

Least Squares Approximations for 4-20 mA Current Loop Transmitter Calibration Based on a Quadratic Equation

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1. Introduction

As usage of 2-wire configuration, or 4 to 20 mA loop signals, is popular to represent temperature, pressure, flow, speed, position, radiation, level, or pH etc., the loop current calibration is an important process to maintain an accurate signal in a plant. In this work, least squares approximation technique is investigated to calibrate an analog output signal from 4 to 20 mA. From a quadratic equation, the linear form approximation was derived, and the full-scale errors were measured.

2. Theory

2.1 Quadratic Equation

A quadratic equation is the simplest combination involving linear and nonlinear properties. Because of this clearly distinguishable property, it is straightforward to check whether the response is linear or not. Equation (1) shows a typical quadratic equation. If the coefficients C_2 , C_1 , and C_0 are known, then the output response, or current signal (mA), can be determined.

$$y_{mA} = \underbrace{C_2}_{\text{Leading Coefficient}} (x_{bit})^2 + \underbrace{C_1}_{\text{Slope}} (x_{bit}) + \underbrace{C_0}_{\text{Intercept}} \quad (1)$$

Under the condition that the leading coefficient C_2 is so small that the leading coefficient is negligible, it is concluded that the response is linear, and Equation (1) can be rewritten as Equation (2). The coefficients C_1 and C_0 represent the slope and intercept point, respectively.

$$x_{bit} = (y_{mA} - C_0) / C_1 \quad (2)$$

If the leading coefficient C_2 is significant, it should be taken into account, and the response will be nonlinear.

2.2 Least Squares Approximation

To estimate coefficients, the least square method is applied [1]. Because the coefficients (C_2 , C_1 , and C_0) are changed according to measurement data, the more data that are sampled, the more accurate the results will be. In this experiment, 33 points are chosen, and the binary digit step is 2^{11} . Thus, the binary input digit varies as 0, 1×2^{11} , 2×2^{11} , 3×2^{11} , 4×2^{11} , ..., and $32 \times 2^{11} - 1$ for initial loop current measurements.

2.3 Full-Scale Error

The full-scale error can be calculated based on Equation (3).

$$Error_{FullScale} = (y_{Measured} - y_{Ideal}) / y_{FullScale} \times 100\% \quad (3)$$

It is assumed that the full-scale current ($y_{FullScale}$) is 20 mA. The measured and ideal currents at the designated points are denoted as $y_{Measured}$ and y_{Ideal} .

2.4 Estimation of coefficients

It is necessary to roughly estimate the coefficients to check whether the results of the least squares approximations are reasonable or not.

The binary digital input (x_{bit}) can be varied from 0 to 65535 (2^{16} bits-1), and the corresponding analog current output signal changes from 3.92 to 20.4 mA, or the range of the analog output signal is approximately 16 mA. Then the coefficient C_1 can be estimated as $2.44E-4$ (16 mA / 65535). Here, C_0 is the minimum current level, and it typically varies with each module. The nonlinear part is negligible as long as the maximum binary digit, or $4.3E+9$ (65535^2), is depreciated by the leading coefficient C_2 . For example, if the leading coefficient (C_2) is $1E-12$, the maximum percentage due to C_2 is then approximately 0.022 % ($1E-12 \times 4.3E+9 / 20mA \times 100\%$). In this experiment, the nonlinear part is neglected when the maximum percentage of $C_2(x_{bit})^2$ at full-scale (20 mA) is less than 1%.

3. Test Results

The specification of the test modules are listed in Table I, and four modules were made for testing. The sampling number is 33 to cover the full range (x_{bit} : $0 \sim 2^{16}$ bits-1), and thus the step of an analog output signal is selected as 0.5 mA. The corresponding range is 3.92 to 20.4 mA. However, the covered range will be slightly different depending on each component characteristic. Figs.1 and 2 show a block diagram and the realized modules, and Fig. 3 shows the measured current signals. To measure the current signals, Agilent 34410A was used. From the initial measurement, the coefficients were calculated by a least squares approximation, and Table II shows the results. It should be noted that the calculated coefficients are similar to the estimated values. Under assumption that the leading coefficient C_2 is negligible, it was left out to be recalculated. This means that the response is almost linear. The recalculated results are shown in Table III. Slight

differences in C_1 and C_0 can be observed between Table II and III owing to the leading coefficient C_2 .

