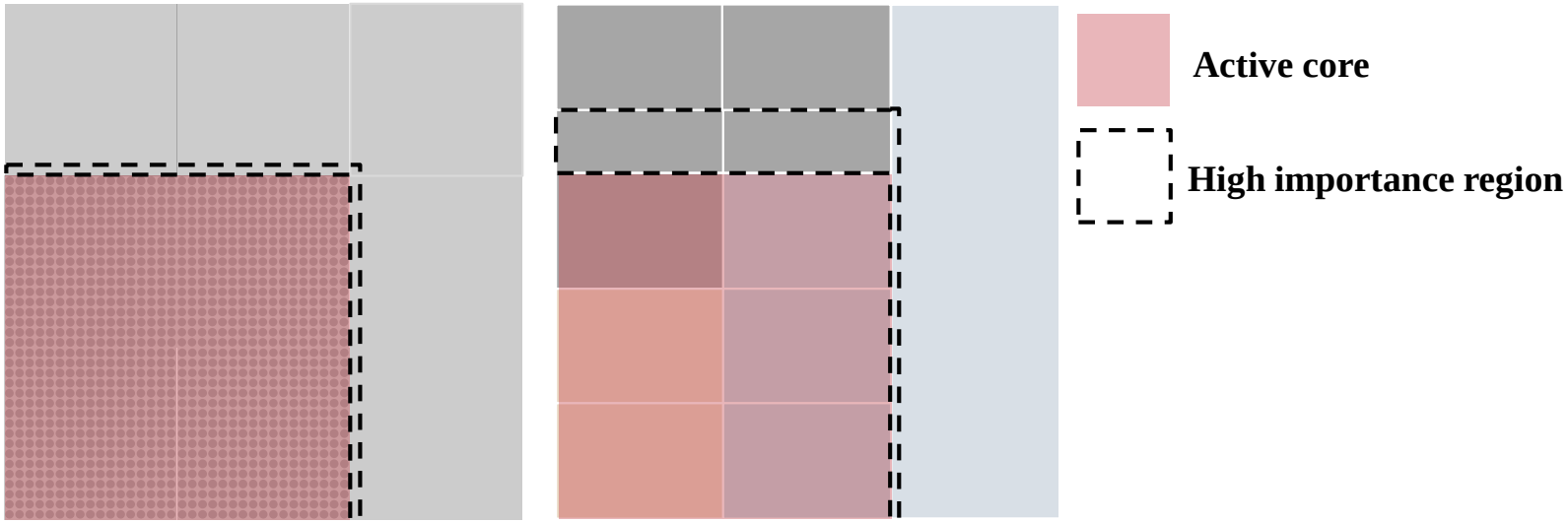
Numerical Results (1)

Computing time

– Computing time was increased to track split neutrons, especially fission neutrons

Parameter	Standard MC	CMFD & DTMC		
		IMP = 1	IMP = 2	IMP = 3
Computing time (min)	49	58	73	81

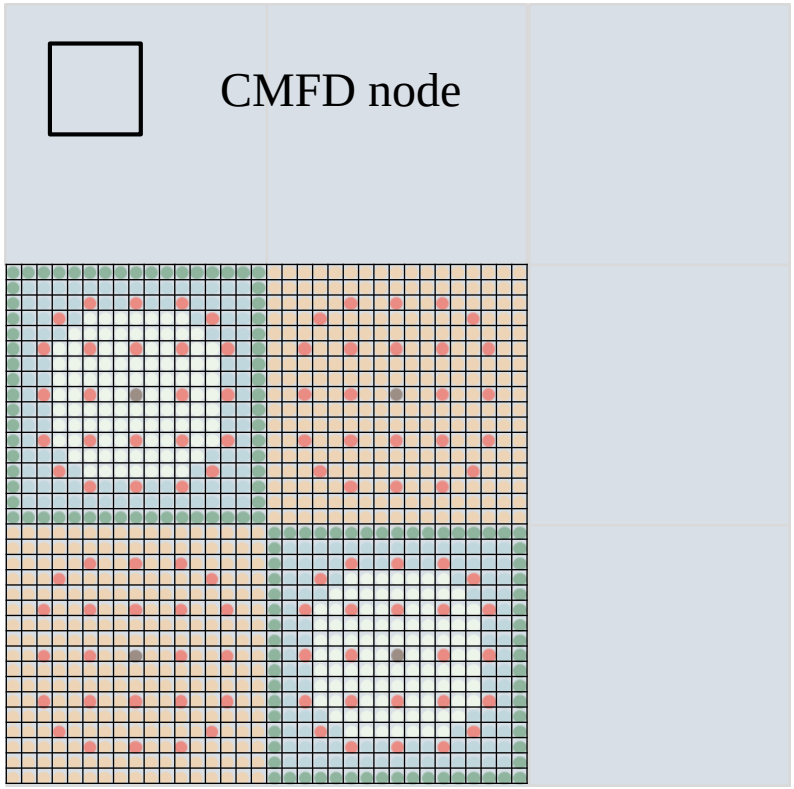
– Higher importance can be applied only in the reflector region to minimize the computing time



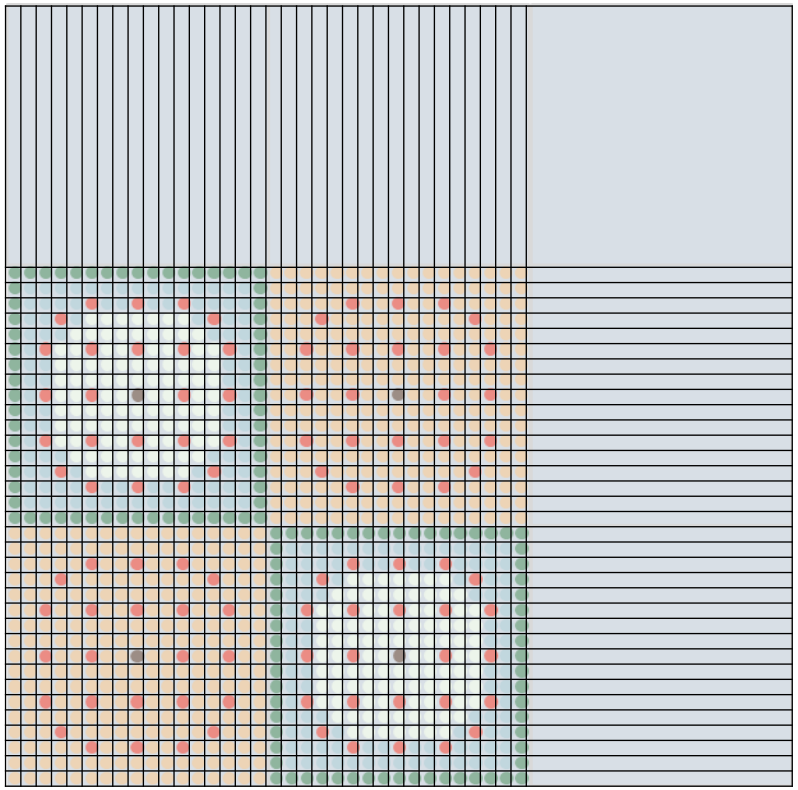
Methods (2)

Vacuum boundary condition with irregular nodes

- **CMFD domain is extended to actual boundary surface**
- **The CMFD parameters in reflector can be more reliable with irregular node**



Albedo boundary condition



Vacuum boundary condition

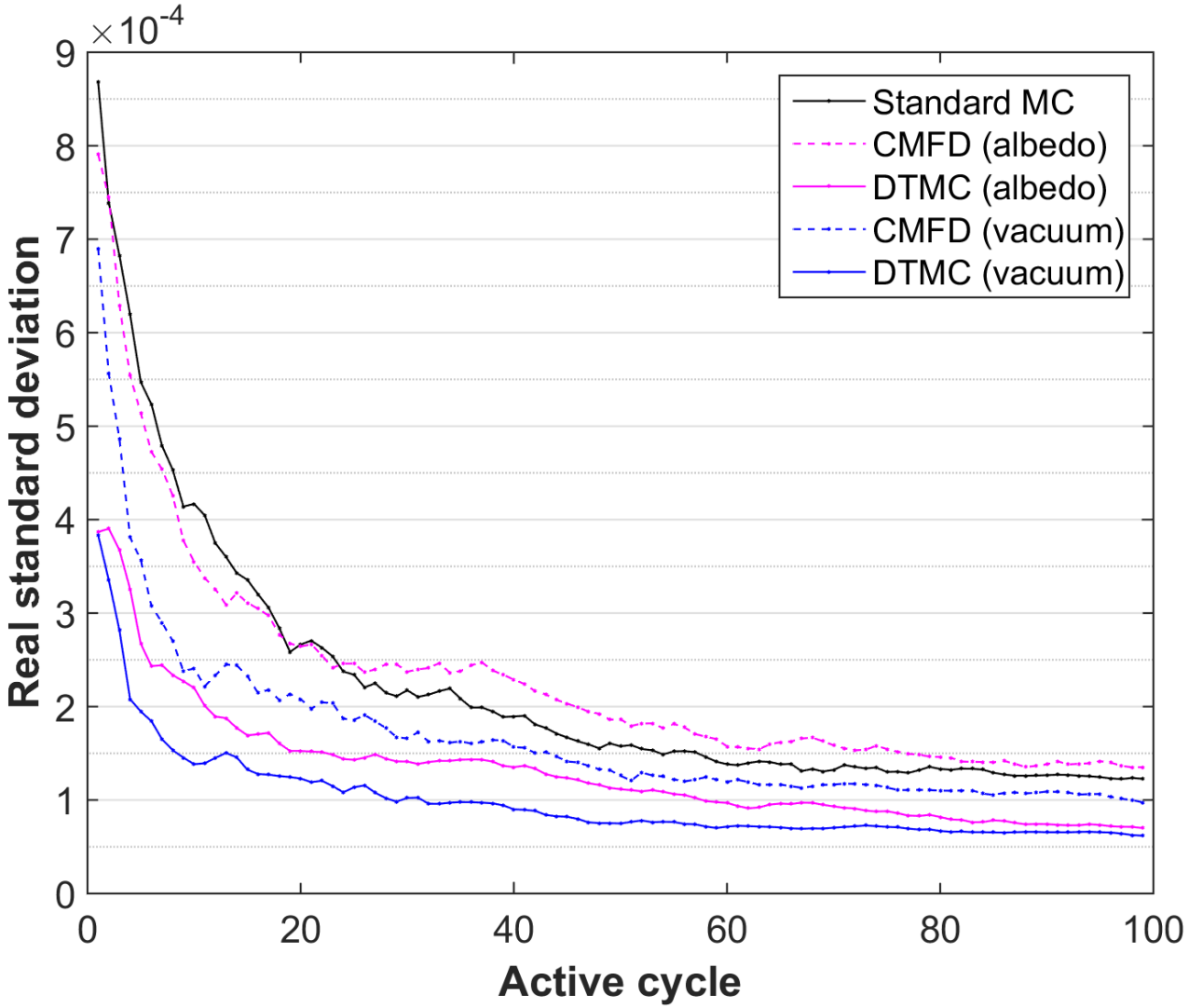
Numerical Results (2)

Multiplication factor

Cycle	Parameter	Standard MC	Albedo BC		Vacuum BC	
			CMFD	DTMC	CMFD	DTMC
1	k_{eff}	1.12867	1.12845	1.12821	1.12805	1.12827
	σ_a (pcm)	-	-	-	-	-
	σ_r (pcm)	86.8	79.1	38.7	69.0	38.4
5	k_{eff}	1.12783	1.12834	1.12826	1.12895	1.12862
	σ_a (pcm)	41.4	41.2	23.0	34.8	17.4
	σ_r (pcm)	54.7	51.4	26.7	35.7	19.5
10	k_{eff}	1.12787	1.12826	1.12842	1.12824	1.12827
	σ_a (pcm)	31.4	31.7	17.0	26.9	13.4
	σ_r (pcm)	41.7	35.5	22.0	24.1	13.8
20	k_{eff}	1.12810	1.12807	1.12824	1.12842	1.12833
	σ_a (pcm)	21.8	23.5	12.3	19.8	9.8
	σ_r (pcm)	26.6	26.4	15.3	20.8	12.3
100	k_{eff}	1.12819	1.12811	1.12818	1.12824	1.12818
	σ_a (pcm)	10.2	11.1	5.9	9.0	4.8
	σ_r (pcm)	12.3	13.5	7.0	9.7	6.2

Numerical Results (2)

Cumulative real standard deviation for k_{eff}



Numerical Results (2)

Power distribution

Cycle	Parameter	Standard MC	Albedo		Vacuum	
			CMFD	DTMC	CMFD	DTMC
1	ε_{avg}	0.625	0.633	0.563	0.515	0.460
	ε_{max}	4.610	4.370	4.218	3.479	3.255
5	ε_{avg}	0.366	0.372	0.356	0.299	0.287
	ε_{max}	2.817	2.768	2.733	2.200	1.977
10	ε_{avg}	0.274	0.274	0.267	0.223	0.217
	ε_{max}	2.095	2.237	2.096	1.656	1.563
20	ε_{avg}	0.198	0.198	0.194	0.163	0.159
	ε_{max}	1.453	1.489	1.449	1.152	1.156
100	ε_{avg}	0.092	0.092	0.091	0.076	0.075
	ε_{max}	0.712	0.687	0.696	0.552	0.538

