# On the Consistency of Non-Drag Interfacial Force Models in Fully Developed Turbulent Bubbly Two Phase Flow

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#### 1. Introduction

Applications of CFD (Computational Fluid Dynamics) codes to two-phase flow allow for safety investigations to get some access to smaller scale flow processes that are not seen by system codes. Many investigations using Euler-Euler two-fluid model have been done concerning the distribution of radial gas profiles in a simple vertical pipe flow in the regime of fully-developed turbulent bubbly flow. Balance of the non-drag forces, namely the lift, wall lubrication and turbulent dispersion, acting perpendicularly to the flow direction determines the establishment of radial gas profiles or, in other words, radial distributions of the bubbles. The original formulation of wall lubrication force model of Antal et al [1] has been widely used in CFD simulation of bubbly flow even though its coefficients were modified case by case in order to balance with lift force coefficient in higher liquid velocity condition. It is worth noting that the coefficients in Antal et al (1991)'s model were obtained in laminar flow condition in which the liquid velocity was assumed to be zero. Therefore, the modification of coefficients may take into account the effect of liquid velocity. From this point of view, the wall lubrication force was extended by considering the effect of liquid velocity. Analysis of void fraction distribution was conducted for radial phase distribution in fully turbulent bubbly two-phase flow by using the EAGLE code and validated against VAWL experimental data [2].

#### 2. Methods and Results

#### 2.1 Wall lubrication force models of Antal et al. (1991)

It is worth recalling the functional form of the wallforce on a sphere deduced by Antal et al. (1991). Since an exact analytic expression has not yet been found for the hydrodynamic force on a moving bubble near a fixed wall, an estimate can be obtained by considering the flow past two cylinders moving at velocity U, whose radii are  $R_0$  and whose centers are separated by a distance  $2y_0$ . By virtue of symmetry, the line of y = 0can be taken as a fixed boundary so that the half of the plane represents the flow past a cylinder moving along a fixed wall. More detailed information of derivation process can be found in Antal et al. (1991). The final expression of wall force was obtained for laminar flow condition with the assumption of  $u_L = 0$  as follows:

$$F_W = \frac{\alpha \rho_L U_R^2}{R_b} \left[ 6\lambda_{n+2} \left( \frac{R_b}{y_0} \right)^n + 3\lambda_{n+4} \left( \frac{R_b}{y_0} \right)^{n+2} - \frac{3\lambda_5}{4} \left( \frac{R_b}{y_0} \right)^3 - 6\lambda_{2n+3} \left( \frac{R_b}{y_0} \right)^{2n+1} \right]$$
(1)

In order to evaluate the exponent, n, in (1), a Taylor series expansion of the term in the brackets was performed about  $y_0 = R_b$ :

$$\left[ 6\lambda_{n+2} \left( \frac{R_b}{y_0} \right)^n + 3\lambda_{n+4} \left( \frac{R_b}{y_0} \right)^{n+2} - \frac{3\lambda_5}{4} \left( \frac{R_b}{y_0} \right)^3 - 6\lambda_{2n+3} \left( \frac{R_b}{y_0} \right)^{2n+1} \right]$$
$$= C_{W1} + C_{W2} \left( \frac{R_b}{y_0} \right) + C_{W3} \left( \frac{R_b}{y_0} \right)^2 + \dots$$
(2)

The resulting constants were evaluated by comparison to a three-dimensional direct numerical simulation of viscous flow past a single bubble. The first two terms in the right hand side of Eq. (2) were found to satisfactorily fit the numerical results.

$$(C_{W1} = -0.06U_R - 0.104$$
 ,  $C_{W2} = 0.147$ ).

2.2 Wall lubrication force coefficients used for CFD analysis in the open literature researches

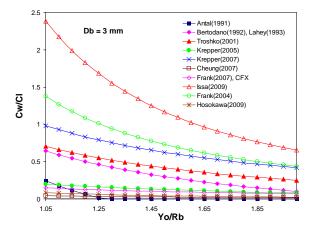


Fig. 1. Coefficients in the open literatures

Figure 1 shows the ratio between wall lubrication and lift force coefficients used for CFD analysis of void distribution in fully-developed turbulent bubbly flow. As can be seen, generally, the coefficients are larger than the Antal's one and they vary considerably from author to author. Such discrepancies imply that the liquid dependency of the non-drag force coefficients in fully-developed turbulent bubbly flow needs to be taken appropriately.