The coefficients in Table III reflect the full range of analog signals (3.92~20.4 mA), and it had to be corrected again by means of a trial and error method because the current increase step is 0.5 mA in the range of interest from 4 to 20 mA. The final results are shown in Table IV, and the full-scale error is as shown in Fig. 4. The measured full-scale errors are less than $\pm 0.005\%$.

4. Conclusions

A quadratic equation is used to calibrate the current loop transmitter. Because the quadratic has both nonlinear and linear parts, it is easy to check whether the response is linear or not from the least squares approximations. The realized modules show that the analog output current signal can be corrected through least squares approximations, and the full-scale errors were less than $\pm 0.005\%$, while showing linear responses.

REFERENCE

[1] Gilbert Strang, *Linear Algebra and Its Application 3rd*, Chap. 3, Harcourt.

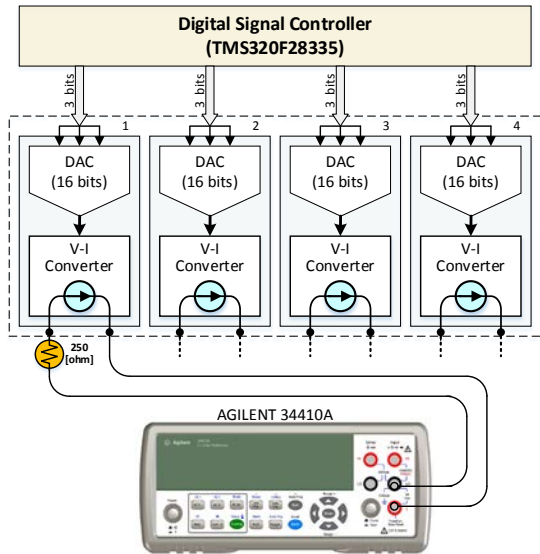


Fig. 1. Block diagram of the current loop test board.



Fig. 2. Realized modules-1/2/3/4.

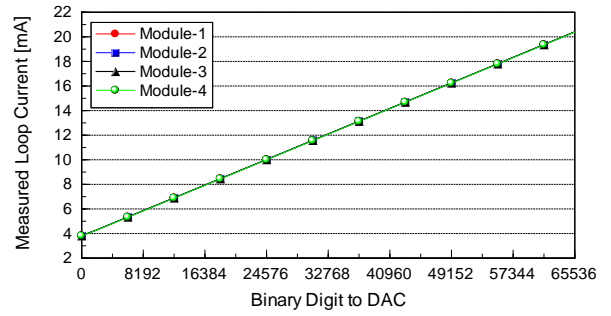


Fig. 3. Measured loop currents before calibration.

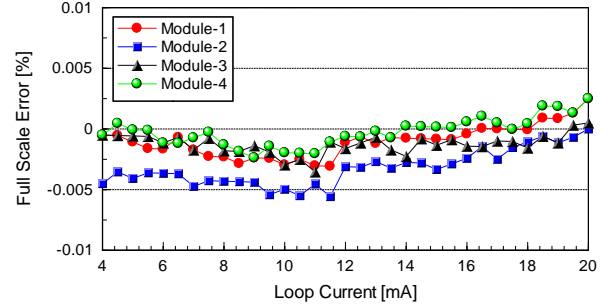


Fig. 4. Full-scale errors after calibration.

Table I: Test Module Specification

DAC Resolution	16 [bits] (0~65535)
V-I Converter Range	3.92~20.4 [mA]
Reference Voltage	4.096 [volt]
Digital Signal Controller	TMS320F28335

Table II: Quadratic Approximation

	C_0	C_1	C_2
Module-1	3.7986	2.5306×10^{-4}	7.8551×10^{-12}
Module-2	3.7953	2.5305×10^{-4}	7.3006×10^{-12}
Module-3	3.7893	2.5304×10^{-4}	7.4299×10^{-12}
Module-4	3.7727	2.5310×10^{-4}	6.8533×10^{-12}

Table III: Linear Approximation

	C_0	C_1	C_2
Module-1	3.7932	2.5357×10^{-4}	0
Module-2	3.7902	2.5352×10^{-4}	0
Module-3	3.7842	2.5354×10^{-4}	0
Module-4	3.7679	2.5355×10^{-4}	0

Table IV: Approximation by Trial and Error

	C_0	C_1	C_2
Module-1	3.7888	2.5368×10^{-4}	0
Module-2	3.7862	2.5362×10^{-4}	0
Module-3	3.7800	2.5363×10^{-4}	0
Module-4	3.7634	2.5365×10^{-4}	0