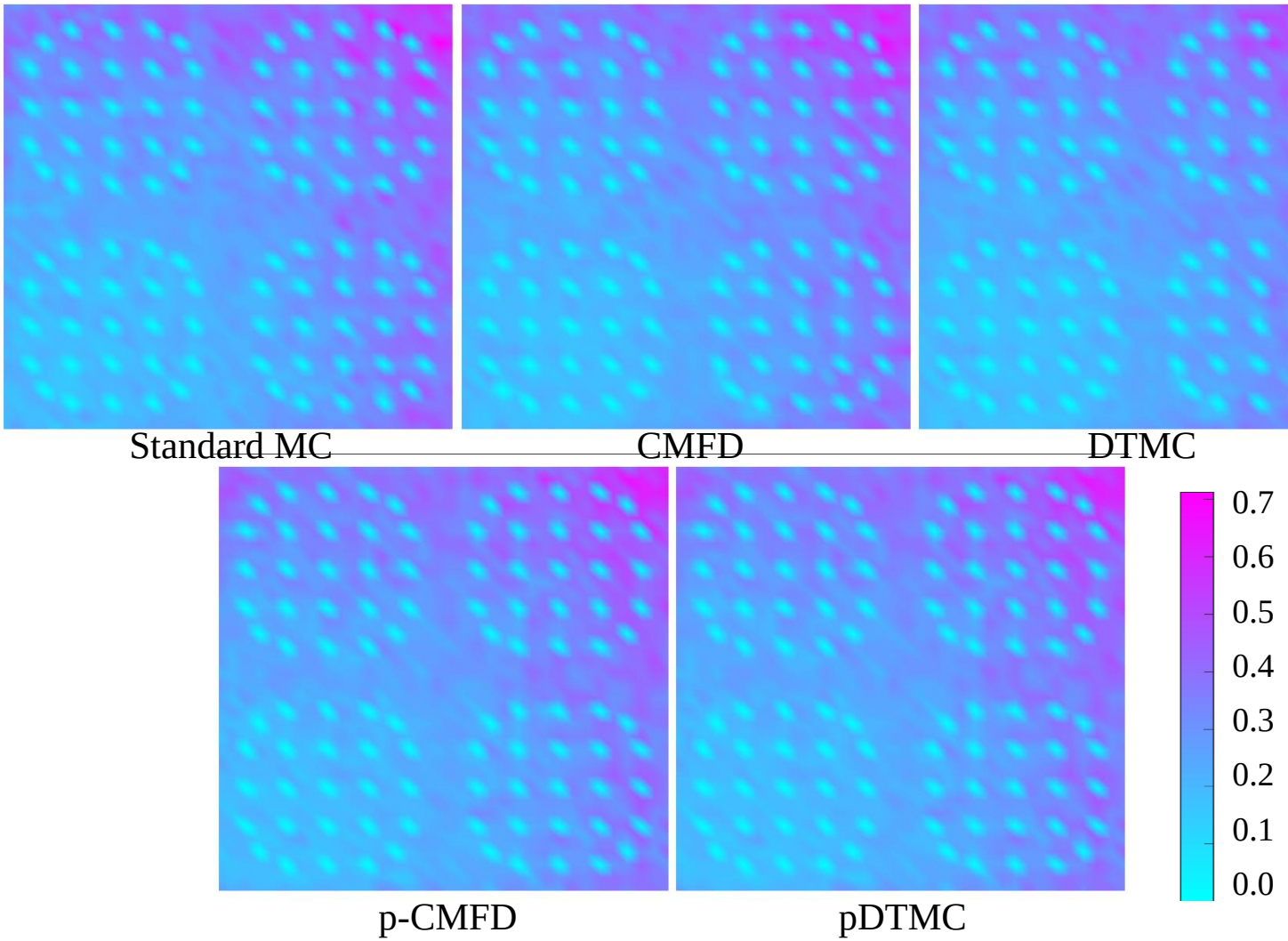
ε_{avg} : average relative error

ε_{max} : maximum relative error

Numerical Results (2)

RMS error distribution for power distribution

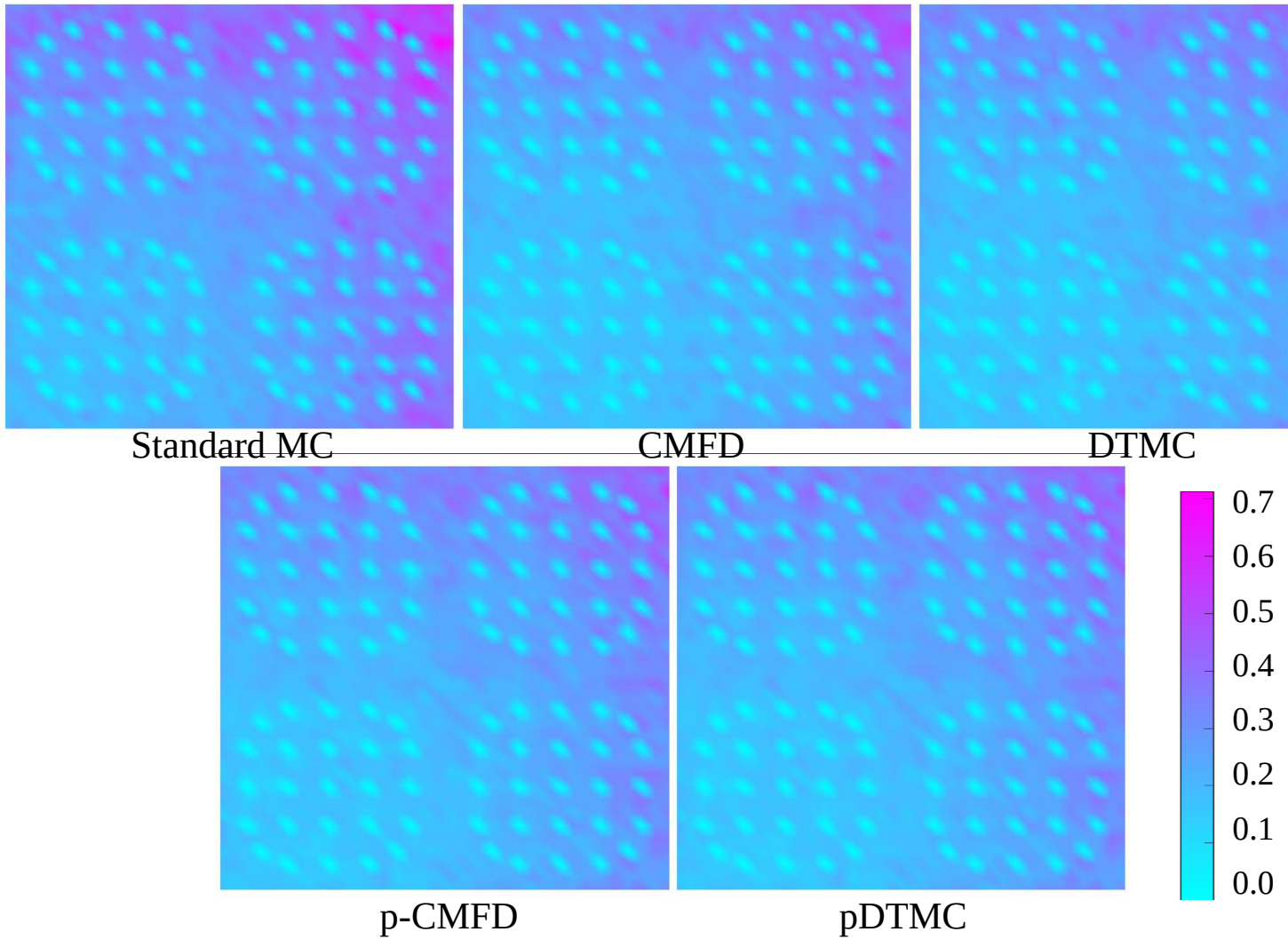
- **Albedo BC**



Numerical Results (2)

RMS error distribution for power distribution

– **Vacuum BC**



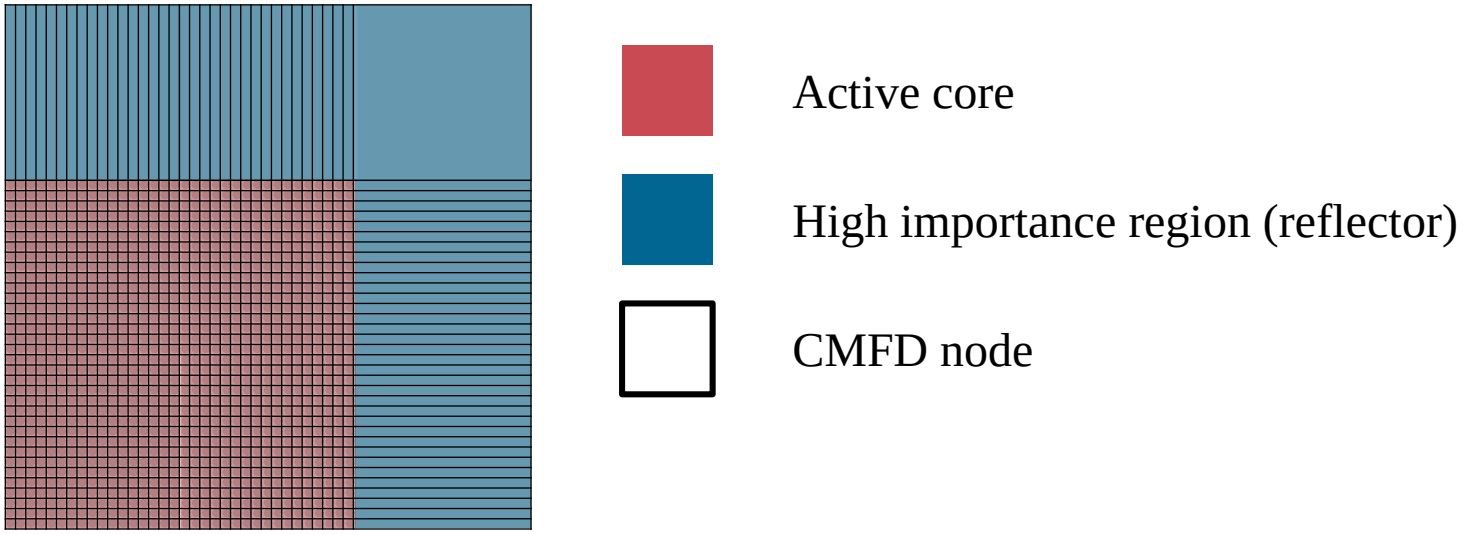
Numerical Results (2)

Computing time

– Computing time was somewhat increased for the CMFD computation

Parameter	Standard MC	Albedo BC		Vacuum BC	
		CMFD	DTMC	CMFD	DTMC
Computing time (min)	47	52		55	
σ_a for k_{eff}	10.2	11.0	5.9	9.0	4.8
Figure-of-merit	3.41E+04	2.65E+04	9.21E+04	3.74E+04	1.39E+05

– Vacuum boundary condition with weight adjustment method can further improve the solution



Concluding Remarks

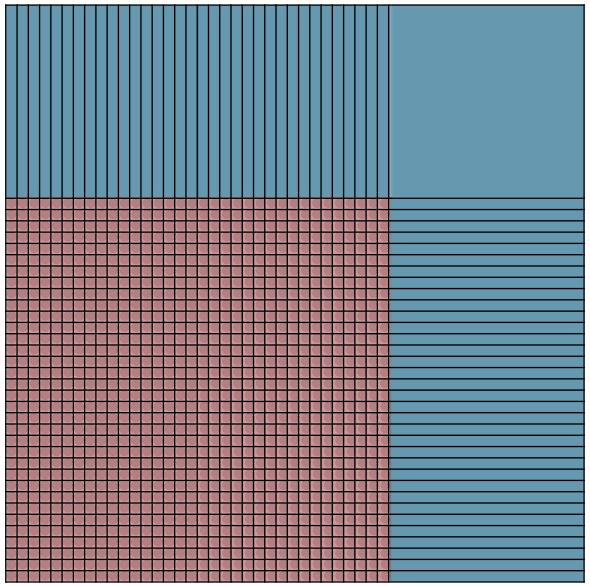
Summary & Conclusions

- **Method I : weight adjustment method**
- The DTMC solution was improved with the optimization of boundary condition
- The power became more reliable especially near boundary with weight adjustment
- The computing time was increased with the higher cell importance
- **Method II : vacuum boundary condition**
- The weight adjustment only within the reflector can minimize the computing time
- The stochastic uncertainty and error were noticeably decreased with vacuum BC
- It is expected that the weight adjustment with vacuum BC can further improve the solution

Concluding Remarks

Future work

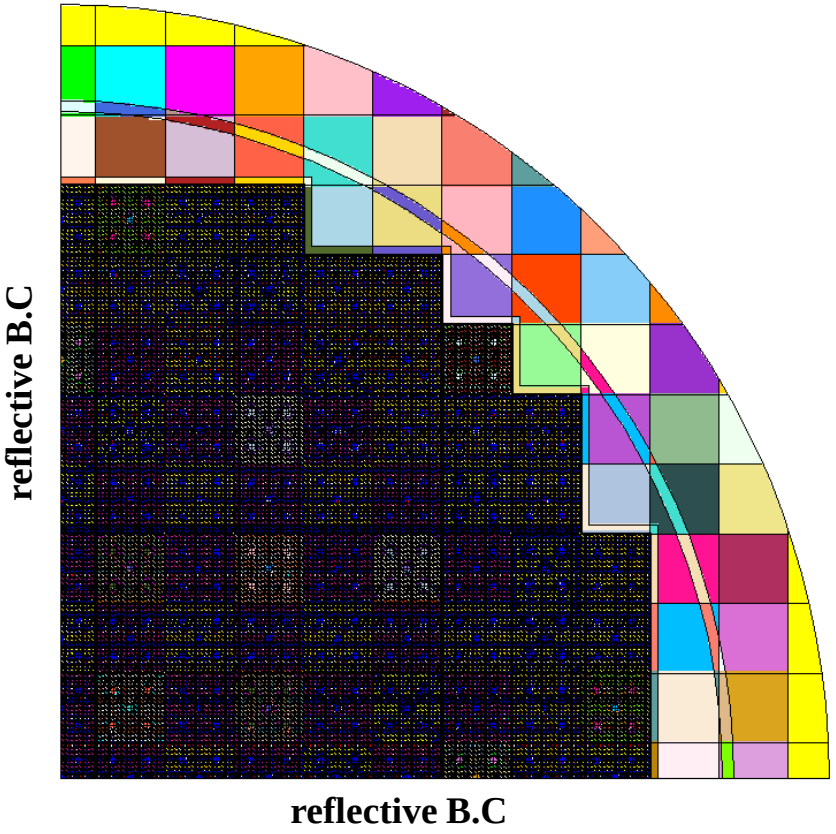
- Vacuum BC & weight adjustment in the reflector region
- Application n big size reactor problem



