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# Large Mass Method As a Substitute of Quasi-Static Decomposition Method for Dynamic Analysis of Rod Subjected to Support Motion

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## 1. Introduction

Structural and mechanical systems are often excited dynamically through support motions rather than by applied external loadings, e.g., piping systems in a nuclear power plant subjected to support motions at their connections to containment buildings and heavy equipment, which in turn are responding to earthquake ground motion inputs at their supports. This kind of problems is characterized by structural dynamic problems with time-dependent boundary conditions.

It is well known that the quasi-static decomposition method is used to obtain a solution using the modal dynamic concept. Mindlin and Goodman[1] developed it and applied to the problem of flexural vibration of beams with time-dependent boundary conditions. The time-dependence is removed by means of the method from the boundary conditions. Then the remaining problem can be solved by any one of classical or numerical methods available for handling the free- or forced-vibration problem.

Clough and Penzien[2] applied the method to the discrete system of a finite element formulation. Meanwhile, two numerical methods, large mass finite element method and large stiffness finite element techniques are frequently used because they are simple to use and are easily incorporated into a commercial program. But ironically, many users address a complaint to the artificial parameter input of so-called 'large mass' and still debate on the validity of the large mass method because its theoretical background has not been provided intensively yet. The questions often asked by users are as follows: Is it a standard method of using an acceleration time history by applying a force to the large mass? How do you calculate mass participation ratio or how sensitive is the magnitude of the large mass that is used? When does a problem with round-off error start to kick in?

Kim and Jhung[3] presented mathematical analysis on large mass method of a single-degree-of-freedom system to answer these questions. They also investigated theoretical analysis of LM method for the multi-degree-of-system and presented some useful theorems on the method. Their conclusion can be summarized as follows: Accurate numerical solutions of a MDOF system, which is subjected to a support motion, can be obtained if the magnitude of the large mass is much larger than the largest structural mass in the MDOF system. Otherwise, it gives erroneous results.

In this paper, Kim's concept will be applied to the axial vibration of rod subjected to support motion to understand or justify the effectiveness of the large mass method.

## 2. Equation of Motion for Axial Vibration of Rod Subjected to Support Motion

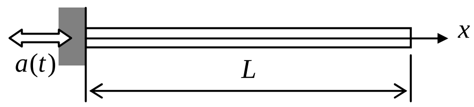


Fig. 1. Rod subjected to support motion $a(t)$

The response of a uniform rod subjected to support excitation shown in Fig. 1 can be described by

$$m\ddot{u}(x,t) = EAu''(x,t) + f(x,t) \quad \text{in } 0 < x < L \qquad (1)$$

where a superimposed dot denotes a time derivative, a superscript prime stands for a spatial differentiation, $EA$ and $m$ denote the axial stiffness and mass per unit length, respectively. $u$ is the axial displacement, which is a function of position $x$ and time $t$. For simplicity, the load $f(x,t)$ is assumed to be zero during support excitation. The boundary conditions are

$$u(0,t) = a(t) \text{ and } u'(L,t) = 0 \qquad (2)$$

where $a(t)$ is a prescribed support motion. The initial conditions are assumed as follows:

$$u(x,0) = u_0(x) \text{ and } \dot{u}(x,0) = \dot{u}_0(x) \qquad (3)$$

## 3. Equation of Motion of Effective Force Model

The solution can be decomposed into two parts:

$$u(x,t) = u_S(x,t) + w(x,t) \qquad (4)$$

where $u_S(x,t)$ denotes the quasi-static solution, and $w(x,t)$ is the dynamic contribution due to the inertia effect. The quasi-static part $u_S(x,t)$ must satisfy

$$EAu_S''(x,t) = 0 \qquad (5)$$

and is subject to boundary conditions:

$$u_S(0,t) = a(t) \text{ and } u_S'(L,t) = 0 \qquad (6)$$

By solving equation (5) with boundary conditions in equation (6), we have

$$u_S(x,t) = a(t) \qquad (7)$$

Then equation (1) is expressed in terms of $w(x,t)$ as follows:

$$m\ddot{w}(x,t) = EAw''(x,t) - m\ddot{a}(t) \qquad (8)$$

The boundary conditions in equation (2) are expressed in terms of $w(x,t)$ as

$$w(0,t) = 0 \text{ and } w'(L,t) = 0 \quad (9)$$

and the initial conditions are written as

$$w(x,0) = u_0(x) - a(0) \quad (10)$$

$$\dot{w}(x,0) = \dot{u}_0(x) - \dot{a}(0) \quad (11)$$

The system expressed by dynamic displacement like equations (8) to (11) is called 'effective force model' in this study.

