Source Expansion Nodal Solution of SP3 Equations with P1 Coarse Mesh Finite Difference Formulation

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1. Introduction

In order to effectively account for the transport effects in core calculations, the SP3 equations are adopted in some of the existing nodal diffusion codes such as PARCS[1] and DYN3D[2]. The advantage of using the SP3 equations comes from the similarity between the SP3 equations and the diffusion or P1 equation that make it possible to use the existing code's architecture and solution methods that were developed for the nodal diffusion equation. The only difference is that there are one more balance equation and one additional unknown, the second angular moment. For the solution of the SP3 equations by the nodal method, the nodal expansion method was first developed [1,2] and the source expansion nodal method(SENM)[3] was introduced as an accurate kernel to capture correctly the drastic variation of the second angular moment near material interfaces. The exponential part of the source expansion nodal solution turned out to be very effective in describing the strong gradient in the second angular flux near the surface and this capability of SENM provides better accuracy than the corresponding NEM solution.

On the other hand, a nodal solution kernel can be formulated locally employing either a one-node or two-node formulation. The one-node formulation requires incoming current conditions[1] while the two-node formulation requires node average fluxes[3]. In principle, these boundary conditions can be provided by the global coarse mesh finite difference (CMFD) solution that includes both zero-th and second angular moment fluxes. Inclusion of the second angular moments in the CMFD system, however, can lead to potential instability because of the large gradient of the second angular moments near each interface. This work is to develop a way not to use the second angular moment in the CMFD equation by keeping the ordinary P1 CMFD formulation.

2. Solution Method

The transverse integrated 1-D SP3 equation can be obtained as follows with the inclusion of transverse-leakages terms on the right hand side as:

$$\begin{bmatrix} -D_0\dfrac{d^2}{dx^2}+\Sigma_r & -2D_0\dfrac{d^2}{dx^2} \\ -\dfrac{2}{5}D_0\dfrac{d^2}{dx^2} & -\dfrac{4}{5}D_0\dfrac{d^2}{dx^2}+\dfrac{3}{5}D_2\dfrac{d^2}{dx^2}+\Sigma_t \end{bmatrix}\begin{bmatrix}\phi_0\\ \phi_2\end{bmatrix}=\begin{bmatrix} q_0-L_0 \\ -\dfrac{2}{5}L_0-\dfrac{3}{5}L_2\end{bmatrix} \quad (1)$$

By assuming a quadratic spatial variation of the transverse leakage source, Eq. (1) can be solved for the entire 1-D domain consisting of several nodes with the boundary conditions specified at the two boundaries. The diagonalization process by similarity transform is performed first, however, to express the solution in terms of the first and second harmonic modal solutions given below:

$$\begin{bmatrix}\psi_1(\xi)\\ \psi_2(\xi)\end{bmatrix}=\begin{bmatrix}\cosh(\kappa_1\xi) & 0\\ 0 & \cosh(\kappa_2\xi)\end{bmatrix}\begin{bmatrix}a_1\\ a_2\end{bmatrix}+\begin{bmatrix}\sinh(\kappa_1\xi) & 0\\ 0 & \sinh(\kappa_2\xi)\end{bmatrix}\begin{bmatrix}b_1\\ b_2\end{bmatrix}+\sum_{i=0}^{4}P_i(\xi)\begin{bmatrix}c_{1,i}\\ c_{2,i}\end{bmatrix} \quad (2)$$

with ξ being the normalized intra nodal spatial variable.

The coefficients of the homogeneous solution of each node are coupled to those of neighboring nodes due to the continuity condition of flux and current at the interface. The diagonalization process and the derivation of a coupled linear system which takes a form of a block bidiaongal matrix as shown below are detailed in a previous work[4].

$$\begin{bmatrix}
\Delta_0 & \Omega_{0,4} & & & & & & & \\
\Omega_{1,1} & \Delta_{1,1} & \Omega_{1,4} & \Omega_{1,5} & & & & & \\
\Omega_{1,2} & \Omega_{1,3} & \Delta_{1,2} & \Omega_{1,6} & & & & & \\
 & & \Omega_{2,1} & \Delta_{2,1} & \Omega_{1,4} & \Omega_{2,5} & & & \\
 & & \Omega_{2,2} & \Omega_{2,3} & \Delta_{2,2} & \Omega_{2,6} & & & \\
 & & & \ddots & \ddots & \ddots & \ddots & & \\
 & & & & \Omega_{n-1,1} & \Delta_{n-1,1} & \Omega_{n-1,4} & \Omega_{n-1,5} \\
 & & & & \Omega_{n-1,2} & \Omega_{n-1,3} & \Delta_{n-1,2} & \Omega_{n-1,6} \\
 & & & & & & \Omega_{n,1} & \Delta_n
\end{bmatrix}\begin{bmatrix}a_1\\ b_1\\ \\ \vdots\\ \\ \\ a_n\\ b_n\end{bmatrix}=\begin{bmatrix}\gamma_{Bot}\\ \gamma_{\phi,1}\\ \gamma_{J,1}\\ \vdots\\ \\ \gamma_{\phi,n-1}\\ \gamma_{J,n-1}\\ \gamma_{Top}\end{bmatrix} \quad (3)$$

This is called a whole 1-D SENM formulation and the linear system of Eq. (2) is formed and solved for each line of nodes. In a two-dimensional problem, these lines of nodes are formed in the x- and y-directions, respectively. The surface current information determined at the interfaces can be used to determine the current correction factor in the CMFD balance equation. The solution of the CMFD problem is then be used to construct the node average fission source and transverse leakage needed in Eq. (1). The intra-nodal fission source shape is updated during the solution of Eq. (1), but the node average value is kept invariant in order to improve stability.

