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Efficient Normal Mode Analysis of the Structures using MOR method

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1. Introduction

It is possible for us to perform the dynamic analysis of the structures, such as normal mode analysis and transient response analysis, having very large amount of finite elements in CAE fields utilizing finite element method. But the dynamic analysis of large system is still very time-consuming even though the great improvement of computing power because we need the finer finite element model to get more accurate results. Therefore, model reduction method still has much advantage when it comes to the computational efficiency. In this paper, the model order reduction(MOR) method based on moment matching is introduced and its application to the normal mode analysis is represented to show the efficiency and accuracy of the MOR method.

2. Model order reduction method

In this section some of the basic theories of MOR used to reduce the model size are described.

2.1 Model reduction of linear large-scale systems

Usually, the dynamic system having huge degrees of freedom requires a lot of computation time and resources to obtain the eigenvalues or the response characteristics. Therefore, MOR method has been used to get the responses of the dynamics systems effectively.[1,2,3]

The general equations of motion of dynamic systems can be represented as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = bu(t) \tag{1}$$

where M , C and K are the mass, damping and stiffness matrices respectively , $bu(t)$ is the load vector and $x(t)$ is a vector of unknown degree of freedoms.

For the normal mode analysis Eq. (1) is converted into eigenvalue problem to obtain the eigenvalue as follows.

$$\{K - \lambda M\}q(t) = 0 \tag{2}$$

where λ is the eigenvalue of the systems and $q(t)$ is the corresponding eigenvectors. The basic concept of MOR is to construct the transformation matrix(T) to satisfy the following relationship (Eq.(3)) to change the state variable $q(t)$ of the large system(N) into the state variable $z(t)$ of the reduced system with small degree of freedoms(n).

$$\begin{gathered} q(t) = Tz(t), \\ \text{where } q(t) \in R^N,\ T \in R^{Nxn},\ z(t) \in R^n \end{gathered} \tag{3}$$

After projecting the transformation matrix (T) to the Eq.(2) and pre-multiplying the transpose of the transformation matrix we get the following reduced equation.

$$\begin{gathered} \{K_r - \lambda M_r\}z(t) = 0, \quad q(t) = Tz(t) \\ \text{where, } K_r = T^T KT,\ M_r = T^T MT \end{gathered} \tag{4}$$

K_r and M_r is the reduce system matrices having small degree of freedoms(n). The most important factor to determine the accuracy and effectiveness depends on how to construct the transformation matrix(T). In mechanical engineering, the transformation matrix is usually chosen from the egienstates of Eq.(2) or by the Guyan method[4]. But moment matching method via Krylov subspaces which is relatively new technique is used in this study[5,6].

2.2 Moment matching method

In Eq.(1) if we neglect the damping term we get following form of equation.

$$M\ddot{x}(t) + Kx(t) = bu(t) \tag{5}$$

Laplace transformation of Eq.(5) creates the following equations such that

$$\begin{gathered} s^2 MX(s) + KX(s) = BU(s) \\ \left(s^2 M + K\right)X(s) = BU(s) \end{gathered} \tag{6}$$

Therefore the transfer function can be written as

$$H(s) = \frac{X(s)}{U(s)} = \frac{B}{s^2 M + K} \tag{7}$$

We can re-write the Eq.(7) for the convenience of expansion.

$$\begin{aligned} H(s) &= \frac{B}{s^2 M + K} = \frac{B}{\left\{K\left(s^2 K^{-1} M + I\right)\right\}} \\ &= \left\{K\left(s^2 K^{-1} M + I\right)\right\}^{-1} B = \left(s^2 K^{-1} M + I\right)^{-1} K^{-1} B \end{aligned} \tag{8}$$

$\left(s^2 K^{-1} M + I\right)^{-1}$ in Eq.(8) is expanded by a Taylor series for s^2 at s=0

$$\left(s^2 K^{-1} M + I\right)^{-1}\Big|_{s=0} = I - s^2 K^{-1} M + s^4\left(K^{-1} M\right)^2 - s^6 (K^{-1} M)^4 + \cdots \tag{9}$$