Active core

High importance region (reflector)

CMFD node



Thank you

for your attention

Backup Slides

CMFD method (1/4)

Basic theory of CMFD method

– The balance equation by integrating the diffusion equation over a node

$$J^{i+1/2} - J^{i-1/2} + \Sigma_a^i \phi^i = S^i \quad (1)$$

where

J : net current

ϕ : neutron flux

Σ_a : absorption cross section

S : fission source

– The net neutron flow within the node is preserved by reference net current

$$J^{i+1/2} = -\tilde{D}^{i+1/2}(\phi^{i+1} - \phi^i) + \hat{D}^{i+1/2}(\phi^{i+1} + \phi^i) \quad (2)$$

where

$$\tilde{D} = \frac{2d^{i+1}d^i}{d^{i+1} + d^i} \text{ : effective diffusion coefficient}$$

$$d = D / \Delta x \text{ : unit diffusion coefficient}$$

– The correction factor is calculated by

$$\hat{D} = \frac{\vec{J}_{MC}^{i+1/2} + \tilde{D}^{i+1/2}(\phi_{MC}^{i+1} - \phi_{MC}^i)}{\phi_{MC}^{i+1} + \phi_{MC}^i} \quad (3)$$

CMFD method (2/4)

Basic theory of p-CMFD method

– The balance equation by integrating the diffusion equation over a node

$$J^{i+1/2} - J^{i-1/2} + \Sigma_a^i \phi^i = S^i \quad (1)$$

– The net neutron flow within the node is preserved by reference two partial currents

$$J^{+,i+1/2} = -0.5\tilde{D}^{i+1/2}(\phi^{i+1/2} - \phi^i) + \hat{D}^{+,i+1/2}\phi^i \quad (4.a)$$

$$J^{-,i+1/2} = +0.5\tilde{D}^{i+1/2}(\phi^{i+1/2} - \phi^i) + \hat{D}^{-,i+1/2}\phi^{i+1} \quad (4.b)$$

– The correction factors are calculated by

$$\hat{D}^{+,i+1/2} = \frac{\vec{J}_{MC}^{+,i+1/2} + 0.5\tilde{D}^{i+1/2}(\phi_{MC}^{i+1} - \phi_{MC}^{i})}{\phi_{MC}^{i}} \quad (5.a)$$

$$\hat{D}^{-,i+1/2} = \frac{\vec{J}_{MC}^{-,i+1/2} - 0.5\tilde{D}^{i+1/2}(\phi_{MC}^{i+1} - \phi_{MC}^{i})}{\phi_{MC}^{i+1}} \quad (5.b)$$

CMFD method (3/4)

CMFD parameters

– **Neutron current**

$$\phi = \int \phi(\vec{r}, E) dE dV \tag{6}$$

– **Neutron flux**

$$J^{+} = \int_{\Omega^{+}} \Omega \cdot \varphi(\vec{r}, \Omega, \mathrm{E}) d\Omega \tag{7.a}$$

$$J^{-} = \int_{\Omega^{-}} \Omega \cdot \varphi(\vec{r}, \Omega, \mathrm{E}) d\Omega \tag{7.b}$$

– **Group constant**

$$\Sigma_{\alpha} = \frac{\int \int \Sigma_{\alpha}(\vec{r}, E) \phi(\vec{r}, \mathrm{E}) dE dV}{\int \int \phi(\vec{r}, \mathrm{E}) \, \mathrm{dEdV}} \tag{8}$$

CMFD method (4/4)

Numerical treatments for CMFD in MC method

– **Boundary condition**

- The correction factor is defined by the ratio of current and flux as in rebalance method

$$\hat{D} = J / \phi \tag{9}$$

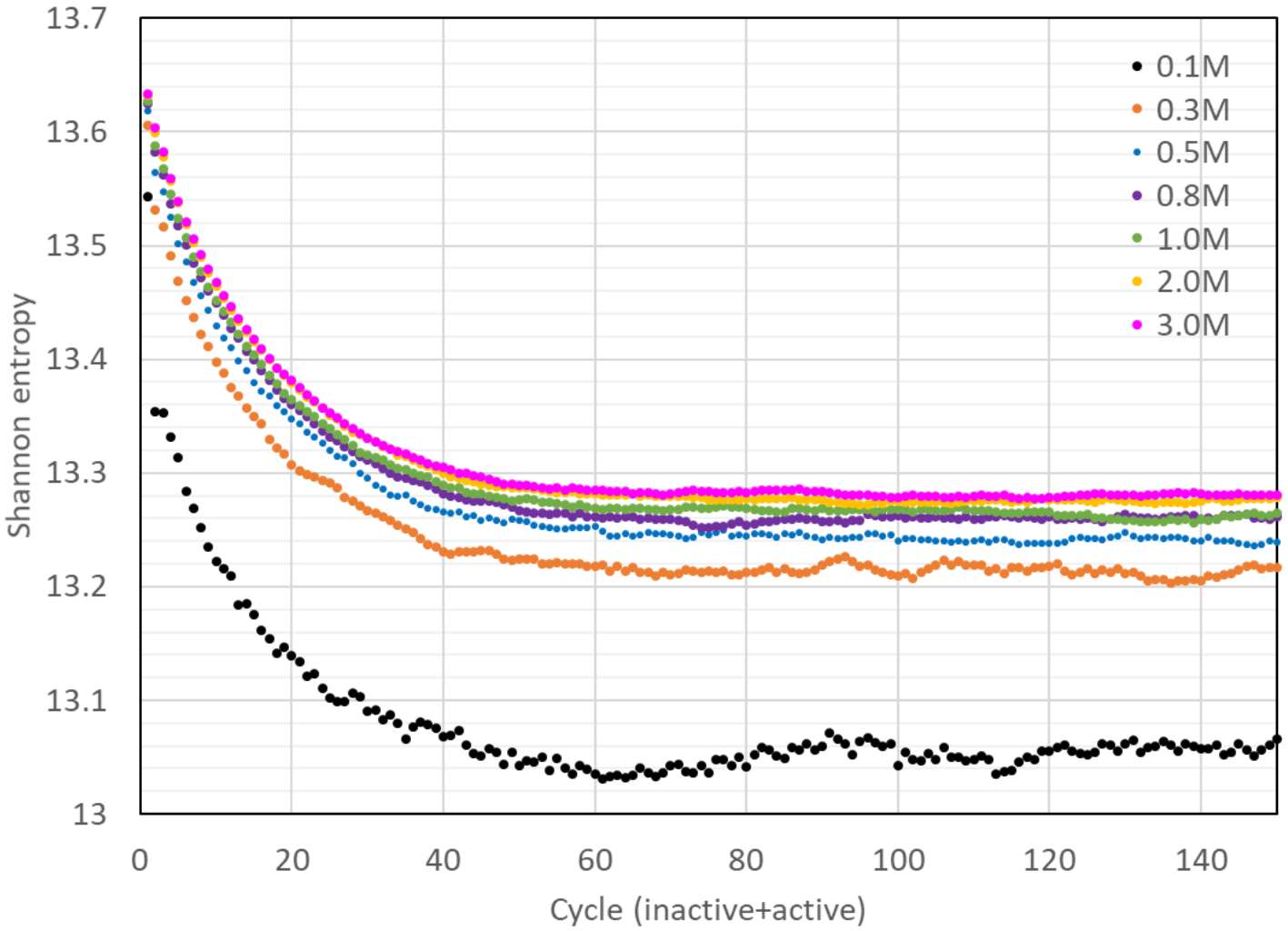
– **Weight adjustment**

- Particles' weight is adjusted by the ratio of the fission source probability

$$w_j^{'} = w_j \times \frac{\nu\Sigma_f \phi_i^{CMFD} \Big/ \sum_i \nu\Sigma_f \phi_i^{CMFD}}{\nu\Sigma_f \phi_j^{MC} \Big/ \sum_j \nu\Sigma_f \phi_j^{MC}} \tag{10}$$

Convergence of FSD

Shannon entropy with different generation sizes



Simulation Code

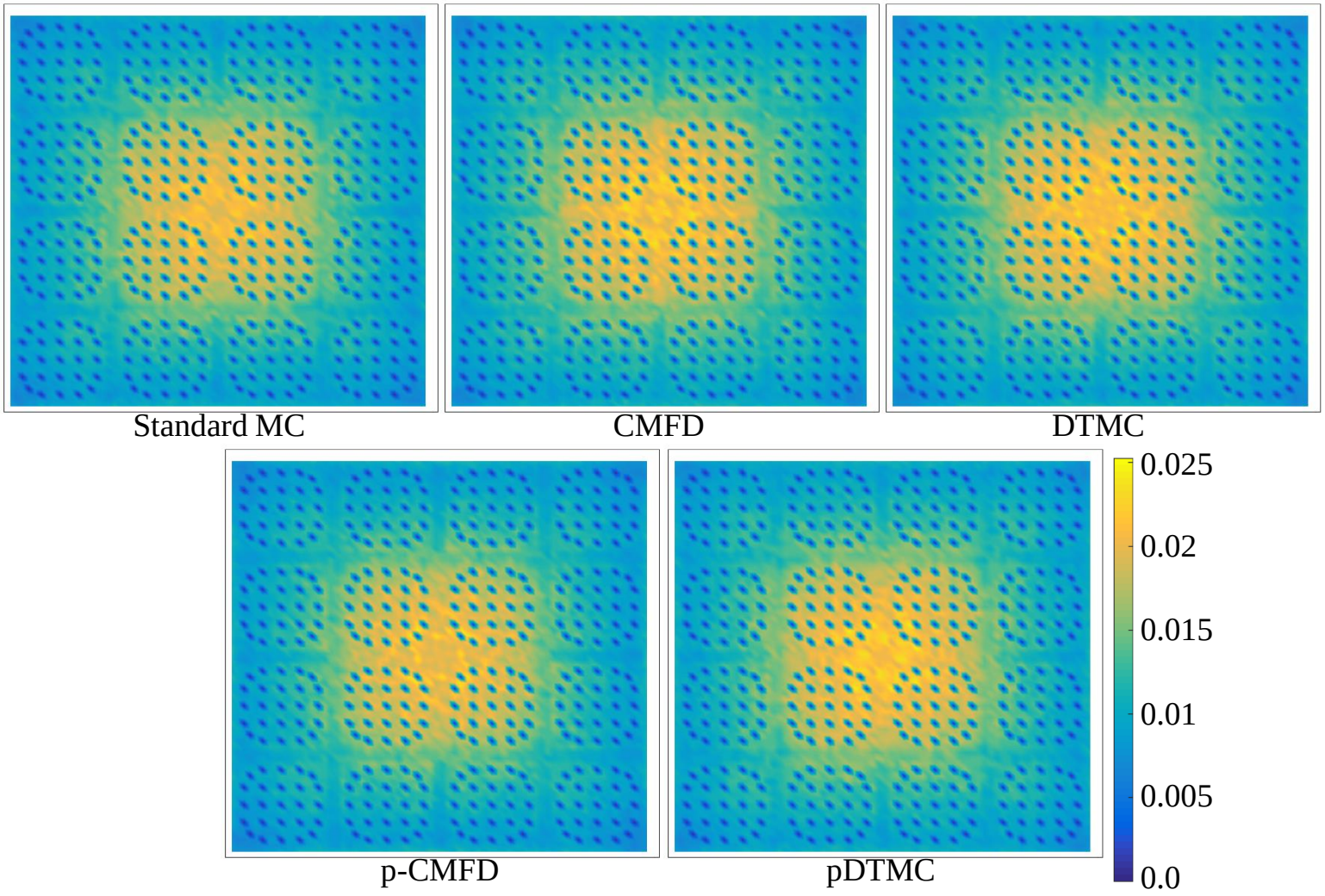
In-house MC code

- In-house MC code for neutronic analysis
- 3D pin-lattice geometry
- Multi-group energy
- Continuous energy (under development)
- CMFD & p-CMFD acceleration scheme

