Analysis of Effective Gadolinium Depletion Model

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1. Introduction

This paper analyzes direct Gd depletion model and effective Gd depletion model in RAST-K 2.0 in order to get accurate solution. RAST-K 2.0 [1] nodal code has been under development for PWR reactor analysis since 2013 by the UNIST CORE research group. In the past, due to the memory limitation and simulation time, most of the nodal codes adapted macroscopic depletion solvers. However, in order to track the amount of major isotopes in the reactor core, the microscopic depletion model needs to be used. Furthermore, these isotopes have dependencies on history variables determined by performing separate single assembly STREAM [2] depletion calculations for each of the history effects [3].

RAST-K 2.0 has three separate burnup chains for heavy nuclides, Xe/Sm fission products, and gadolinium burnable absorbers. A Chebyshev Rational Approximation Method (CRAM) [4] is adopted in the depletion solver of RAST-K 2.0 for both heavy nuclide and fission product burnup chains [5][6].

The majority of pressurized water reactors (PWRs) in Korea use gadolinium, which has a large neutron absorption cross section, as burnable absorber for excessive reactivity control. The direct numerical solution for Gd chain does not follow STREAM solution. Thus, the depletion calculation of gadolinium requires special treatments due to non-linear absorption cross section change as burnup. This paper analyzes two different approaches for the treatment of Gd depletion to solve the limitation of direction solution for linear Gd chain.

2. Gd Depletion Solvers

Nodal codes which adopts microscopic depletion module use separate burnup chains for: heavy nuclides, fission products and high burnable absorbers. A linear chain of Gd isotopes is as shown in Fig. 1. ^{155}Gd and ^{157}Gd have the highest absorption cross sections among the 7 isotopes.
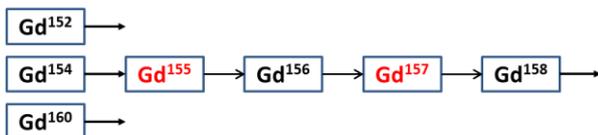

Fig. 1. Gd isotopes chain.

The first depletion model is a direct numerical solution solved by linear chain, written as:

$$\frac{dN_m(t)}{dt} = N_{m-1}(t)\sigma_{m-1}(t)\varphi(t) - N_m(t)\sigma_m(t)\varphi(t),$$
$$(m = {}^{154}\text{Gd to } {}^{158}\text{Gd}), \tag{1}$$

where N_m is the number density of isotope m, as a function of time (t). $\sigma_m(t)$ is the microscopic absorption cross section of isotope m, and $\varphi(t)$ is neutron flux [7].

The second depletion model is effective Gd isotope depletion. Five Gd isotopes from ^{154}Gd to ^{158}Gd are replaced with a single effective Gd [3][8]. The number density ($N_{\text{Gd}^{\text{eff}}}$) and microscopic cross section ($\sigma_{\text{Gd}^{\text{eff}}}$) for the effective Gd isotope are defined as:

$$N_{\text{Gd}^{\text{eff}}} = 5N_{\text{Gd}^{154}} + 4N_{\text{Gd}^{155}} + 3N_{\text{Gd}^{156}} + 2N_{\text{Gd}^{157}} + N_{\text{Gd}^{158}}, \tag{2}$$

$$\sigma_{\text{Gd}^{\text{eff}}} = \frac{\Sigma_{\text{Gd}^{154}} + \Sigma_{\text{Gd}^{155}} + \Sigma_{\text{Gd}^{156}} + \Sigma_{\text{Gd}^{157}} + \Sigma_{\text{Gd}^{158}}}{N_{\text{Gd}^{\text{eff}}}}. \tag{3}$$

These definitions depend only on the structure of burnup chain, not on cross sections [8].

3. Gd Depletion Results

The test cases for this analysis are summarized in Table I. The assembly geometry is based on a 16×16 Combustion Engineering (CE) type fuel assembly. The 2D fuel assembly consists of uranium oxide fuel pins and gadolinia fuel pins.

The reference data is generated by STREAM. For the depletion calculation, fuel pins and gadolinia pins are divided into 3 and 10 rings, respectively. STREAM adopts quadradic depletion for the Gd isotope because of spatial self-shielding effect [7].

For this single assembly depletion test, STREAM does not use critical spectrum calculation to compare with RAST-K 2.0. Fuel temperature and moderator temperature are fixed as 850 K and 584 K respectively. Boron concentration is given as 700 ppm. Boundary conditions for all directions are reflective. The RAST-K 2.0 searches eigenvalue (k_{inf}) instead of critical boron concentration.

Table I: Specification of Test Cases.

Case	Fuel Pin (^{235}U wt.%)	Fuel Pins	Gadolinia Pin (^{235}U/Gd$_2$O$_3$ wt.%)	Gd Pins
1	3.4/2.9	124/100	0.7/6.0	12
2	4.7/4.1	164/52	2.2/8.0	20

k_{eff} differences between STREAM and RAST-K 2.0 are shown in Figs. 3-4. The direct Gd chain solution is not consistent according to the test cases. The Gd

