Discussion on Scaling Analysis on the Fluid-Solid Coupled Domain in Sodium Fast Reactor

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1. Introduction

In the test simulation of nuclear reactor system the modeling of the heat transfer from core to coolant is very important. And the role of the heat transfer is more important if it is the single phase system as like SFR (Sodium Fast Reactor). For the test simulation of the prototypic behavior a scaling law is used, and the one suggested by Ishii et al is known to have a wide applicability and an ease of implementation[1]. However Ishii's et al.'s approach is featured by that the similarity of a solid-fluid heat transfer is not exactly satisfied, because modified Stanton number and Biot number are now easily conserved.

This study discusses the similarity of the core-coolant heat transfer and the design of the model core.

2. Scaling Methods

Constrain in this derivation of scaling law is

- Length ratio: 1/5
- Area ration: 1/5
- Reference power: 7% of normal full power
- Model electric heater: SS 316 (assumption)

Scaling approach in this study is composed of top-down approach and bottom-up approach which was proposed by Zuber et al., and Ishii et al.[2,1]. In the top-down approach governing equations are set up including following fluid energy equation, solid conduction equation, and solid-fluid boundary condition, and dimensionless number are derived.

$$\rho c_p \left\{ \frac{\partial T_i}{\partial t} + u_i \frac{\partial T_i}{\partial z} \right\} = -k \frac{\partial^2 T_i}{\partial z^2} + \frac{4h_i}{D_{Hi}} \left(T_{s,i} - T_i \right) \quad (1)$$

$$\left(\rho_s c_{ps} \frac{\partial T_s}{\partial t} \right)_i + \left(k_s \nabla^2 T_s \right)_i - \dot{q}'''_{s,i} = 0 \quad (2)$$

$$-\left(k_s \frac{\partial T_s}{\partial y} \right)_i = h_i \left(T_{s,i} - T_i \right) \quad (3)$$

Some of the important dimensionless numbers in solid-fluid heat transfer are presented in Table 1.Time ration number is very important to conserve the ratio of time constant in prototype and that in model because it describes the ratio of fluid transport time constant and solid conduction time. Similarity requirement can be achieved by letting the conduction depth ratio

$$\delta_{i,R} = \sqrt{\alpha_{s,R} \left(l_0 / u_0 \right)_{i,R}} = \alpha_{s,R}^{1/2} l_{0,R}^{1/4} = l_{0,R}^{1/4} \quad (4)$$

Table I: Dimensionless Numbers

Dimensionless No.	Expression
Modified Stanton No.	$St_i \equiv \left(\frac{4hl_0}{\rho c_p u_0 D_H} \right) = \frac{wall\ convection}{axial\ convection}$
Time Ratio No.	$T_i^* \equiv \left(\frac{l_0 / u_0}{\delta^2 / \alpha_s} \right) = \frac{transport\ time}{conduction\ time}$
Biot No.	$Bi_i \equiv \left(\frac{h\delta}{k_s} \right) = \frac{wall\ convection}{conduction}$
Heat Source No.	$Q_{si} \equiv \left(\frac{\dot{q}'''_s l_0}{\rho_s c_{ps} u_0 \Delta T_0} \right)_i = \frac{heat\ source}{axial\ energy\ change}$

In order to meet the requirement of modified St. No. and Bi No. at the same time following requirement should be met.

$$\delta_{i,R} = k_{s,R} \sqrt{l_{0,R}} \Big/ D_{Hi,R} = \sqrt{l_{0,R}} \Big/ D_{Hi,R} \quad (5)$$

Considering equations (4) and (5) the hydraulic diameter ratio should be

$$D_{Hi,R} = \left(k_s \rho_s c_{ps} \right)_R^{1/2} l_{0,R}^{1/4} = l_{0,R}^{1/4} \quad (6)$$

From the formulation of hydraulic diameter following wetted perimeter requirement is derived.

$$\xi_R = a_R / D_{H,R} = \frac{a_R}{\left(k_s \rho_s c_{ps} \right)_R^{1/2} l_{0,R}^{1/4}} = \frac{a_R}{l_{0,R}^{1/4}} \quad (7)$$

Under the same solid property the ratio of hydraulic diameter becomes 1/1.5, and the ratio of wetted perimeter 1/16.1. Such a geometry is impossible. Thus, similarity requirement of St No. and Bi No. cannot be achieved at the same time. Distortion is inevitable.

3. Wall Heat Transfer Model in SFR

Forced convective heat transfer is expected in SFR. For laminar flow

$$Nu = hD_H / k = 4.36 \quad (8)$$

For turbulent flow

$$Nu = c_1 + c_2 Pe^n \quad (9)$$

0.8 is used for n usually.

