The Effects of the Equivalent Mass Technique in the Criticality Benchmark Problems with Unstructured Tetrahedral Mesh

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1. Introduction

In the reactor physics calculation, solutions for the neutron transport equation are obtained mostly by the discrete ordinates method, referred as an S_N method. A number of computer codes that use S_N method require regular mesh (such as rectangular, cylindrical or spherical) to model the problems geometry.

Using such a specific regular mesh leads to the simplest difference equations but may require an excessive number of mesh points to describe complicated geometries adequately.

The MUST (**M**ulti-group **U**nstructured geometry **S_N** **T**ransport) code [1] uses unstructured tetrahedral elements so that it can be applied to solve complicated geometry.

However, even the simple criticality benchmark problems (i.e., Godiva and VERA1B) can be difficult ones due to a curved surface.

When a curved surface is meshed with tetrahedral elements, original volume may not be conserved because curved surface is modeled with several faces of tetrahedral elements [2].

Instead of conserving volume, in this paper, an equivalent mass technique is applied to the criticality benchmark problems and the effects of it are shown.

2. Method and results

2.1 Volume Change by Using Unstructured Tetrahedral Mesh

Meshing curved geometry with tetrahedral elements causes volume change.

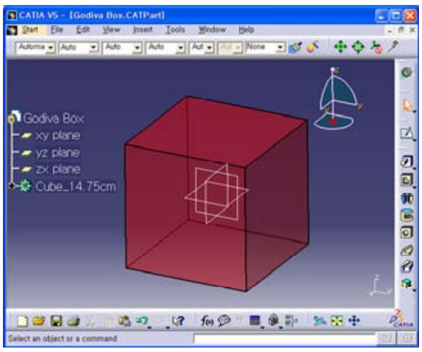

Fig. 1. CAD Modeling of Godiva-Box.

This is happened because a curved surface is modeled with several plain surfaces that are one of the four faces of tetrahedral element.

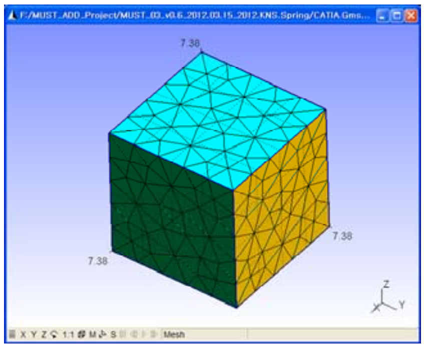

Fig. 2. Meshed Godiva-Box geometry.

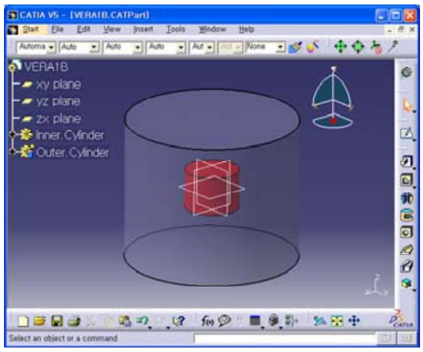

Fig. 3. CAD Modeling of VERA1B.

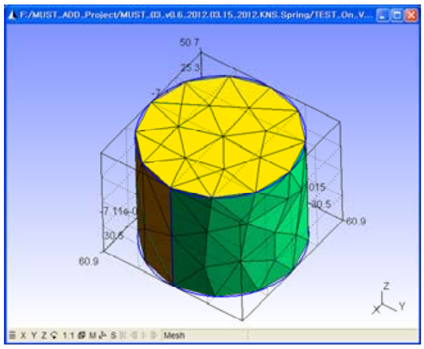

Fig. 4. Meshed VERA1B geometry.

In the Figs. 1 and 2, the volume change does not happened because the boundary of the Godiva-Box consists with six planes. However, perfect cylinder geometry in Fig. 3 has been changed to a polygon cylinder due to using tetrahedral elements.

To minimize volume change, more tetrahedral elements are needed. However, it cause huge computational burden.

2.2 Equivalent Mass Technique

As we can see in the Fig. 4, volume change is inevitable as long as we use tetrahedral elements for meshing geometry with curved surface.

Volume conservation by adjusting positions of nodes while meshing geometry can be one of the solutions for this problem. However, it may not be easy.