Therefore, the expansion of the transfer function of H(s) is given as

$$H(s)\big|_{s=0} = \left[I - s^2K^{-1}M + s^4\left(K^{-1}M\right)^2 - s^6(K^{-1}M)^4 + \cdots \right]K^{-1}B$$
$$= \sum_{i=0}^{\infty}(-1)^i\left(K^{-1}M\right)^i K^{-1}Bs^{2i} \tag{10}$$

Where the coefficient $(-1)^i\left(K^{-1}M\right)^i K^{-1}F$ is defined as the moment of the transfer function. And then we seek a transformation matrix (T) that yields the same first *q* moments for the transfer function of the reduced system. The Arnoldi algorithm[7] reduces the $N \times N$ matrix $K^{-1}M$ to a small $n \times n$ block upper Hessenberg matrix H_n and during this transformation creates a matrix T such that

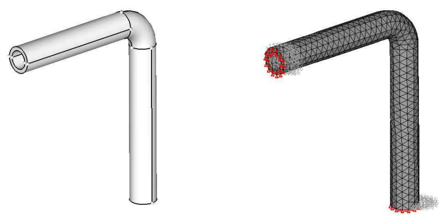
$$\begin{aligned} &\mathrm{colspan}(T) = K_n\left(K^{-1}M, K^{-1}B\right) \\ &= \mathrm{span}\left(K^{-1}B, K^{-1}MK^{-1}B, \cdots \left(K^{-1}M\right)^{n-1}K^{-1}B \right) \\ &T^TK^{-1}MT = H_n \\ &T^TT = I_n \end{aligned} \tag{11}$$

It can be shown that by using these matrices the corresponding moments in the full and reduced system match up to the *n*th moments.

2.3 Normal mode analysis using MOR

Simple pipe example model is taken to demonstrate the efficiency and accuracy of the MOR method.

The geometry and FE models are shown in Fig.1 respectively.



(a) (b)

Fig. 1 Simple pipe model

The computation time and natural frequency of first bending mode are compared in Table I and the first mode shape is shown in Fig.2.

Table I: Comparison of result (1st mode)

	TDOF	Time(sec)	Freq. (Hz)
Full model	3771	1393.67	244.85
MS	n=30	14.62	244.85
Reduced model	30	7.87	244.85
※ MS : mode superposition			244.86 (ANSYS)

The computation time of reduced model using MOR is much less than that of full model or mode superposition method. But the accuracy of solution is very good even compared to the solution of ANSYS.

Of course, the accuracy of MOR decreases in higher modes over 20 in this example as shown in Fig. 3 because only 30 order of the system is considered in this example and the expansion point is $s = 0$ in Eq. (10).

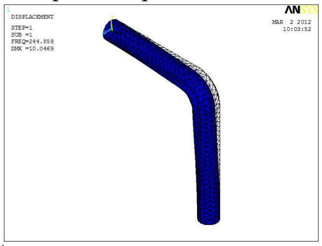


Fig. 2 1st mode shape computed using ANSYS

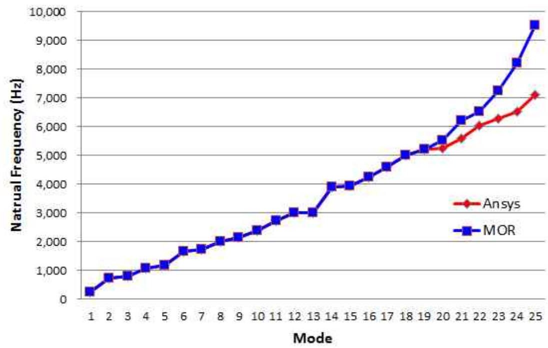


Fig. 3 Comparison of natural frequency

3. Conclusions

We have presented a novel approach to compute a reduced order model using model order reduction method based on moment matching. It was shown that this approach works very well for normal model analysis; in addition, it proved its efficiency and accuracy with the numerical example.

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