## 4. Equation of Motion for Large Mass Model

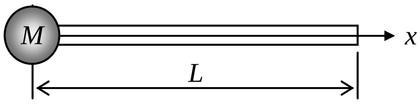


Fig. 2. Large mass model for the model in Fig. 1, where $M$ denotes a large mass

The large mass model corresponding to the model in Fig. 1 is shown in Fig. 2. The equation of motion is:

$$m\ddot{u}(x,t) = EAu''(x,t) \text{ in } 0 < x < L \quad (12)$$

The boundary conditions are

$$EAu'(0,t) = M\ddot{u}(0,t) \text{ and } EAu'(L,t) = 0 \quad (13)$$

and the initial conditions are same as equation (3).

## 5. Conditionally Approximate Equivalence between EF Model and LM Model

*5.1. How do you calculate mass participation ratio?*

To answer the question, consider the integral representation of the equation of motion of the rod subjected to external force $F(t) = M\ddot{a}(t)$ at support, i.e.,

$$\int_0^L m\ddot{u}(x,t)dx = \int_0^L EAu''(x,t)dx + M\ddot{a}(t) \quad (14)$$

or

$$\int_0^L m\ddot{u}(x,t)dx = EAu'(x,t)\Big|_{x=L} - EAu'(x,t)\Big|_{x=L} + M\ddot{a}(t) \quad (15)$$

Using the boundary conditions in equation (13), equation (15) can be written as

$$\int_0^L m\ddot{u}(x,t)dx + M\ddot{u}(0,t) = M\ddot{a}(t) \quad (16)$$

From the above equation (16), we see that the condition

$$\int_0^L m\ddot{u}(x,t)dx << M\ddot{u}(0,t) \quad (17)$$

yields $\ddot{u}(0,t) \to \ddot{a}(t)$, which is the desired result. Though the discussion gives an insight on the role of the large mass and on the possibility of the existence of appropriate magnitude of large mass, it cannot answer the question still.

*5.2. Is it a standard method of using an acceleration time history by applying a force to the large mass?*

To answer the question, assume that the appropriate large mass, which gives $\ddot{u}(0,t) \cong \ddot{a}(t)$, exists and consider the system equation of large mass model expressed by relative displacement $v(x,t)$ which is defined as

$$v(x,t) = u(x,t) - u(0,t) \quad (18)$$

The equation (18) implies that

$$v(0,t) = 0 \text{ and } v'(L,t) = 0, \quad (19)$$

$$\dot{v}(0,t) = 0 \text{ and } \ddot{v}(0,t) = 0 \quad (20)$$

$$v(x,0) = u(x,0) - u(0,0) = u_0(x) - a(0), \quad (21)$$

$$\dot{v}(x,0) = \dot{u}(x,0) - \dot{u}(0,0) = \dot{u}_0(x) - \dot{a}(0) \quad (22)$$

The equation of large mass model in equation (12) is expressed in terms of $v(x,t)$ as

$$m\ddot{v}(x,t) + m\ddot{u}(0,t) = EAv''(x,t) \quad (23)$$

Since $\ddot{u}(0,t) \cong \ddot{a}(t)$ from the assumption, equation (23) is written as

$$m\ddot{v}(x,t) \cong EAv''(x,t) - m\ddot{a}(t) \quad (24)$$

which is an approximate from of the equation of motion of EF model in equation (8). Its boundary conditions are given by equation (19) and initial conditions are provided by equations (21) and (22). Thus, the boundary and initial conditions are coincident with those of EF model in equations (9) to (11), respectively.

The above discussion shows that the large mass model can give good approximate solutions if an appropriate large mass is chosen.

The second boundary condition of large mass model in equation (13) becomes the second one in equation (19) and the first boundary condition in equation (13) becomes equilibrium equation at $x = 0$.

## 6. Conclusion

Even though the discussions are confined to the axial vibration of rod subjected to support motion, it shows that the dynamic analysis using large mass method can be approximately equivalent to the analysis employing quasi-static decomposition method.

There does not exist how to calculate mass participation ratio or large mass ratio. Appropriate large mass ratio should be chosen through numerical tests or experiences. But because of its simplicity and relative ease to incorporate into a commercial program, it is worthwhile to investigate the theoretical justification or supplementation of the large mass method.

## REFERENCES

[1] R. D. Mindlin and L. E. Goodman, "Beam Vibrations with Time-Dependent Boundary Conditions," *Journal of Applied Mechanics, ASME*, pp. 377-380, 1950.

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[3] Y.-W. Kim and M. J. Jhung, "Mathematical Analysis Using Two Modeling Techniques for Dynamic Responses of a Structure Subjected to a Ground Acceleration Time History," *Nuclear Engineering and Technology*, Vol. 43, No. 4, pp. 361-374, 2011.