amount of Case 1 is loaded smaller than Case 2 and the direct numerical solution is ±50 pcm difference compared to STREAM. If the Gd amount increases, the direct numerical solution results in differences of larger than 1,000 pcm during depletion as shown in Fig. 3. On the other hand, the effective Gd depletion model provides consist solutions (within 20 pcm difference) when compared to the STREAM results. Furthermore, this depletion model is not dependent on the amount of Gd during the depletion.
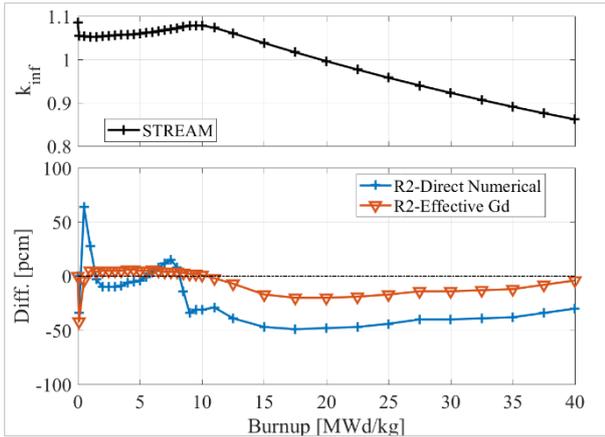

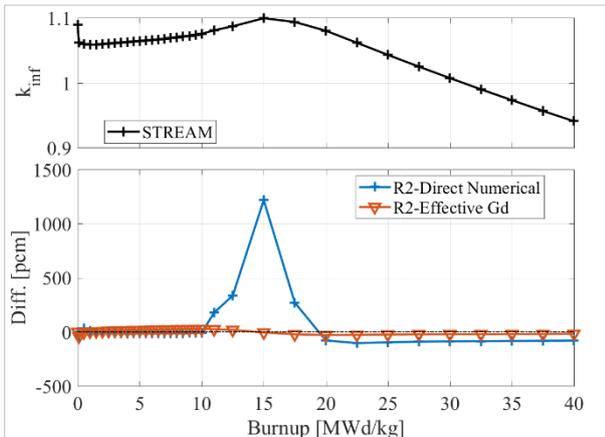Fig. 2. Depletion results of gadolinia assembly for Case 1.

Fig. 3. Depletion results of gadolinia assembly for Case 2.

^{155}Gd and ^{157}Gd have the highest absorption cross sections among the Gd isotopes, so amounts of these isotopes decline significantly. The number density change is related to absorption cross section. Their microscopic absorption cross sections change a lot as a function of burnup, as shown in Figs. 4-5. The cross sections increase rapidly at the specific burnup point because of spectrum softening, and the increment is non-linear.

On the contrary, the cross section variation of the effective Gd along the burnup is very small (blue lines in Figs. 4-5). The effective Gd approach leads to linear absorption cross section increments as a function of

burnup. Therefore, the effective Gd depletion model provides consistent results during the depletion.

In order for the direct numerical solution to provide acceptable accuracy, the number of burnup points needs to be finer than the current burnup points. Interpolated cross section with provided burnup points can be different with true solution owing to non-linearity of absorption cross section for both ^{155}Gd and ^{157}Gd isotopes. The quadratic depletion has similar problem. the quadratic depletion model estimates the absorption reaction rate change with the quadratic formula, however, the estimation can be different if burnup points are wide not to cover the curve of cross section change. Therefore, finer burnup points should be provided from STREAM for direct numerical solution to cover non-linear cross section change as burnup. This leads to more calculation burden in STREAM in order to calculate more burnup steps. Therefore, the effective Gd depletion method is the appropriate model for RAST-K 2.0.
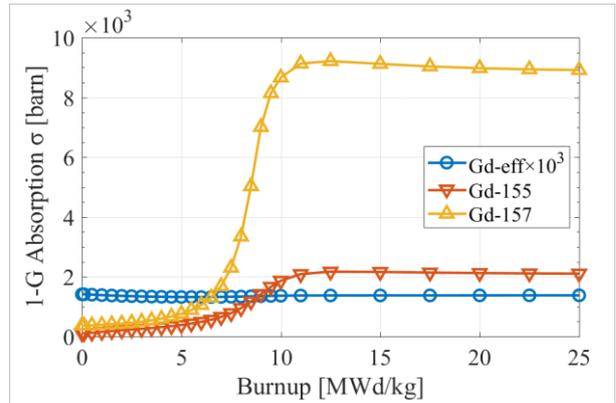

Fig. 4. 1 group microscopic absorption cross section change as burnup for Case 1.
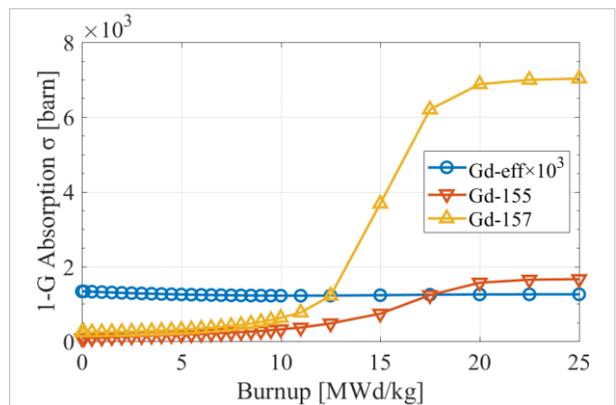

Fig. 5. 1 group microscopic absorption cross section change as burnup for Case 2.

4. Conclusions

This paper analyzes two kinds of Gd depletion models: direct Gd linear chain depletion and effective Gd depletion. The effective Gd depletion model provides a high accuracy solution within 20 pcm due to the linear change of absorption cross section as a function of burnup. The direct Gd linear chain depletion model needs finer burnup steps to get acceptable accuracy, because absorption cross section of individual Gd isotope changes non-linearly during depletion. Therefore, the effective Gd depletion model is suitable for RAST-K 2.0.

5. Acknowledgement

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