Unavailability of k-out-of-n Reactor Protection Systems in Consideration of CCF

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1. Introduction

Many safety-critical applications including nuclear power plants are equipped with *k*-out-of-*n* or specific-voting-logic redundant safety signal generation systems for ensuring both safety and economy. In order to determine the configuration of these safety signal generation systems and to analyze the risk from these systems, common cause failures (CCF) must be considered carefully. It is known that CCF events are main contributors to unavailability of that kind of system [1, 2], therefore CCFs should be considered carefully in calculating the unavailability.

2. Unavailability of k-out-of-n RPS Configuration

Suppose that the n channels are identical and concurrent operation of k channels initiates the reactor trip. When there are greater than or equal to $n-k+1$ failed channels in a system, this *k*-out-of-*n* system is unavailable. Independent failures and CCFs in these channels can be assumed to be symmetric, and we use single CCF event for all the causes of CCF for simplicity.

2.1 The unavailability from the CCF

If $i \geq n-k+1$ channels fail to perform given safety function because of common cause, it will cause the system failure. The probability of *i* channels' failure caused by the CCF ($Q_{CCF(i)}$), can be expressed as

$$Q_{CCF(i)} = \binom{n}{i} q_{(i)}$$

where $q_{(i)}$ implies the probability of the CCF of specific *i* channels. Then the availability caused by the CCF is

$$U_{CCF} = \sum_{i=n-k+1}^{n} \binom{n}{i} q_{(i)}$$

2.2 The unavailability from the independent failures

If there is no CCF, independent failure of $i \geq n-k+1$ channels result in system unavailability. The probability of independent failure is

$$P(independent\ failure) = \frac{q_1}{q_0+q_1}$$

where *q* is channel failure probability and $q_0=1-q$. Then the probability of *i* channels' failure caused by independent failures ($Q_{IND(i)}$), and the unavailability (U_{IND}) can be expressed as

$$\mathrm{Q}_{IND(i)} = \overline{Q_{CCF}} \binom{n}{i} \left(\frac{q_1}{q_0+q_1}\right)^{i} \left(\frac{q_0}{q_0+q_1}\right)^{n-i}$$

$$where\ \overline{Q_{CCF}} = 1 - \sum_{j=2}^{n} \binom{n}{j} q_{CCF(j)}$$

$$U_{IND} = \overline{Q_{CCF}} \sum_{i=n-k+1}^{n} \binom{n}{i} \left(\frac{q_1}{q_0+q_1}\right)^{i} \left(\frac{q_0}{q_0+q_1}\right)^{n-i}$$

2.3 The unavailability from the combination of CCF and independent failures

Combination of CCF of *i* channels and independent failures of *j* channels will result in system unavailability if $i+j \geq n-k+1$. The probability of this event is

$$\mathrm{Q}_{COMB(i,j)} = Q_{CCF(i)} \binom{n-i}{j} \left(\frac{q_1}{q_0+q_1}\right)^{j} \left(\frac{q_0}{q_0+q_1}\right)^{n-i-j} = \binom{n}{i} q_i \binom{n-i}{j} \left(\frac{q_1}{q_0+q_1}\right)^{j} \left(\frac{q_0}{q_0+q_1}\right)^{n-i-j}$$

then the unavailability (U_{COMB}) is

$$\mathrm{U}_{COMB} = \sum_{i=2}^{n-k} \binom{n}{i} q_{CCF(i)} \times \left\{ \sum_{j=n-k+1-i}^{n-i} \binom{n-i}{j} \left(\frac{q_1}{q_0+q_1}\right)^{j} \left(\frac{q_0}{q_0+q_1}\right)^{n-i-j} \right\}$$

3. Application to 2-out-of-4 RPS Configuration

For 2-out-of-4 configuration, failure of 3 channels result in the system failure. First, CCF of 4 or 3 channels comprise U_{CCF}.

$$U_{CCF} = P(C_{ABCD}) + \{P(C_{ABC}N_D) + \cdots + P(C_{BCD}N_A) + P(C_{ABC}I_D) + \cdots + P(C_{BCD}I_A)\} = q_4 + 4q_3$$

The probability of independent failure of 4 or 3 channels is

$$U_{IND} = \{P(I_A I_B I_C I_D)\} + \{P(I_A I_B I_C N_D) + \cdots + P(N_A I_B I_C I_D)\}$$
$$= (q_0 + q_1 - 3q_2 - q_3)\left(\frac{q_1}{q_0+q_1}\right)^4 + 4(q_0 + q_1 - 3q_2 - q_3)\left(\frac{q_0}{q_0+q_1}\right)^1 \left(\frac{q_1}{q_0+q_1}\right)^3$$

,where we used

$$\overline{Q_{CCF}} = \{(1-q_0)+q_0\} - \binom{4}{4} q_4 - \binom{4}{3} q_3 - \binom{4}{2} q_2 = \left\{ \left(\sum_{i=1}^{4} \binom{4-1}{i-1} q_i \right) + q_0 \right\} - q_4 - 4q_3 - 6q_2$$

$$= q_0 + q_1 - 3q_2 - q_3$$

CCF of 2 channels and independent failure of one or two channels causes system unavailability.

$$U_{COMB} = \{P(C_{AB}I_C N_D) + \cdots + P(C_{CD}N_A I_B)\} + \{P(C_{AB}I_C I_D) + \cdots + P(C_{CD}I_A I_B)\}$$
$$= 12q_2\left(\frac{q_0}{q_0+q_1}\right)^1\left(\frac{q_1}{q_0+q_1}\right)^1 + 6q_2\left(\frac{q_1}{q_0+q_1}\right)^2$$

