Feasibility Study for Applicability of the Wavelet Transform to Code Accuracy Quantification

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1. Introduction

A purpose of the assessment process of large thermal-hydraulic system codes is verifying their quality by comparing code predictions against experimental data. This process is essential for reliable safety analysis of nuclear power plants. Extensive experimental programs have been conducted in order to support the development and validation activities of best estimate thermal-hydraulic codes [1].

So far, the Fast Fourier Transform Based Method (FFTBM) has been used widely for quantification of the prediction accuracy regardless of its limitation that it does not provide any time resolution for a local event. As alternative options, several time windowing methods (running average, short time Fourier transform, and etc.) can be utilized, but such time windowing methods also have a limitation of a fixed resolution. This limitation can be overcome by a wavelet transform because the resolution of the wavelet transform effectively varies in the time-frequency plane depending on choice of basic functions which are not necessarily sinusoidal [2].

In this study, a feasibility of a new code accuracy quantification methodology using the wavelet transform is pursued.

2. Methods and Results

2.1 Wavelet transform

A good introduction to basic theory is given by several authors [3, 4]. The wavelet transformation is defined as follows:

$$W_i^x(s) = \sum_{j=0}^{N-1} x_j \Psi(t_j - t_i, s)\,, \tag{1}$$

where time t_i is certain time, scale s that is related to frequency, x_j is data at time t_j, $\Psi(t_j\text{-}t_i,\ s)$ is localized wavelet that is stretched and translated function of a chosen mother wavelet Ψ_0.

$$\Psi(t_j - t_i, s) = c(s)\Psi_0\left(\frac{t_j - t_i}{s}\right). \tag{2}$$

c(s) is a normalization factor. Torrence[3] suggested as follow:

$$c(s) = \left(\frac{\Delta t}{s}\right)^{1/2}. \tag{3}$$

In this study, the Morlet mother wavelet (eq. 4), which is easy to analyze and requires short calculation time, is considered [5].

$$\Psi_0(\theta) = \pi^{-1/4} e^{iw_0\theta} e^{-\theta^2/2}. \tag{4}$$

For w_0=6, this gives Fourier period λ=1.03s, indicating that the wavelet scale is almost equal to the Fourier period [3]. If one considers arbitrary scales and length of the time series, it is continuous wavelet transformation (CWT). Because the wavelet function Ψ and wavelet transform $W_i(s)$ are in general complex, the wavelet power spectrum (WPS) is defined as $|W_i(s)|^2$.

2.2 Cross wavelet transform and wavelet coherency

WPS can be extended to compare different two time series $x(t_i)$ and $y(t_i)$ using a wavelet cross spectrum (WCS) as the product of the two data :

$$\begin{aligned} W_i^{xy}(s) &= \left\langle W_i^x(s) W_i^y(s)^* \right\rangle \\ &= \left| W_i^{xy}(s) \right| e^{i\Phi_i(s)} \end{aligned}, \tag{5}$$

where * denotes the complex conjugation, < > denotes the smoothing operation in both time and scale, and the phase $\Phi_i(s)$ means the delay between the two data at time t_i on a scale s. WCS informs the common power of two data without normalization to the single WPSs. This makes misunderstanding. For example, CWT has peak if one of WPS has flat region and the other WPS has strong peaks because CWT is multiple of two WPSs. The wavelet coherency (WCO) voids this problem by normalization:

$$R_i^2 = \frac{\left|\left\langle s^{-1} W_i^{xy}(s) \right\rangle\right|^2}{\left\langle s^{-1} \left|W_i^x(s)\right|^2 \right\rangle \cdot \left\langle s^{-1} \left|W_i^y(s)\right|^2 \right\rangle}. \tag{6}$$

WCO takes values between 0 and 1, with 1 indicating the highest coherence and 0 the lowest. Although the CWT has the problem, its phase spectrum is still valuable [4].