Numerical Results

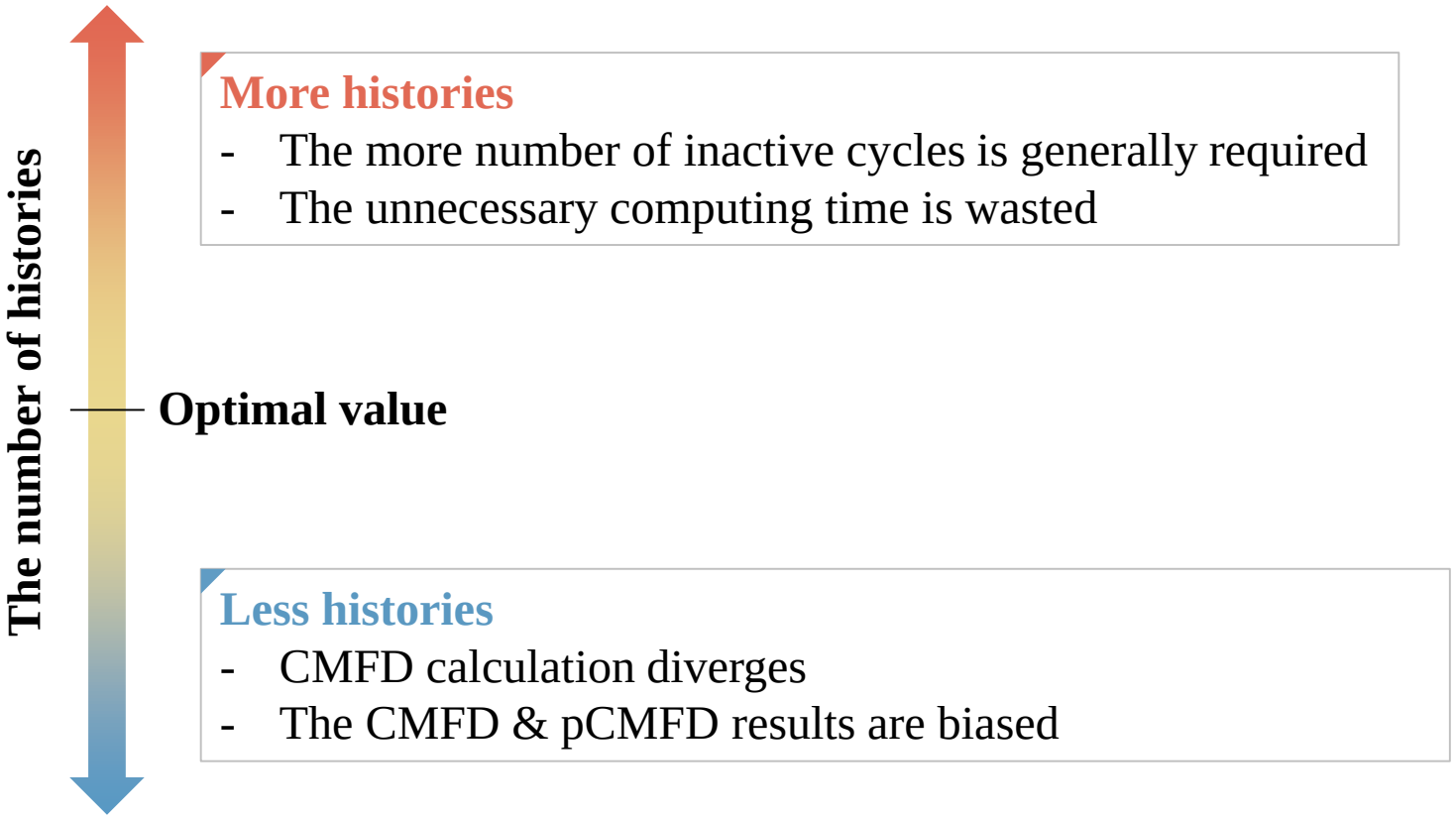
Real standard deviation distribution of pin power

– **1st layer (near mid-plane)**



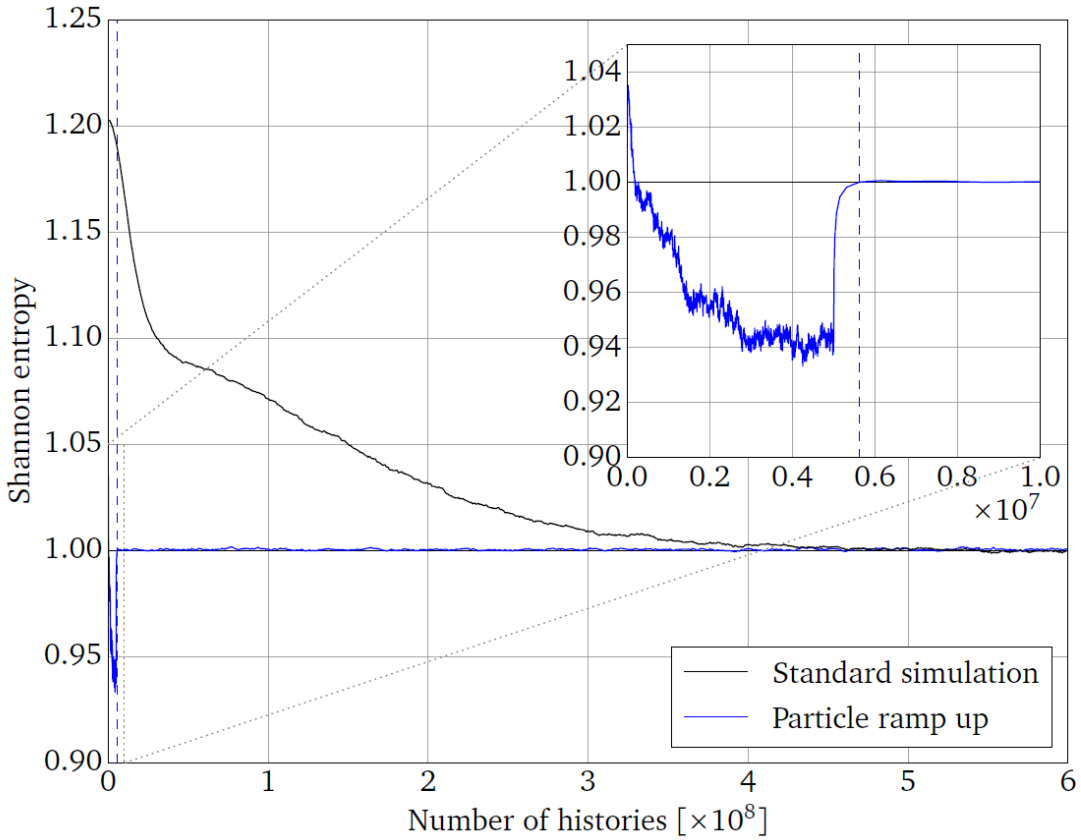
Optimal generation size in MC & CMFD calculation

- Enhances the efficiency of the simulation
- Guarantees the convergence of the calculation



Particle ramp-up method (A. L. Lund, P. K. Romano, A. R. Siegel)

- A procedure for accelerating convergence of the source distribution
- Roughly converge using fewer particles per generation and increase the number of particles



❖ **Procedure**

N_0 initial particles

↓

New source is rescaled to N_i particles

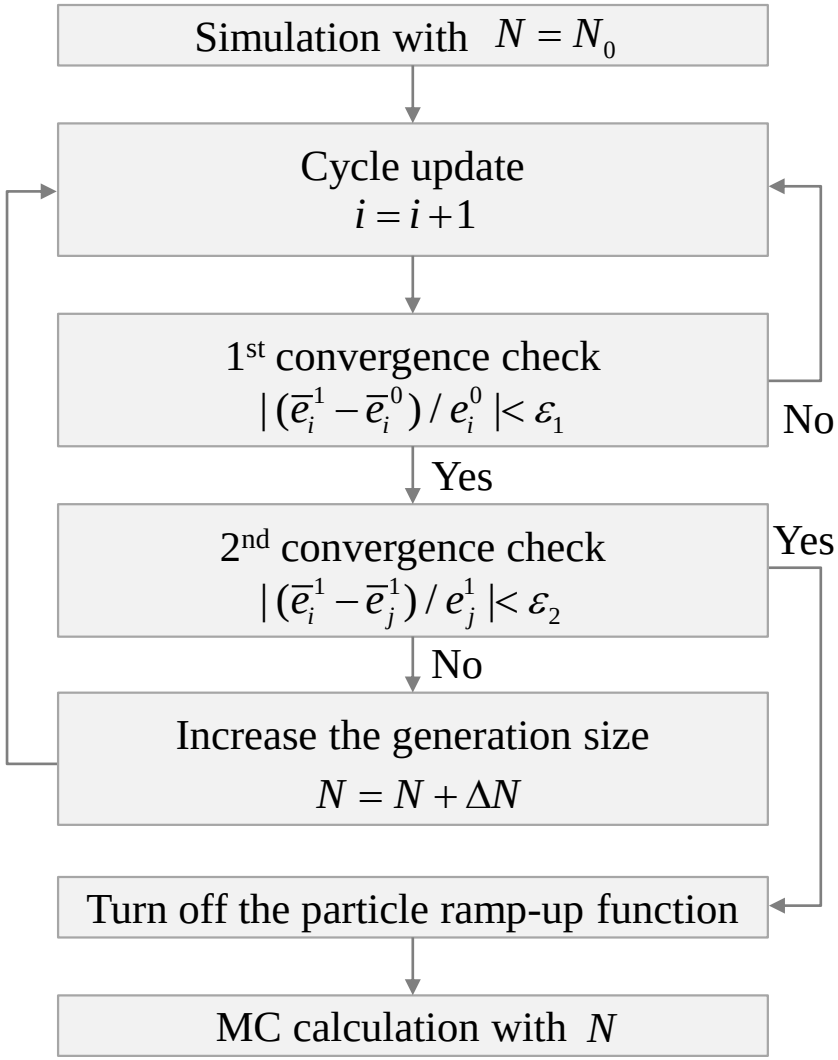
↓

The expected number of fission neutrons is biased by a factor of **two**

↓

Once the generation size reaches N, begin active generation

Algorithm of modified particle ramp-up technique



- N_0 : initial number of histories
- i : cycle number
- e : Shannon entropy

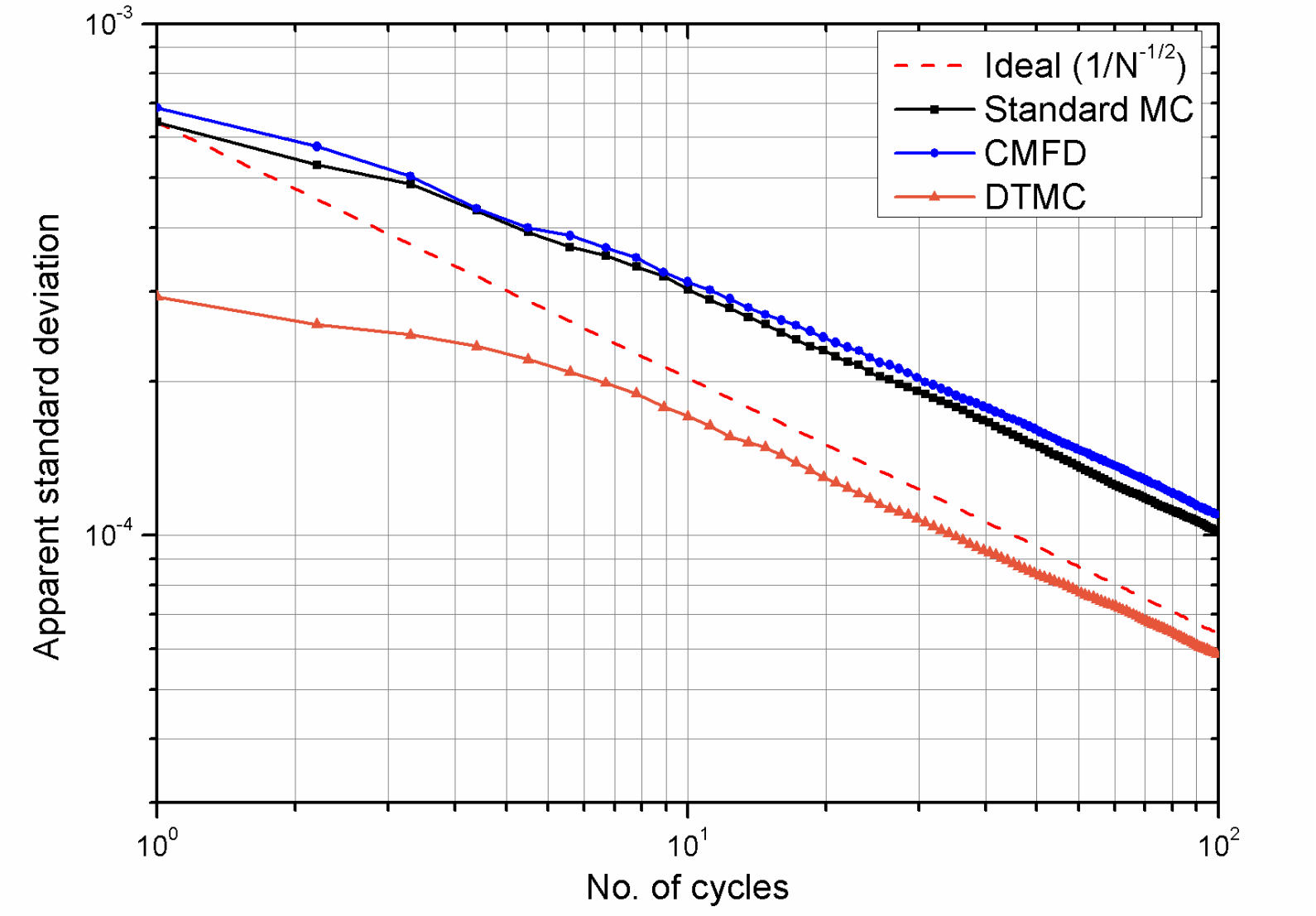
$$\bar{e}_i^1 = \frac{e_{i-l+1} + \cdots + e_i}{l}$$

$$\bar{e}_i^0 = \frac{e_{i-2l+1} + \cdots + e_{i-l}}{l}$$

- l : accumulation length
- ΔN : increment of generation size
- j : cycle number before N changes
- ε_1 : convergence criterion
- ε_2 : stopping criterion