2.3 Liquid velocity dependence of wall lubrication force

Table 1.	Liquid	depend	lency o	of wall	lu	bricatio	on i	force
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n	Formulation				
1	$F_{W} = \frac{\alpha \rho_{L} U_{R}^{2}}{R_{b}} \left[ 4 \left( \frac{R_{b}}{y_{0}} \right) - 2 \left( \frac{R_{b}}{y_{0}} \right)^{3} \right]$				
	$\frac{\alpha \rho_L U_R^2}{R_b} \left[ \left( \frac{u_L}{U_R} \right) \left( \frac{R_b}{y_0} \right) + \left\{ -\frac{6}{5} \left( \frac{u_L}{U_R} \right) - \frac{1}{5} \left( \frac{u_L}{U_R} \right)^2 \right\} \left( \frac{R_b}{y_0} \right)^3 \right]$				
2	$F_{W} = \frac{\alpha \rho_{L} U_{R}^{2}}{R_{b}} \left[ \frac{9\pi}{8} \left( \frac{R_{b}}{y_{0}} \right)^{2} + \frac{5\pi}{4} \left( \frac{R_{b}}{y_{0}} \right)^{4} - \frac{2}{5} \left( \frac{R_{b}}{y_{0}} \right)^{3} - \frac{96}{35} \left( \frac{R_{b}}{y_{0}} \right)^{5} \right]$				
	$+\frac{\alpha\rho_L U_R^2}{R_b} \left[ \frac{9\pi}{32} \left( \frac{u_L}{U_R} \right) \left( \frac{R_b}{y_0} \right)^2 + \frac{15\pi}{128} \left( \frac{u_L}{U_R} \right) \left( \frac{R_b}{y_0} \right)^4 \right]$				
	$-\left(\frac{48}{35}\left(\frac{u_L}{U_R}\right) + \frac{6}{35}\left(\frac{u_L}{U_R}\right)^2\right)\left(\frac{R_b}{y_0}\right)^5\right]$				
3	$F_{W} = \frac{\alpha \rho_{L} U_{R}^{2}}{R_{b}} \left[ \frac{14}{5} \left( \frac{R_{b}}{y_{0}} \right)^{3} + \frac{48}{35} \left( \frac{R_{b}}{y_{0}} \right)^{5} - \frac{256}{105} \left( \frac{R_{b}}{y_{0}} \right)^{7} \right]$				
	$+\frac{\alpha\rho_L U_R^2}{R_b} \left[\frac{4}{5} \left(\frac{u_L}{U_R}\right) \left(\frac{R_b}{y_0}\right)^3 + \frac{12}{35} \left(\frac{u_L}{U_R}\right) \left(\frac{R_b}{y_0}\right)^4\right]$				
	$-\left(\frac{128}{105}\left(\frac{u_L}{U_R}\right) + \frac{16}{105}\left(\frac{u_L}{U_R}\right)^2\right)\left(\frac{R_b}{y_0}\right)^7\right]$				

The analytical wall lubrication force formulations taking into account the effect of liquid velocity are extended based on the original form and summarized in Table 1. It should be noted that these formulations is derived using several simplifying assumptions (e.g. inviscid flow theory) and therefore it cannot be directly used to calculated the repelling force, but only to give the appropriate form of the wall-force closure law. Figures 2 a, b and c show clearly the increasing trend of coefficient upon increasing the liquid velocity. This fact can be explained for the discrepancy on the coefficients described in Fig.1.

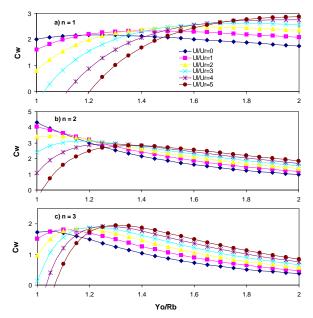


Fig. 2. Liquid dependency of the coefficients

# 2.4 Numerical analysis with the EAGLE code

Numerical analysis of void fraction distribution in fully-developed turbulent bubbly flow were performed by using the CFD EAGLE code and validated against VAWL experimental data. The wall lubrication force coefficients taking into account the effect of liquid velocity were evaluated by obtaining the void fraction profiles that fit experimental data reasonably well. Typical results were showed in Fig. 3.

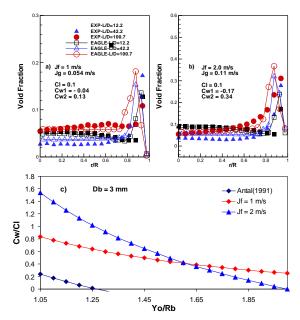


Fig. 3. Numerical analysis of wall lubrication force

#### 3. Conclusions

Results presented in this study show that the effect of liquid velocity on the wall lubrication force should be taken into account appropriately in order to ensure the consistency of the mechanistic modeling of two phase flow.

# ACKNOWLEDGEMENTS

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