4. Local Scaling for Solid-Fluid Heat Transfer

Simple relation between heat generation in the heat structure and heat transfer to fluid can be setup.

$$\dot{q}_s = V_s \left\langle \rho_s c_{ps} \right\rangle \frac{d\left\langle T \right\rangle}{dt} + a_s h\left(T_s - T_f\right) \quad (10)$$

Dimensionless form of the above equation is

$$\frac{\dot{m}_0 h_0}{V_s \left\langle \rho_s c_{ps} \right\rangle} \frac{\tau_0}{\Delta T_0} \dot{q}_s^* = -\frac{d\left\langle \theta \right\rangle}{dt^*} + \frac{\tau_0}{\Delta T_0} \frac{a_s h\left(T_s - T_f\right)}{V_s \left\langle \rho_s c_{ps} \right\rangle} \quad (11)$$

Thus for the similarity following requirement should be met.

$$\left(\frac{\dot{m}_0 h_0}{V_s \left\langle \rho_s c_{ps} \right\rangle} \frac{\tau_0}{\Delta T_0} \dot{q}_s^* \right)_R = 1 \quad (12)$$

$$\left(\frac{\tau_0}{\Delta T_0} \frac{a_s h\left(T_s - T_f\right)}{V_s \left\langle \rho_s c_{ps} \right\rangle} \right)_R = 1 \quad (13)$$

From equation (12) following power density ratio is obtained

$$\dot{q}_{s,R} = \frac{V_{s,R}}{V_{0,R}} \left\langle \rho_s c_{ps} \right\rangle_R \left(a_{0,R} l_{0,R}^{1/2} \right) \quad (14)$$

Core flow are is given by

$$a_{s,core} = n_{ass} n_{rod/ass} \pi d_{rod} l_{rod} = n_{ass} \left[\frac{3\sqrt{3}}{2}(mp)^2 - \left\{ \left(3m^2 - 3m + 1\right) \frac{\pi}{4} d_{rod}^2 \right\} \right] \quad (15)$$

m is the rod number in outer array in the assembly of hexagon, and p is pitch between neighboring rods. Converting the uranium fuel to SS 316 material of electric heaters, the rod diameter should be

$$d_{rod,R} = \alpha_{s,R}^{1/2} l_{0,R}^{1/4} \left(\approx 1/2.59\right) \quad (16)$$

Using this rod diameter ratio and core flow area ratio requirement, rod number ration in outer array should be determined with satisfying following relation.

$$a_{core,R} = n_{ass,R} \left[\frac{3\sqrt{3}}{2}(mp)^2 - \left\{ \left(3m^2 - 3m + 1\right) \frac{\pi}{4} d_{rod}^2 \right\} \right]_R = a_{0,R} \quad (17)$$

For turbulent force convective heat transfer condition (equation (9)) Bi No. similarity reduces to

$$D_{H,R}^{n-1} = \left(k_s \rho_s c_p\right)_R^{1/2} l_{0,R}^{-n/2-1/4} \quad or \quad D_{H,R} \approx 1/102 \quad (18)$$

Pitch ratio is calculated from

$$D_H = \frac{4\left[\frac{\sqrt{3}}{4} p^2 - \frac{\pi}{8} d_{rod}^2 \right]}{\pi d_{rod}} \quad (19)$$

Equations (16), (18), and (19) produce rod diameter 2.86mm, and pitch 2.74mm which is impractical value.

For very slow flow practical values can be obtained via similar method above.

The core power is calculated as like

$$\begin{aligned} \dot{q}_{s,R} &= \frac{V_{s,R}}{V_{0,R}} \left\langle \rho_s c_{ps} \right\rangle_R \left(a_{0,R} l_{0,R}^{1/2} \right) \\ &= V_{s,R} \rho_{s,R} c_{ps,R} \Big/ \sqrt{l_{0,R}} \\ &= V_{s,R} \dot{q}_{s,R}''' \\ &= n_{ass,R} n_{rod/ass,R} d_{rod,R}^2 \left\langle \rho_s c_{ps} \right\rangle_R l_{0,R}^{1/2} \left(\approx 1/143.3\right) \end{aligned} \quad (20)$$

Prototypic normal power is 1548MWth, and 7% of the normal power is considered and then the required model power is 756kW. When the same solid properties are assumed 1939kW is required. Such a discrepancy is caused by the smaller solid volume of core. That is, the fluid volume ratio is 1/125, and the solid volume ratio is 1/478. This large gap in fluid volume and solid volume is because the rod number should be integer rather than the derived real number, so approximation is inevitable.

For the more exact model power the steady form of equation (10) is used and following relation is derived.

$$\begin{aligned} \dot{q}_{s,R} &= a_{s,R} h_R \left(T_s - T_f\right)_R \\ &= \left(n_{ass,R} n_{rod/ass,R} d_{rod,R} l_{0,R}\right)\left(1/D_{H,R}\right)(1) \end{aligned} \quad (21)$$

Resultant power is 1208kW.

5. Conclusions

The similarity of the solid-fluid heat transfer and design of model core is discussed through the intensive similarity analysis on the core fuel rod. It was re-identified that perfect modeling method of the core heat transfer is impossible. However, the method in this study provides somewhat useful analysis.

REFERENCES

[1] Ishii, M. et al, The Three-Level Scaling Approach with Application to the Purdue University Multi-Dimensional Integral Test Assembly (PUMA)", Nuclear Engineering and Design, Vol.186, pp.177-211., 1998

[2] Zuber, N., et al., An Integrated Structure and Scaling Methodology for Severe Accident Technical Issue Resolution: Development of Methodology, Nuclear Engineering and Design, Vol.186, pp.1–21, 1998