Numerical Results

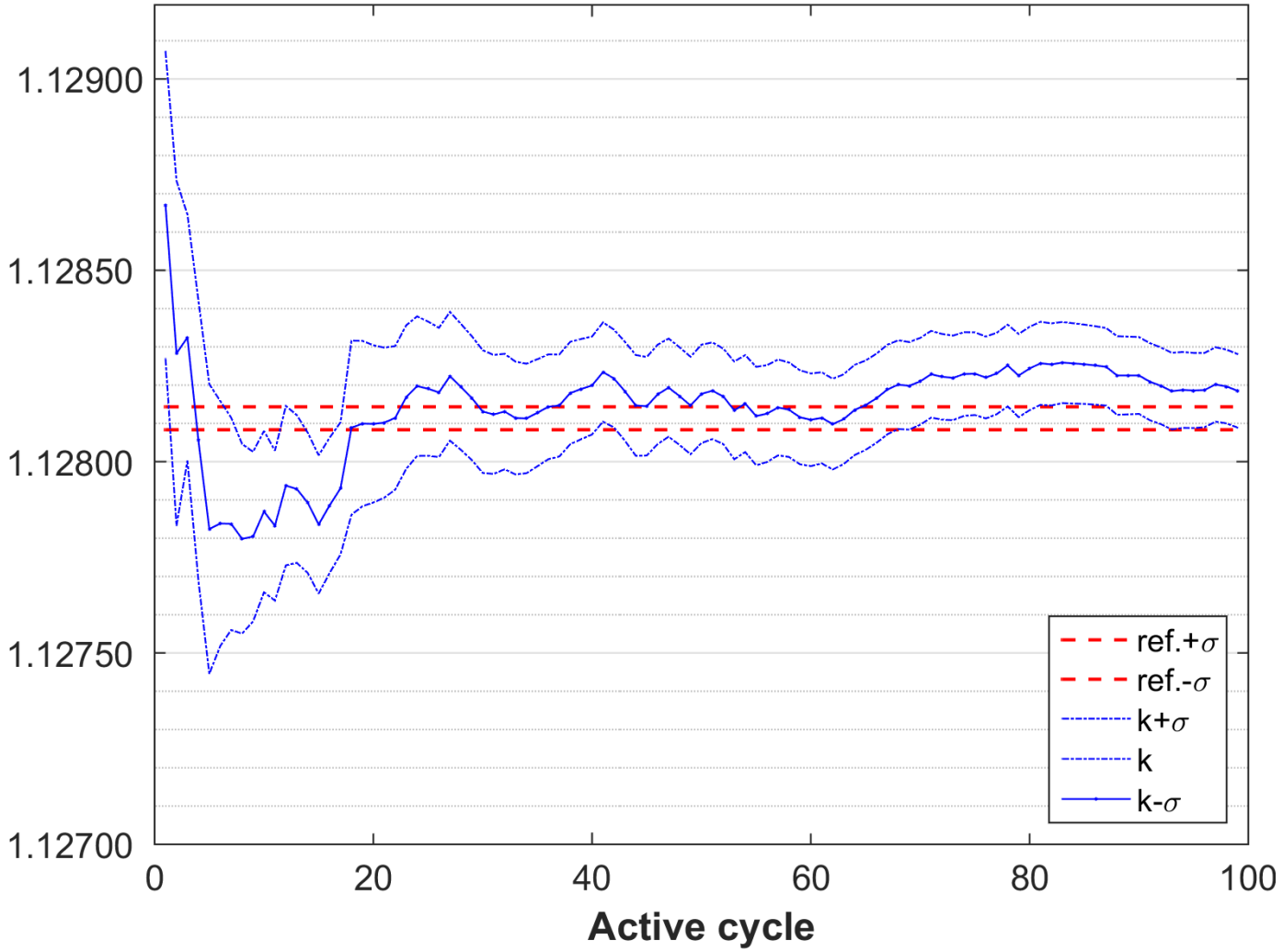
Apparent standard deviation of k_{eff}



Numerical Results

Cycle-wise multiplication factor with stochastic uncertainty

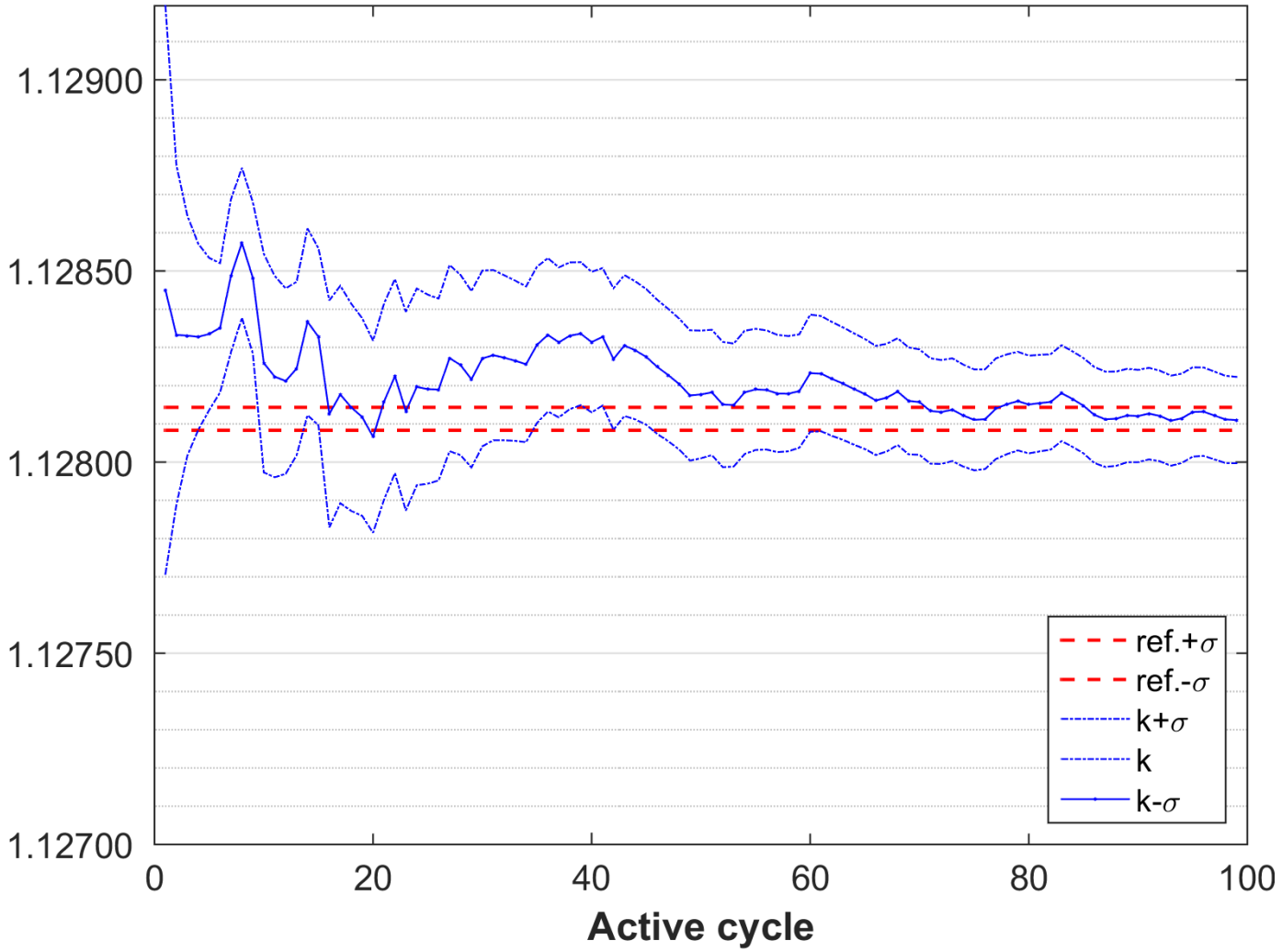
– **Standard MC**



Numerical Results

Cycle-wise multiplication factor with stochastic uncertainty

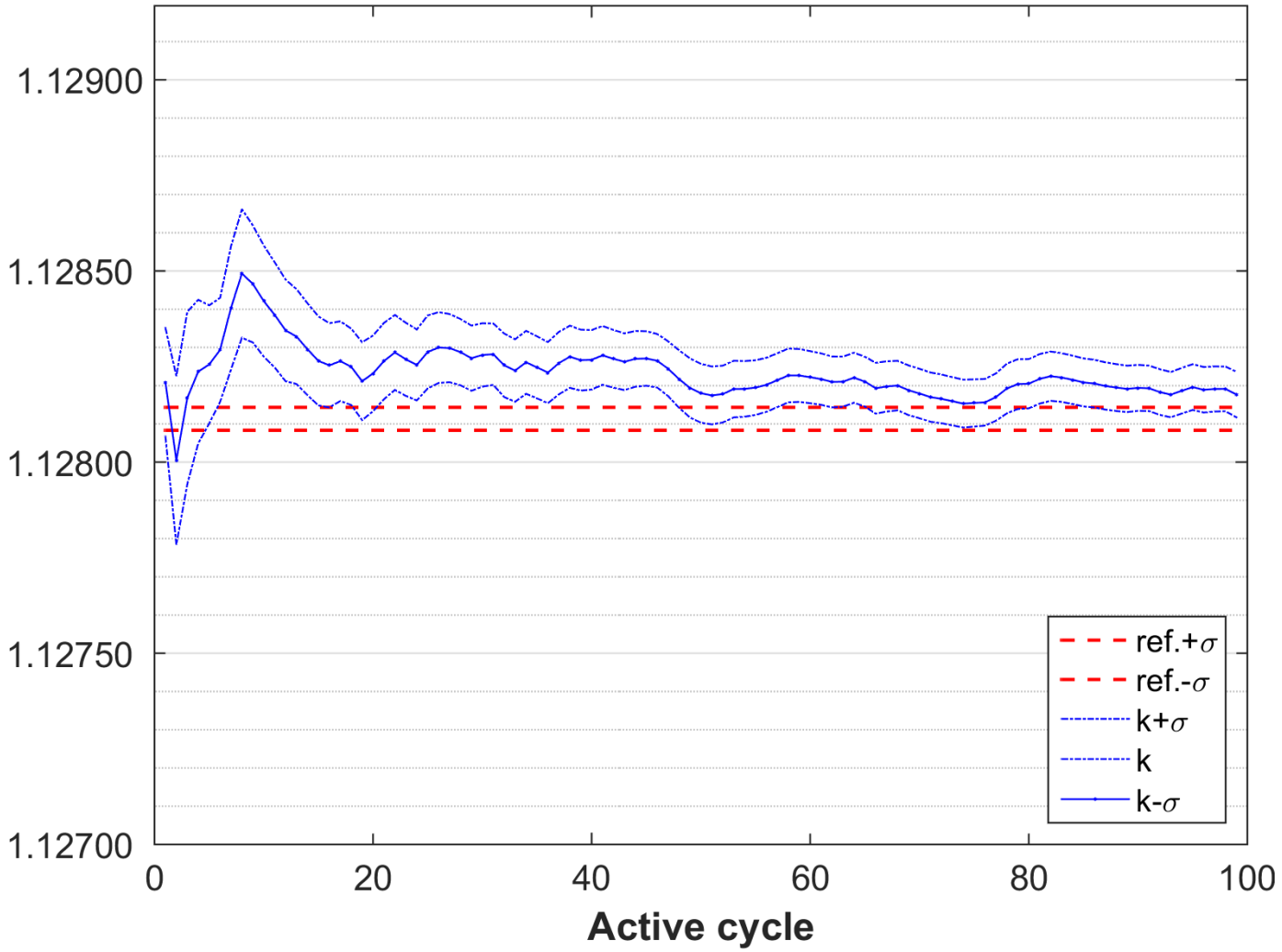
– **CMFD**



Numerical Results

Cycle-wise multiplication factor with stochastic uncertainty

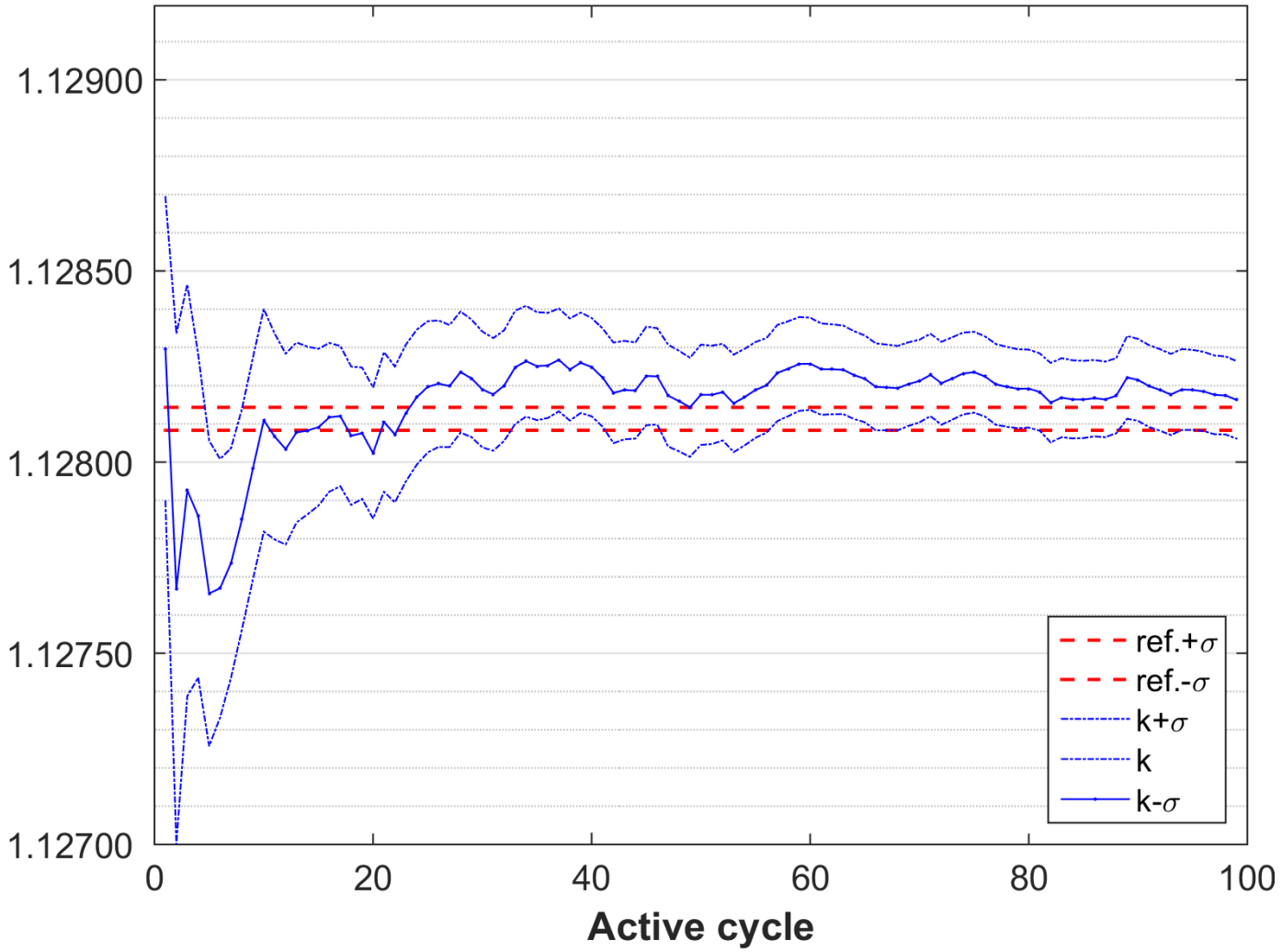
– **DTMC**



Numerical Results

Cycle-wise multiplication factor with stochastic uncertainty

