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Well-posedness, Convergence and the Stability of the Semi-implicit Upwind Numerical Solver for the Single Pressure Multi-fluid Model

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1. Introduction

Single pressure multi-fluid models [1] have complex characteristics [2]. This means that it is not properly posed for the initial value problem[3]. Authors[4] made the following conclusions for this issue:

(1) Hyperbolicity of the corresponding system of partial differential equations is not a necessary condition for the development of a numerical model for multi-phase flow, but whether or not it is hyperbolic can provide guidance relative to initial conditions, boundary conditions, and expected high frequency behavior of the model.

(2) A necessary condition for a well-posed numerical model is stability in the von Neumann sense, i.e. growth factor less than 1.0 for the shortest wave-length, 2Δx.

(3) The smallest node size used for convergence studies should be of the order of the characteristic dimension of the average description.

In this paper, the semi-implicit solution scheme of SPACE[5] is studied whether it is in accordance with these conclusions.

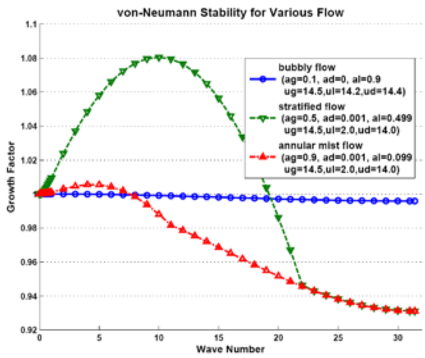

Fig. 1. von-Neumann stability for various flow.

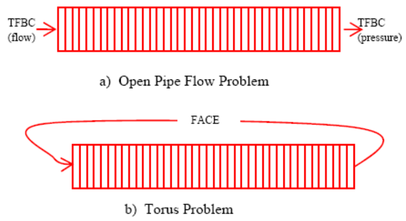

Fig. 2. Cell and faces for the non-linear stability study.

2. Linear Stability Analysis

The result of the linear dispersion analysis[4] of the three-fluid model shows the growth factors are unlimited as the wave number approaches infinity. This is the manifestation of the ill-posedness of the differential model of the multi-fluid flow.

Numerical diffusion introduced by the upwind scheme is a generally accepted method for the regularization. von-Neumann Stability of various flow regimes are compared against each other on Fig. 1. Stratified flow and annular mist flow have growth factors greater than 1.0 for low wave numbers. But even the most unstable flow such as stratified flow has growth factor less than 1.0 for the highest wave number which corresponds to the wavelength of 2Δx. It means that the numerical solver with the semi-implicit upwind scheme results in a numerically well-posed model. Unstable long wavelengths may be acceptable in order to simulate the Kelvin-Helmholtz type instability.

TABLE I. Data for the non-linear study.

Variable	Torus	Open pipe flow		
		Initial	Boundary	
			Flow	Pres
P(Mp)	0.1	0.1	-	0.1
T(°K)	372.0	372.0	372.0	372.0
α_g	0.5+0.005* $sin(2n\pi x/L)$	0.5	0.5(0-T_i sec) 0.5+0.005* $sin(2\pi t)$ (T_i-T_e sec)	0.5
α_l	1.0- α_g	0.5	0.5	0.5
α_d	0.0	0	0	0
v_g(m/s)	14.5*0.5/ α_g	14.5	14.5	-
v_l(m/s)	2*0.5/ α_l	2	2	-
v_d(m/s)	0.0	0.0	0	-

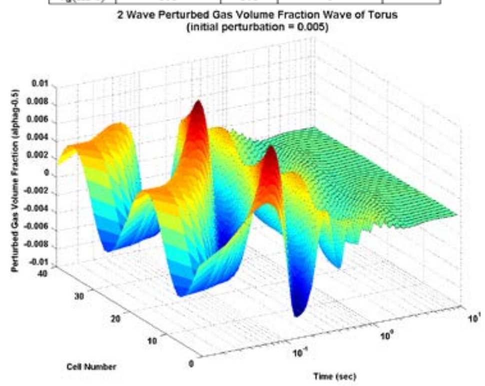

Fig. 3. 2-Wave perturbed α_g behavior in torus problem.

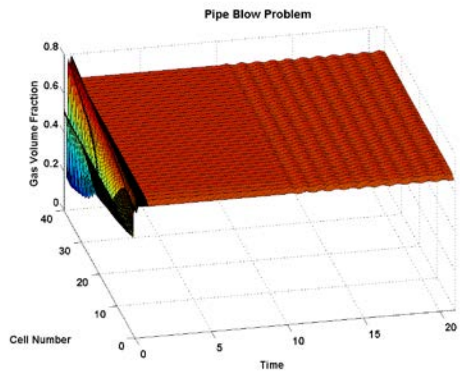

Fig. 4. Short open pipe flow problem.