In the P1 CMFD formulation, the second angular moments are excluded and thus there is no way to determine the transverse leakage information for the second angular moment. This problem can be resolved in two ways. One is to completely neglect L_2 in Eq. (1) and the other is to use the previous step's solution of Eq. (1) with L_2 included. In this case, the whole 1-D solution for ϕ_2 determined for the perpendicular direction is used. The following flow chart describes the whole solution sequence in the case of a two-dimensional problem.

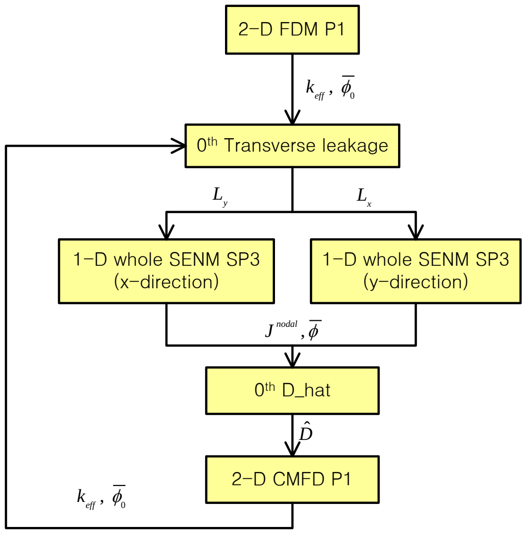

Fig. 1. Schematic of the Solution Sequence for 2-D

3. Performance Examination

The verification of the SENM SP3 kernel based P1 CMFD formulation was done for the Takeda[5] fast reactor problem and the KAIST4G[6] benchmark problem which are both 2-dimensional. The two methods of treating L_2 were examined. The $L_2 = 0$ method is denoted by #1 and the other by #2. The reference transport solution was obtained by the nTRACER MOC code and the reference SP3 solution was obtained with a fine mesh finite difference method solution for the SP3 equations. Different mesh sizes were tried for the nodal solution. In order to examine the accuracy of the SP3 solution relative to the diffusion (P1) solution, the k-effectives of the P1 and SP3 solutions were also compared.

As shown in Tables I and IV, the SP3 k-effective values are found to be much closer than P1 to the reference nTRACER values particularly for the Takeda problem involving large leakage. Tables II, III and V show that the SP3 nodal solution becomes more accurate as the node size gets smaller and also L_2 is included.

Table I. k-eff's for Takeda Problem #2 with Various Solvers

CR	nTRACER	P1 FDM	SP3 FDM
in	1.03109	1.02595(-514)	1.03026(-83)
OUT	1.06367	1.06063(-304)	1.06359(-8)

* differences given in parenthesis in pcm

Table II. SP3 SENM vs. Fine Mesh SP3 for Takeda #2

Case I : Control Rod In					
Reference*		Mesh size [cm]	Error [pcm]		Time [sec]
k-eff	Time [sec]		# 1	#2	
1.03026	931	5	20	7	10.8
		2.5	9	0	38.0

Case II : Control Rod Out					
Reference*		Mesh size [cm]	Error [pcm]		Time [sec]
k-eff	Time [sec]		# 1	#2	
1.06359	1244	5	7	0	9.6
		2.5	4	-2	42.7

* Reference SP3 FDM with the mesh size of 0.27 cm

Table III. Max relative power error (%) in the fuel region for the Takeda Problem

CR	5cm	2.5cm
In	0.29	0.11
Out	0.13	-0.05

Table IV. k-eff Comparison for the KAIST4G Problem

nTRACER	P1 FDM	SP3 FDM
1.06757	1.06644(-113)	1.06783(26)

Table V. SP3 SENM vs. Fine Mesh SP3 for KAIST4G

Reference*		Mesh size [cm]	Error [pcm]		Time [sec]
k-eff	Time [sec]		# 1	#2	
1.06783	2678	21	36	17	0.34
		10.5	20	10	0.81
		5.25	5	1	2.55

* SP3 FDM, 0.16cm

It is also noted that the computing time for the SP3 nodal solutions is trivial compared to the fine mesh FDM solutions as expected.

4. Conclusions

The solution of the SP3 equation was obtained successfully with the whole 1-D SP3 SENM kernel embedded in the P1 CMFD formulation. The SP3 solution turned out to be much more accurate for the fast reactor problems having considerable leakage. With the proposed SP3 nodal method, the computing time could be reduced by more than a factor 100 compared to the FDM cases.

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