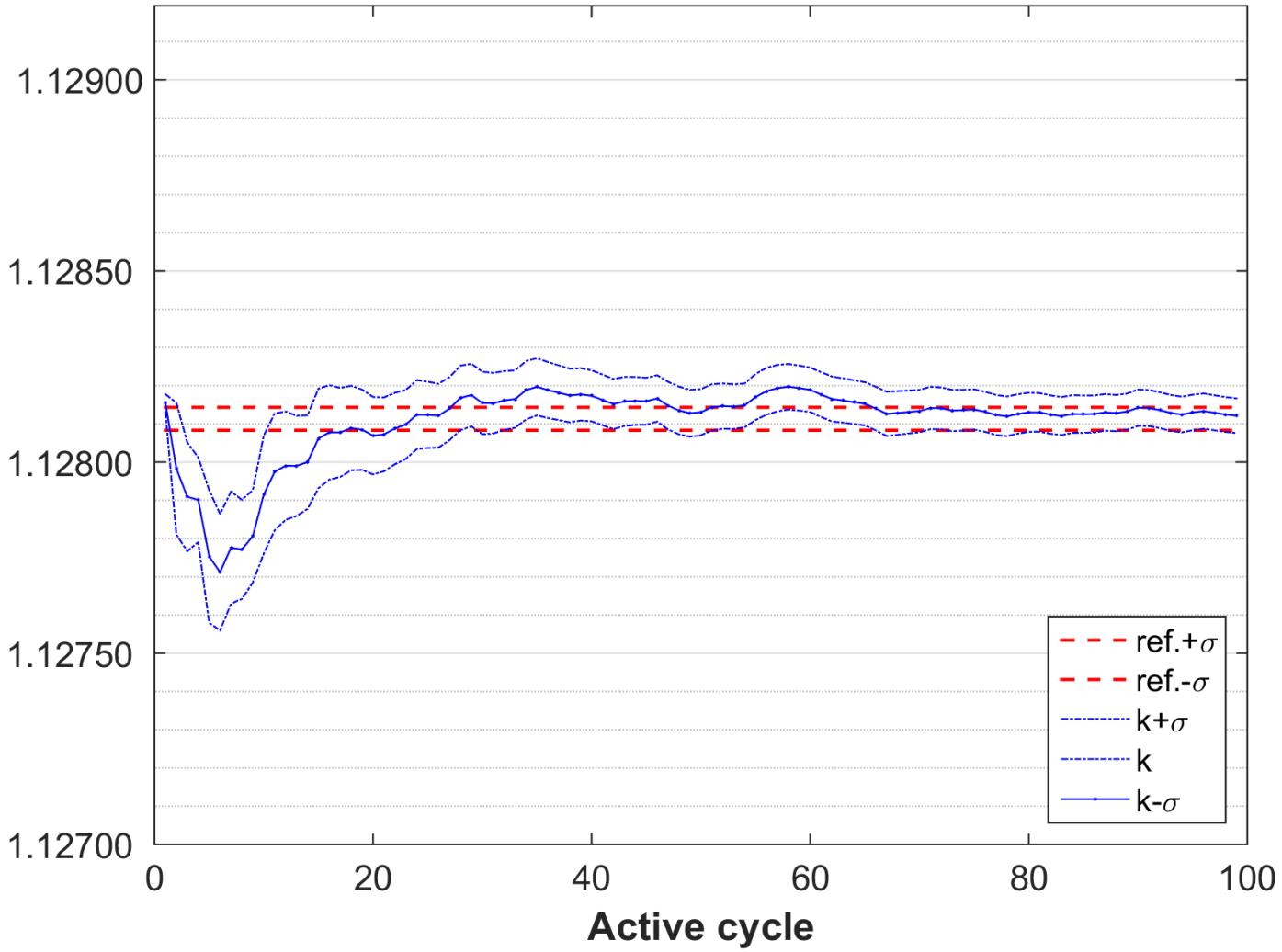
– **p-CMFD**



Numerical Results

Cycle-wise multiplication factor with stochastic uncertainty

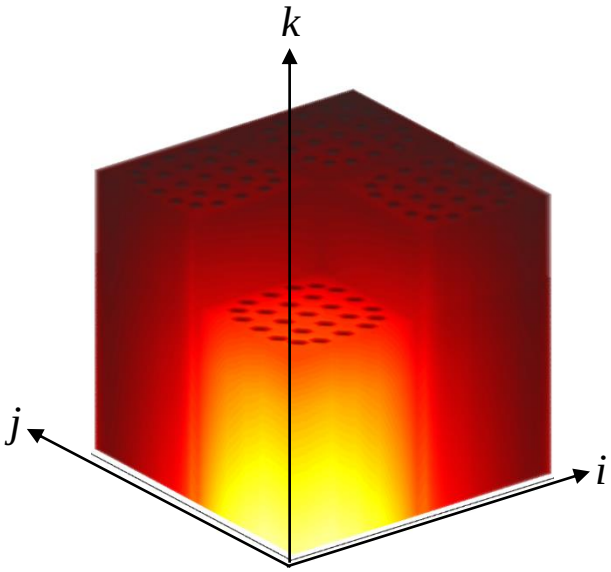
– **pDTMC**

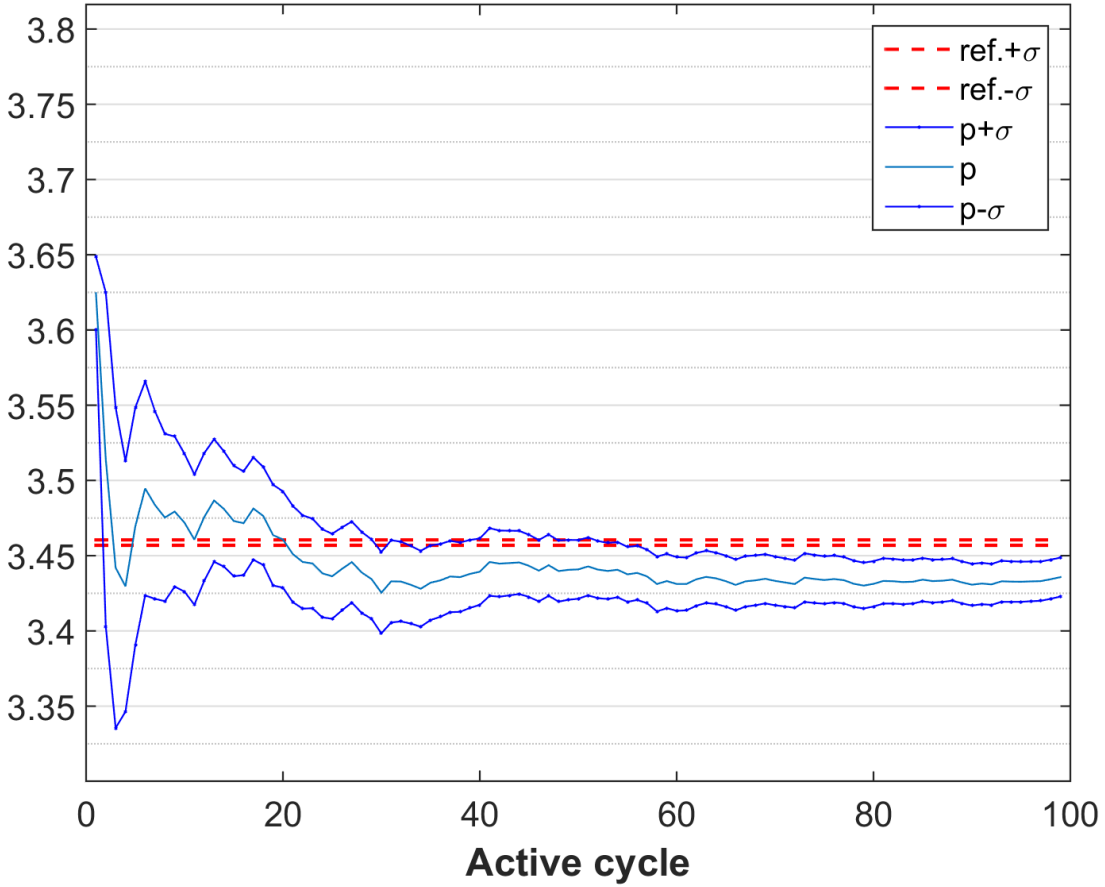


Numerical Results

Cycle-wise normalized power level at specific cell 1

- **Center region; (*i*,*j*,*k*) = (1,2,1)**
- **Standard MC**

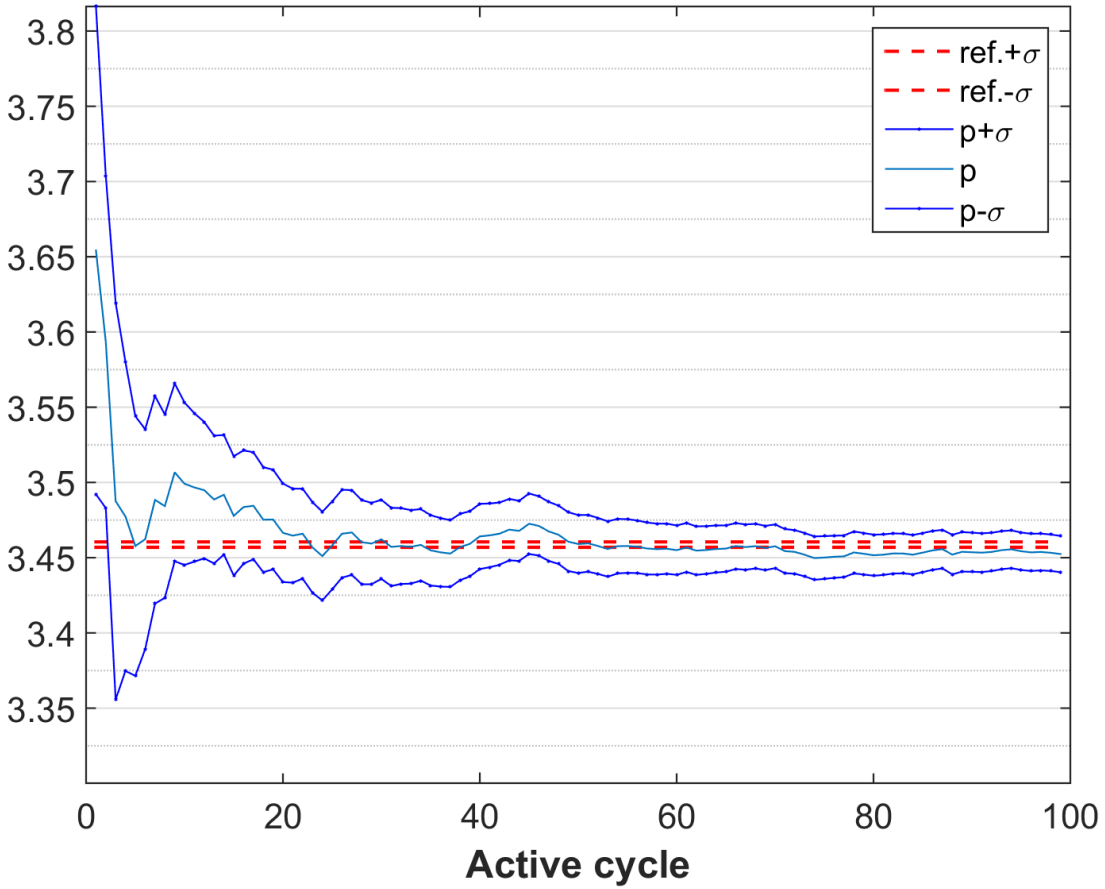




Numerical Results

Cycle-wise normalized power level at specific cell 1

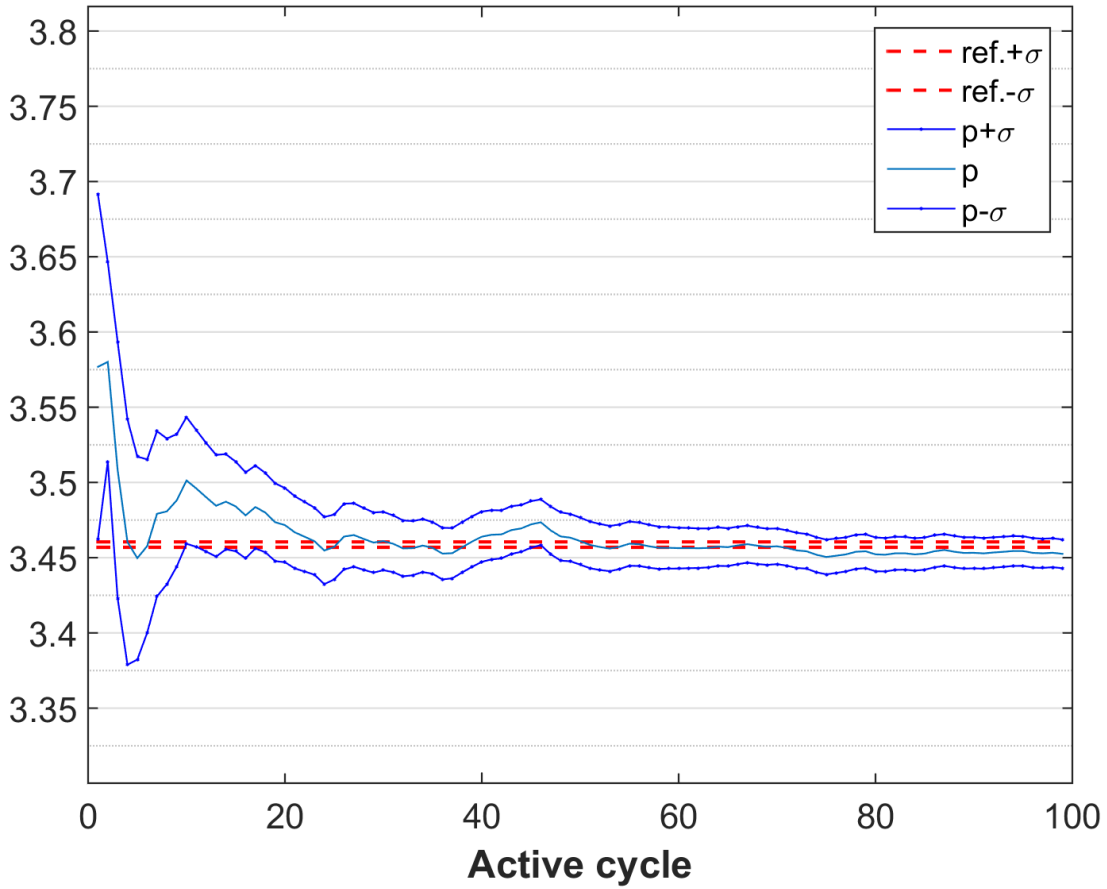