3. Non-Linear Stability Analysis

Open pipe flow and the torus problem(Fig.2 & table I.) is used to study the non-linear stability. Especially, torus problem[6] is equivalent to the von-Neumann stability study used in the previous section. At the beginning of the simulation, the perturbed amplitude in the torus problem is growing as expected by the von-Neumann stability results for the stratified flow. But the growth halts around 0.3 seconds(Fig. 3). Perturbation of the α_g is propagated along the torus. It eventually decays out after several seconds later.

Short open pipe(2m long) flow problem is studied to see the response in the time domain. Input variables are defined at table I. During the initial period (0.0 sec ~ 2.0 sec), as shown in Fig. 4, the imbalance in the pipe between the initial and boundary conditions make a strong transient along the pipe. Initial transient decays down to stable flow and stays stable up to 10.0 seconds. Then, the constant gas volume fraction (0.5) of the input face starts the variation with 1.0 seconds period with the amplitude of 0.005. The cyclic change of the gas volume fraction does not cause any unstable behavior. Instead, flow in the pipe follows the input cycle fairly well.

Long open pipe(40m) is studied with the same cell size(0.1m) to see the possible excitation of the long wave which is predicted in the linear stability analysis.

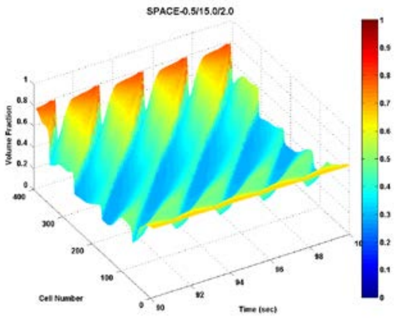

Fig. 5. Waves in the long pipe flow problem in SPACE.

Steam/liquid mixture of volume fraction 0.5 is fed into the long open pipe inlet with the outlet pressure fixed at 1.0 bar. Even though the boundary conditions are constant in time, the gas volume fraction fluctuates in the pipe as shown in Fig. 5. The evaluation of the results shows that the period of the oscillation is related with the average transit time of the two phase mixture.

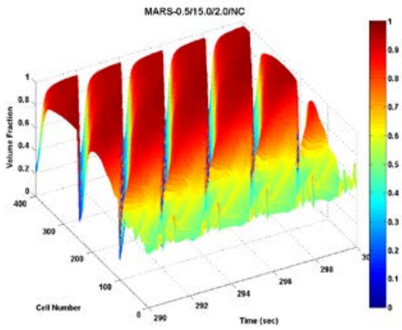

Fig. 6. Waves in the long pipe flow problem in RELAP5.

Similar test cases are run to see whether this behavior can be found in RELAP5[7]. Most of the run conditions are same with the SPACE runs except the non-condensable gas, air, is introduced in RELAP5 runs. Even stronger fluctuations are found in RELAP5 compared to those of SPACE as shown in Fig. 6. Interestingly enough, steady state solutions can't be obtained with the constant boundary conditions in both cases,.

4. Discussions and Conclusions

von-Neumann stability analysis of the single pressure three field multi-fluid model has been executed to confirm that the $2\Delta x$ stability requirements are met for whole flow regimes. Therefore, it can be said that the numerical model used for SPACE code is well-posed.

The growth factor for the bubbly flow regime with physically reasonable interfacial drag correlations is less than 1.0 for the whole wave lengths if mesh size is greater than the bubble size. Considering the fact that general code users use the mesh size greater than the pipe diameter, then, it may be said stable in usual sense. But the growth factor of the stratified flow is $1+O(\Delta t)$ for the most dangerous wave. Therefore, it may grow. In the real safety analysis code such as SPACE the growth is limited by various reasons. Frequency cascading[6] is taken as an effective mechanism for this limitation.

But, if the amplitude, for example, of gas volume fraction goes far beyond the linearization range, most probably flow regime change sets in. Some part of the flow regimes may have more favorable physical parameters such as higher interfacial drag forces. Then, the growth factor may be reduced. Extreme cases are constructed by the multi-fluid sections bounded by the single phase sections that have real characteristics and stable[8]. Therefore, if the code has robust mechanisms to handle the flow regime change, the stability may not be of great concern. Considering the fact that the Kelvin Helmholtz type instability is physically occurring in the two-phase flow and the fact that code calculation results depend on the use of the numerical diffusion that is not easy to control, the local quantification of the convergence and the accuracy of the dependent variables, such as volume fraction, are not justified. Instead, they are quantified by the averaged value such as time and/or volume averaged value of volume fraction.

During the small LOCA simulation with one-dimensional code such as RELAP5, it is easy to experience the case when the prediction error of the local volume fraction at a certain location of the system is near 1.0. But the localized error does not prevent users from using the code for small LOCA simulation. This is mainly due to the fact that the prime purpose of the safety code is to assess the mass and energy transport in the system on the average point of view. The objective of the code is not to follow the detailed phase of the Kelvin Helmholtz instability. Instead, it is aiming at calculating the system with the instability. The accuracy of the multi-fluid code should be judged only based on the validation tests against proper experimental data.

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