It is notable that, in this example, the sum of the probabilities of events that has independent failures is $4q_1$, which indicates the probability of independent failure of each channel.

$$\binom{4}{1} q_1 = \{P(C_{ABC}I_D) + \cdots + P(C_{BCD}I_A)\}$$
$$+\{P(C_{AB}I_C N_D) + \cdots + P(C_{CD}N_A I_B)\}$$
$$+2\{P(C_{AB}I_C I_D) + \cdots + P(C_{CD}I_A I_B)\}$$
$$+\{P(I_A N_B N_C N_D) + \cdots + P(N_A N_B N_C I_D)\}$$
$$+2\{P(I_A I_B N_C N_D) + \cdots + P(N_A N_B I_C I_D)\}$$
$$+3\{P(I_A I_B I_C N_D) + \cdots + P(N_A I_B I_C I_D)\}$$
$$+4P(I_A I_B I_C I_D)$$
$$= 4q_3\frac{q_1}{q_0+q_1} + 12q_2\left(\frac{q_0}{q_0+q_1}\right)^1\left(\frac{q_1}{q_0+q_1}\right)^1$$
$$+12q_2\left(\frac{q_1}{q_0+q_1}\right)^2$$
$$+4(q_0+q_1-3q_2-q_3)\left(\frac{q_0}{q_0+q_1}\right)^3\left(\frac{q_1}{q_0+q_1}\right)^1$$
$$+12(q_0+q_1-3q_2-q_3)\left(\frac{q_0}{q_0+q_1}\right)^2\left(\frac{q_1}{q_0+q_1}\right)^2$$
$$+12(q_0+q_1-3q_2-q_3)\left(\frac{q_0}{q_0+q_1}\right)^1\left(\frac{q_1}{q_0+q_1}\right)^3$$
$$+4(q_0+q_1-3q_2-q_3)\left(\frac{q_1}{q_0+q_1}\right)^4$$
$$= 4\frac{q_1}{q_0+q_1}\left\{q_3 + 3q_2\left(\frac{q_0}{q_0+q_1}\right)^1 + 3q_2\left(\frac{q_1}{q_0+q_1}\right)^1 + (q_0+q_1-3q_2-q_3)\left(\frac{q_0}{q_0+q_1}+\frac{q_1}{q_0+q_1}\right)^3\right\}$$
$$= 4\frac{q_1}{q_0+q_1}\{q_3 + 3q_2 + (q_0+q_1-3q_2-q_3)\} = 4q_1$$

In this study, we used the Alpha Factor model for estimating the CCF probability, $q_{(i)}$. The alpha parameters were determined based on the field experience of the RPS [1], $\alpha_1^{(4)}$=9.06e-01, $\alpha_2^{(4)}$=6.15e-02, $\alpha_3^{(4)}$=2.25e-02 and $\alpha_4^{(4)}$ =1.06e-02.

Figure 1 illustrates the unavailability of 2-out-of-4 system. It shows that the independent failures dominate the system unavailability when q is large, but the CCF dominates when the channel failure probability q is small.

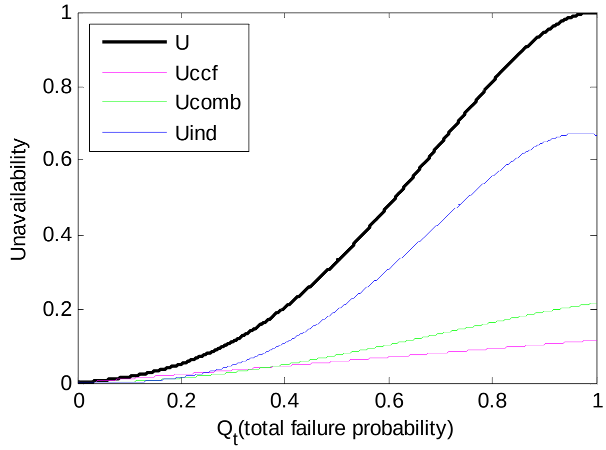

Fig. 1. The unavailability of 2-out-of-4 system

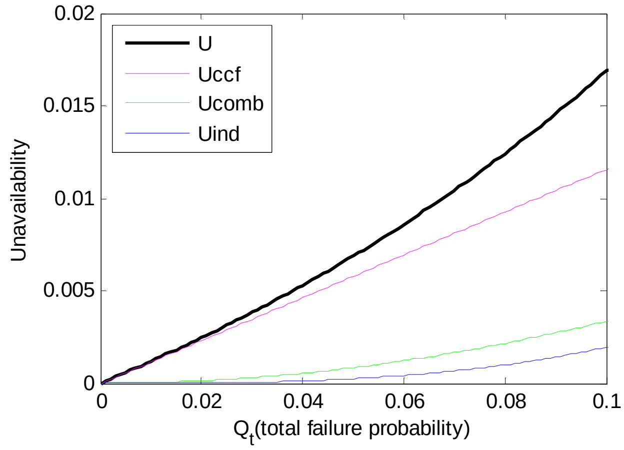

Fig. 2. The unavailability of 2-out-of-4 system when q<0.1

4. Conclusions

In most cases, the probability of CCF is estimated using relatively simple model. To consider the effect of CCF more precisely, analytic equations to calculate the probability of event that contains CCF are developed and 2-out-of-4 RPS configuration was investigated as example. It is also shown in this example, CCF dominates the system unavailability.

REFERENCES

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