2.3 Results and discuss

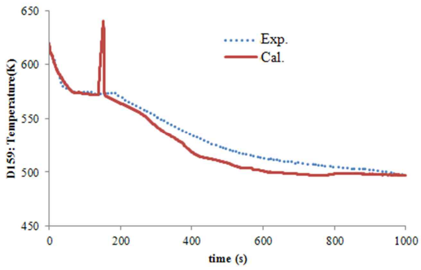

Fig. 1. Calculation and experiment data for maximum wall temperature at elevation 10 (DSP-02).

To check applicability of the wavelet transform to assessment of accuracy of thermal-hydraulic system codes, one temperature data of DSP-02 (Fig. 1.) is demonstrated using MATLAB with the cross wavelet coherence package [5]. Temperatures' behaviors have similar pattern except at 150s – calculation data has a strong peak. Fig. 2 shows WPS for both experiment and calculation. The WPS of calculation shows that most signal are distributed on long period (low frequency) and short period signal is at only 150s while WPS of experiment doesn't have short period signal. This information reflects the characteristics of data on Fig. 1.

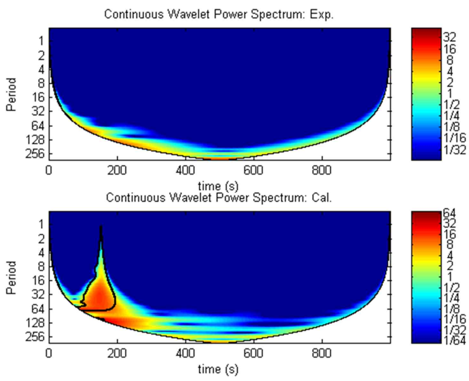

Fig. 2. Continuous wavelet power spectrum for each data

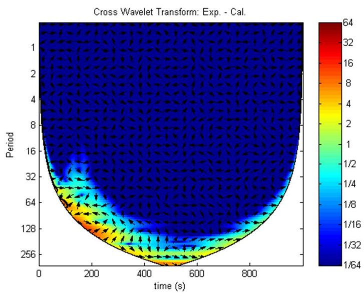

Fig. 3. The cross wavelet transform for two data. The relative phase relationship is shown as arrows (with in-phase pointing right, anti-phase pointing left, and calculation leading experiment by 90° pointing straight down)

Two data are compared using WCS on Fig.3. As mentioned in section 2.2, the WCS has a short period signal at 150s when only calculation result has a strong peak. Therefore, the WCS is not valuable for power spectrum but only the phase information (arrows).

Two data are also applied to WCO (Fig. 4.). As WCO, two data are similar at most time on long period and different at 150s; this pattern can be expected from Fig. 1. Data on low period region look different, but this can be ignored because there are low values, which is meaningless, on this region as Fig. 2. The arrows on low coherence region are not draw because they are meaningless by same reason. The arrows for 200s-500s on around 128s period point upside, and this means calculation signal on this region delays 90°. The arrows for 300s-500s on around 256s period point down side, and this means signal on this region leads 90°.

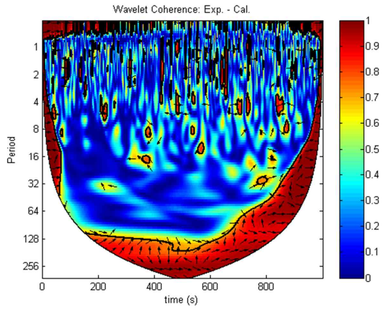

Fig. 4. Wavelet coherence for two data. Arrows indicate the relative phase relationship.

3. Conclusions

The wavelet transform was applied to assessment of thermal-hydraulic system code with a temperature data of DSP-02. Wavelet transform gives coherence for time-frequency domain and phase relationship between two data. The cross wavelet transform was found to give detailed information for similarity of two signals with varying time resolution and it also showed a possibility to be used as a code accuracy quantification method. However, further work is required for practical application, which can consider the effects of multiple parameters together. In addition, comparing process with validated methods (e.g. FFTBM) will be performed.

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