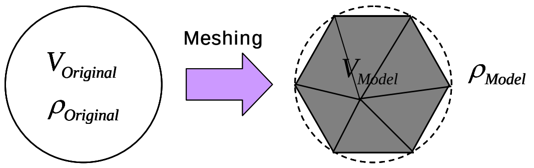

Fig. 5. Concept of an equivalent mass technique.

Instead of volume conservation, mass conservation can be one of the options we can have. To have equivalent mass, modifying model density (ρ_{Model}) by multiplying volume ratio ($V_{Original}/V_{Model}$) to the original density like Eq. (3).

$$M_{Original} = M_{Model} \tag{1}$$

$$V_{Original}\,\rho_{Original} = V_{Model}\,\rho_{Model} \tag{2}$$

$$\rho_{Model} = \frac{V_{Original}}{V_{Model}}\rho_{Original} \tag{3}$$

2.3 Tests and Results

To see the effects of the equivalent mass technique, three criticality benchmark problems (Godiva, Godiva-Box, and VERA1B) are used. As reference calculations, MCNP, spherical geometry option with ONEDANT, and cylindrical geometry option with TWODANT are used for the Godiva-Box, Godiva, and VERA1B respectively.

Equivalent mass technique is tested with MUST code with 4×4 Chebyshev-Legendre quadrature and 1.0e-5 error criteria. LANL-30 group library based on the ENDF-B/VII is used for the DANTSYS and MUST code.

Table I: The k_{eff} results (Godiva-Box)

Reference k_{eff} (MCNP)		1.00002±0.00062			
Element No.	$\frac{V_{Original}}{V_{Model}}$	No correction		Correction with Equivalent Mass Technique	
		k_{eff}	Diff. (pcm)†	k_{eff}	Diff. (pcm)†
1362	1.0	0.99981	-21	N/A	
9894	1.0	1.00051	49		
74296	1.0	1.00064	62		

†Difference (pcm) = (k_{eff}–$k_{eff,\ Reference}$)×10^5

Table II: The k_{eff} results (Godiva)

Reference k_{eff} (DANTSYS, 1D, S_{16})		1.00293			
Element No.	$\frac{V_{Original}}{V_{Model}}$	No correction		Correction with Equivalent Mass Technique	
		k_{eff}	Diff. (pcm)†	k_{eff}	Diff. (pcm)†
545	1.063	0.98275	-2018	1.03318	3025
2479	1.021	0.99436	-858	1.01195	902
4208	1.016	0.99585	-708	1.00934	641

†Difference (pcm) = (k_{eff}–$k_{eff,\ Reference}$)×10^5

Table III: The k_{eff} results (VERA1B)

Reference k_{eff} (DANTSYS, 2D, S_{16})			0.99202			
Element No.	$\frac{V_{Original}}{V_{Model}}$		No correction		Correction with Equivalent Mass Technique	
	Inner Cyl.	Outer Cyl.	k_{eff}	Diff. (pcm)†	k_{eff}	Diff. (pcm)†
1111	1.044	1.047	0.96832	-2370	0.99978	776
5023	1.017	1.015	0.98462	-740	0.99687	485

†Difference (pcm) = (k_{eff}–$k_{eff,\ Reference}$)×10^5

The benchmark results are listed in the Table I, II, and III.

In the Godiva-Box problem, k_{eff} results are very close to the reference calculation (within one sigma range) and mass correction is not needed since there is no volume change.

However, in the Godiva problem, k_{eff} difference is much larger than that of Godiva-Box problem due to volume change. With no correction, k_{eff} increases as mesh numbers are increased. That is because volume deficiency is getting smaller as mesh numbers are increased. With correction, k_{eff} decreases as mesh numbers are increased.

In the VERA1B problem, k_{eff} results with equivalent mass technique shows better results.

3. Conclusions

A volume change while meshing curved geometry with tetrahedral elements is inevitable. Instead of conserving volume, an equivalent mass technique is applied to the three criticality benchmark problems and the results are shown.

The k_{eff} value converges to the reference calculation results with/without equivalent mass technique. However, the converging directions are different.

The ultimate way to solve this problem seems that generating mesh which can cover curved and plain surfaces. In addition, developing solver that can deal with these meshes is also needed.

Acknowledgement

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