- **Center region; (*i*,*j*,*k*) = (1,2,1)**
- **CMFD**



Numerical Results

Cycle-wise normalized power level at specific cell 1

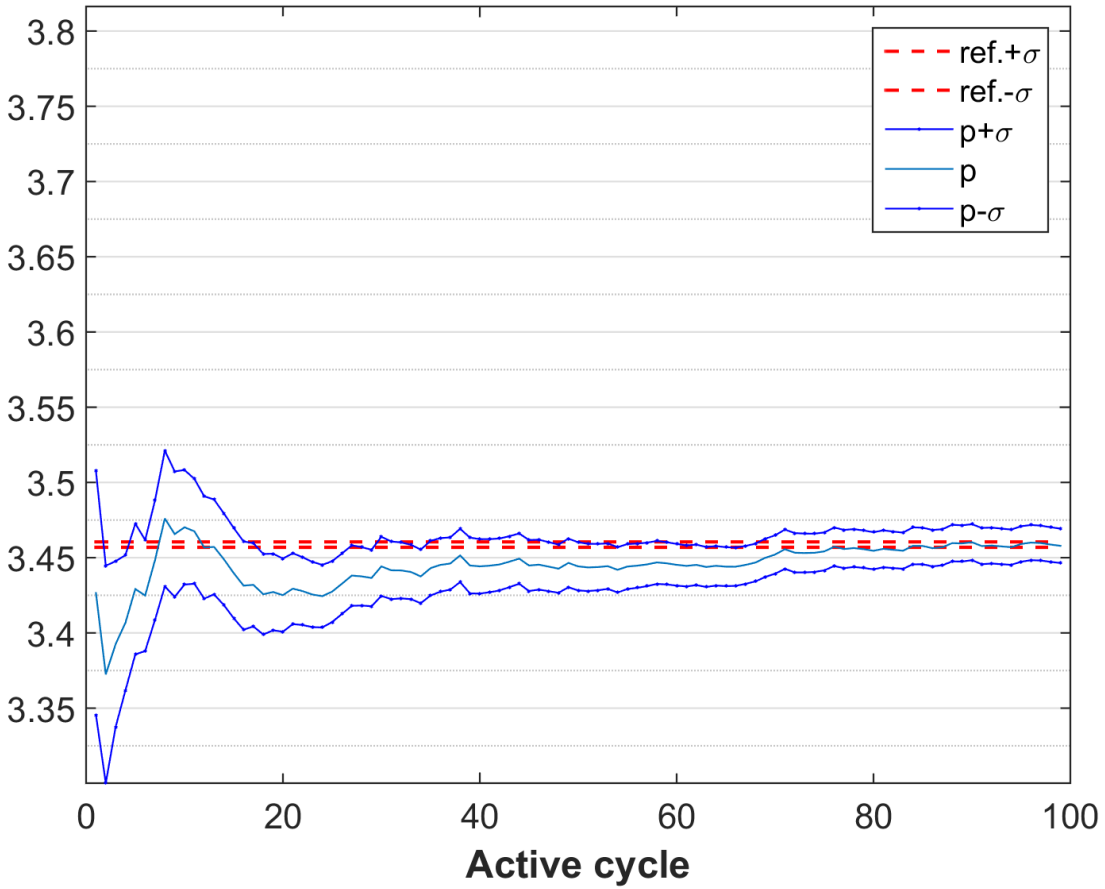
- **Center region; (*i*,*j*,*k*) = (1,2,1)**
- **DTMC**



Numerical Results

Cycle-wise normalized power level at specific cell 1

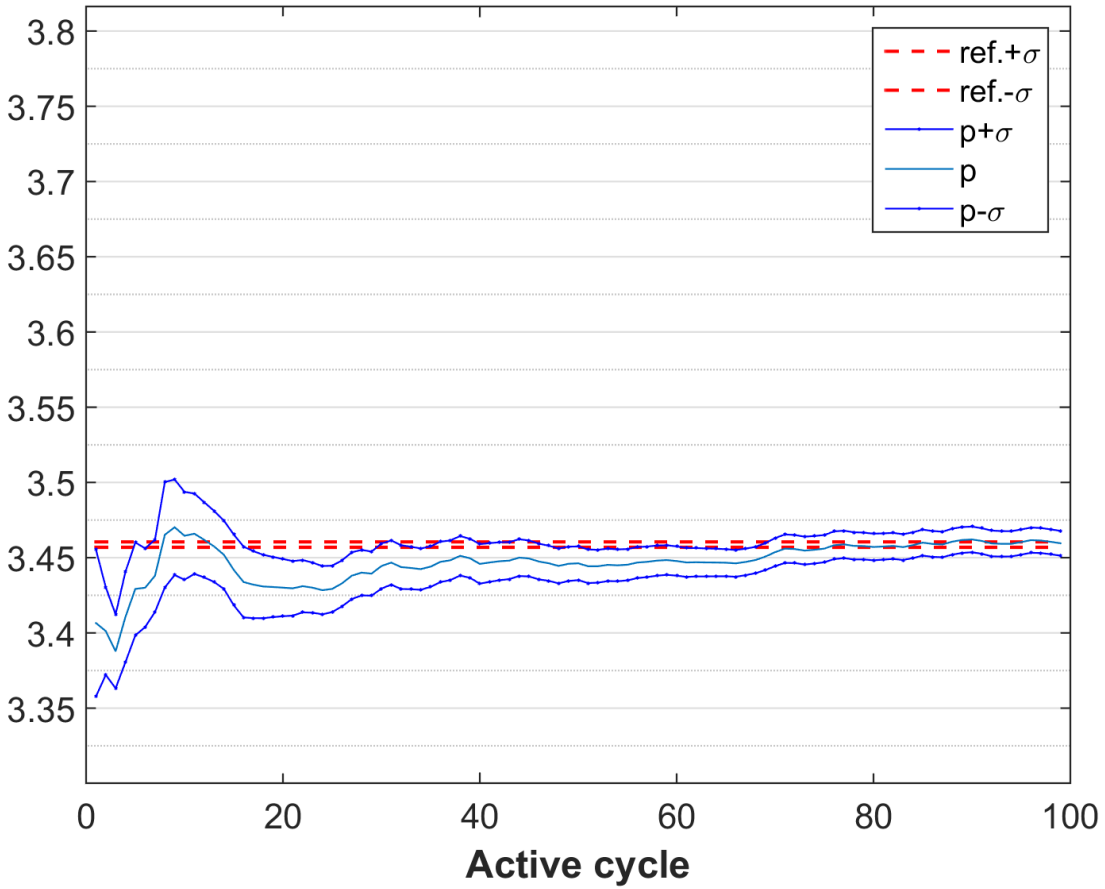
- **Center region; (*i*,*j*,*k*) = (1,2,1)**
- **p-CMFD**



Numerical Results

Cycle-wise normalized power level at specific cell 1

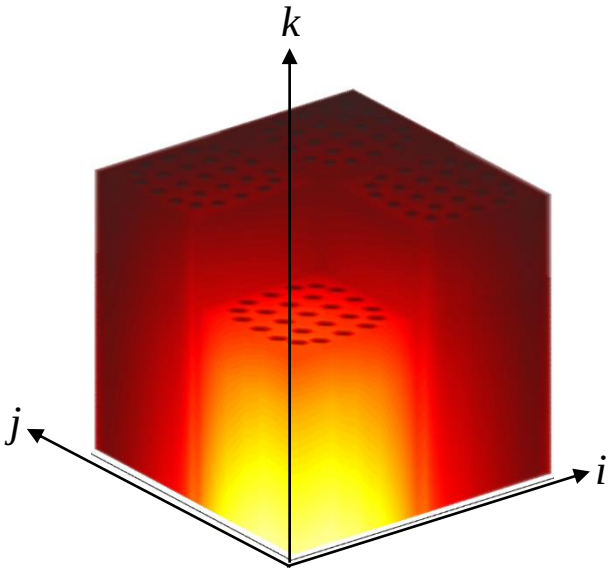
- **Center region; (*i*,*j*,*k*) = (1,2,1)**
- **pDTMC**

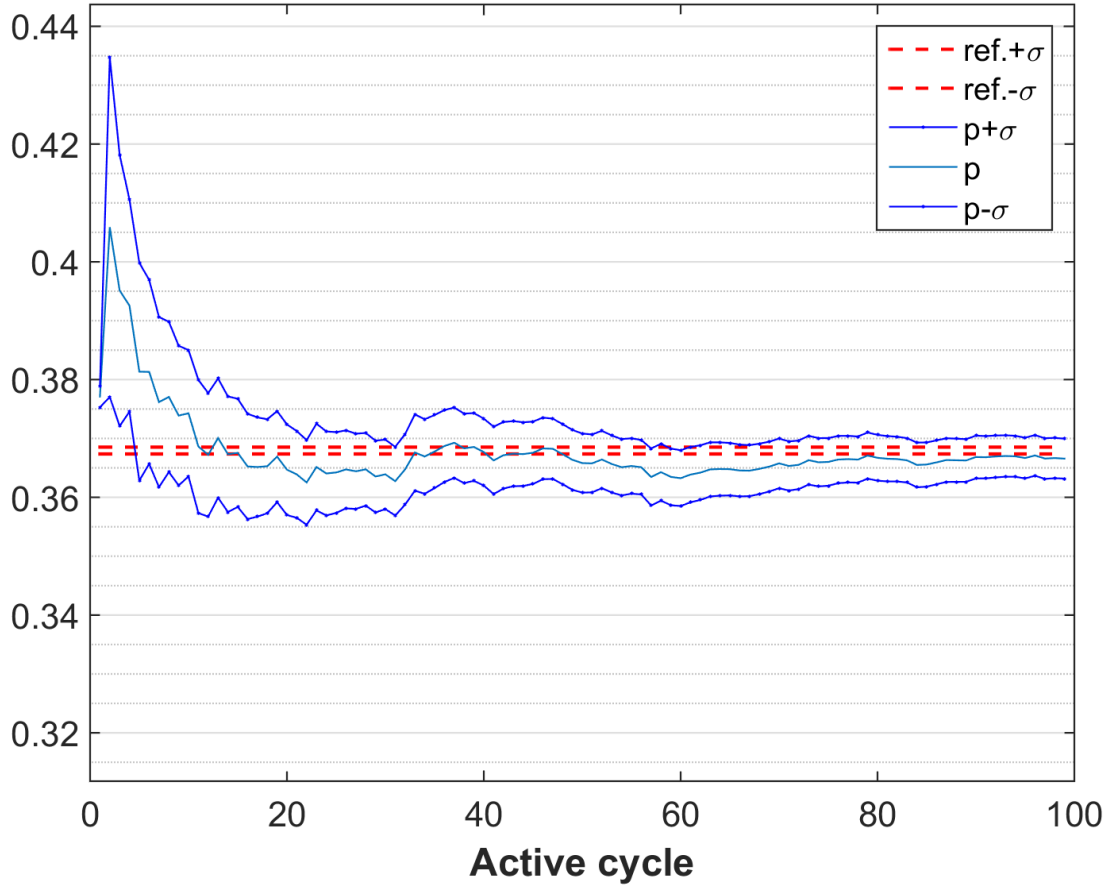


Numerical Results

Cycle-wise normalized power level at specific cell 2

- **Near boundary region; (*i*,*j*,*k*) = (34,34,2)**
- **Standard MC**

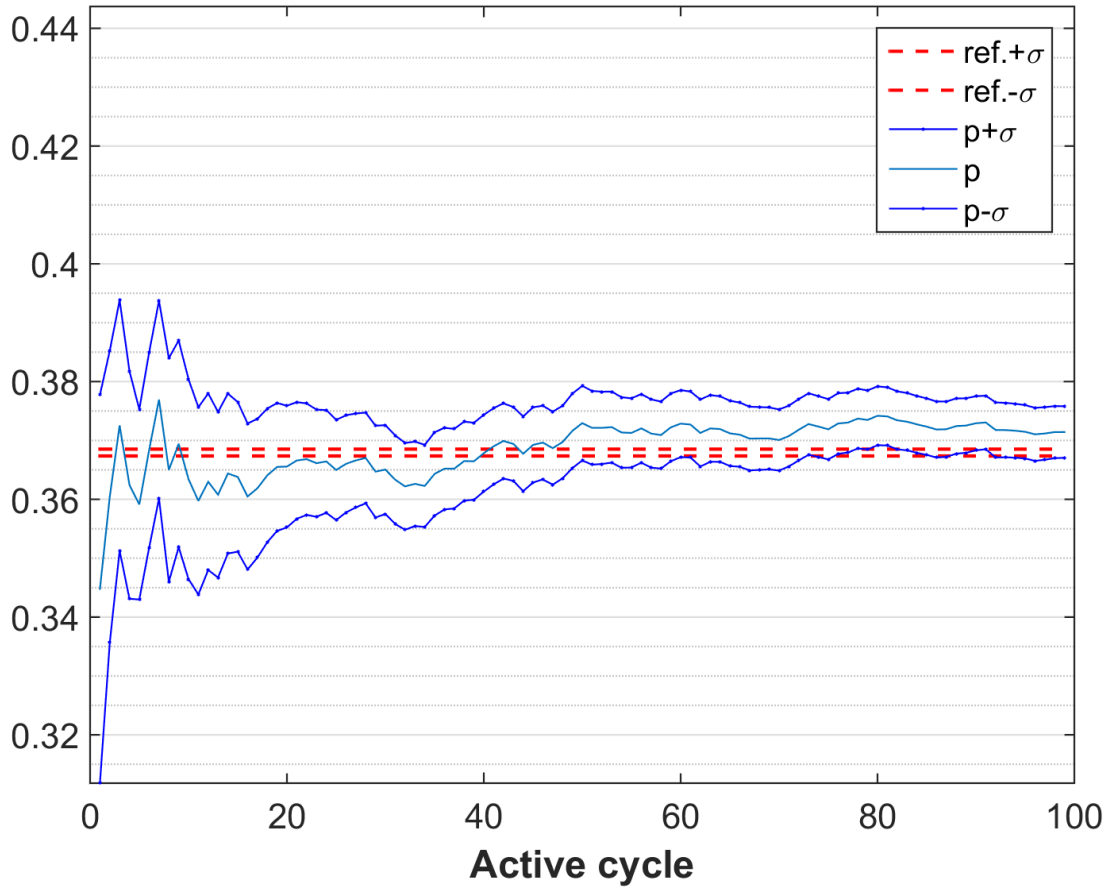




Numerical Results

Cycle-wise normalized power level at specific cell 2

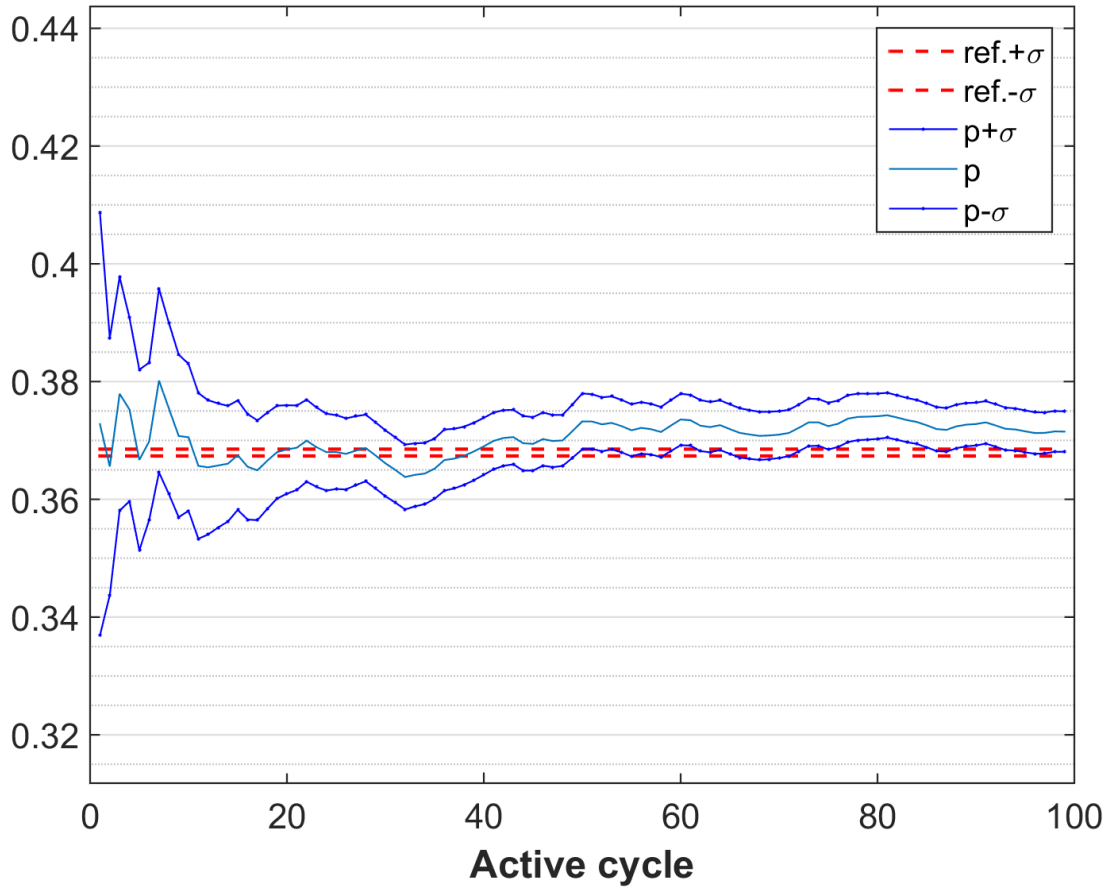
- **Near boundary region; (*i*,*j*,*k*) = (34,34,2)**
- **CMFD**



Numerical Results

Cycle-wise normalized power level at specific cell 2

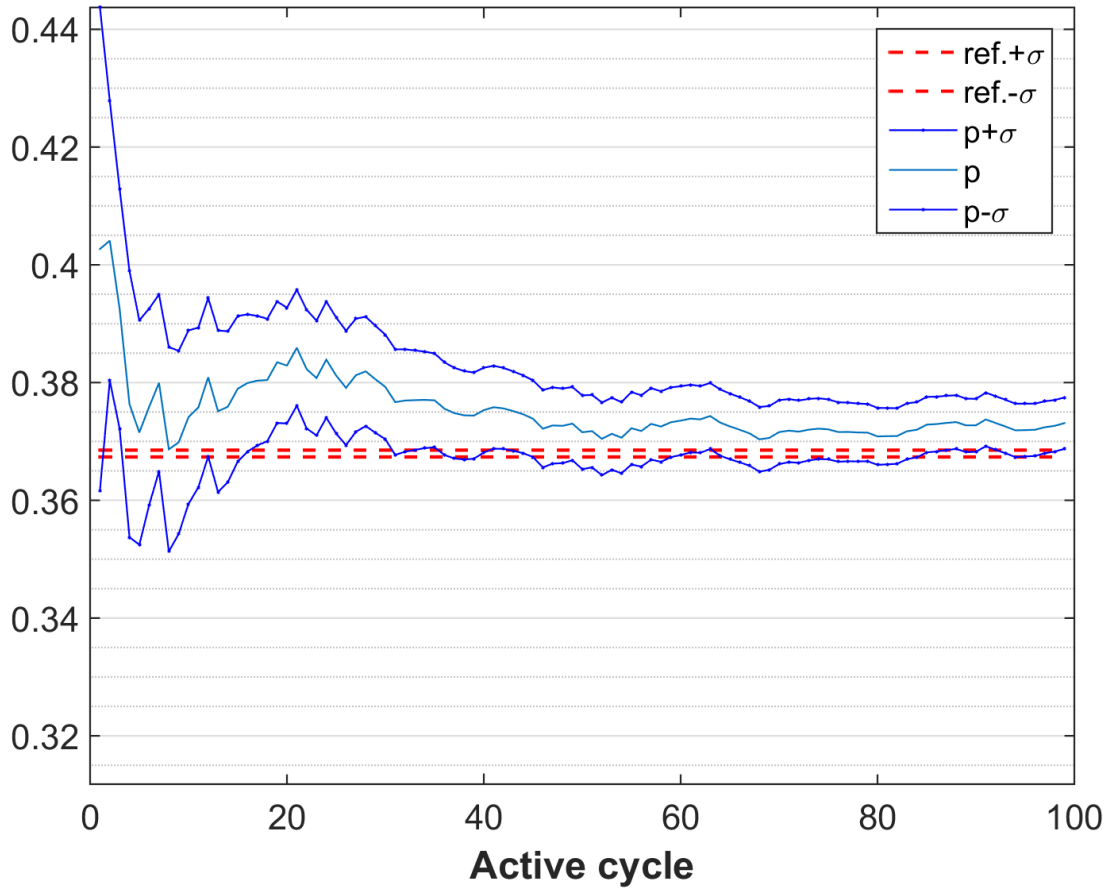
- **Near boundary region; (*i*,*j*,*k*) = (34,34,2)**
- **DTMC**



Numerical Results

Cycle-wise normalized power level at specific cell 2

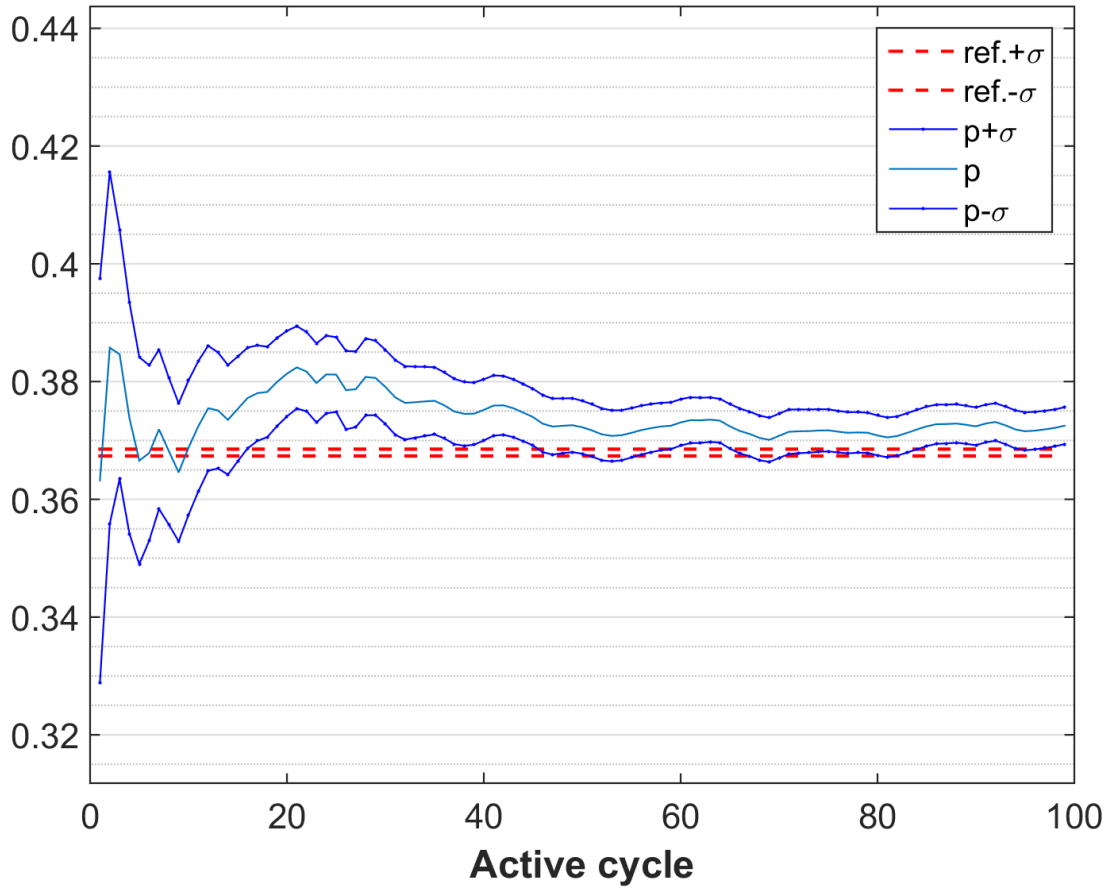
- **Near boundary region; (*i*,*j*,*k*) = (34,34,2)**
- **p-CMFD**



Numerical Results

Cycle-wise normalized power level at specific cell 2

- **Near boundary region; (*i*,*j*,*k*) = (34,34,2)**
- **pDTMC**



Computing time and figure-of-merit (FOM) for k_{eff}

– The computing time of the MC simulation is estimated with a single core

Method	k_{eff}	σ_a (pcm)	Time (min)	FOM
Reference	1.12808	1.7	1867	3.09E+4
Stand-alone MC	1.12819	9.6	46	3.86E+4
CMFD	1.12813	10.8	54	2.62E+4
DTMC	1.12807	5.4		1.05E+5
p-CMFD	1.12812	10.7	52	2.76E+4
pDTMC	1.12811	5.0		1.22E+5
CRX w/o p-CMFD	1.12822	-	4,708*	-
CRX w/ p-CMFD	1.12822	-	871*	-

* *Indirect estimation*
: *calculation time is indirectly estimated under the assumption that the parallel efficiency is 60%*

– **CRX :** conventional deterministic code

- Whole core 3D transport code based on 2D/1D fusion method
- 32 radial cells per modular fuel pin cell and 36 radial cells per modular reflector pin cell
- 3.57 cm in the axial direction, and 8 azimuthal and 3 polar angles per octant with 50 rays per cell per angle