Scaling analysis of a thermal-hydraulic test loop for single-phase natural circulation under dynamic motion condition

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1. Introduction

Floating reactors in the ocean may be exposed to external forces depending on the surrounding environment. The reactor is likely to be shaken by waves or tides, and various components on the plant are subjected to dynamic conditions by being accelerated and it can be expressed as six-degree of freedom (6-DOF) as shown in Fig. 1.

Specifically, as the working fluid in the component is subjected to additional acceleration under dynamic condition, thermal-hydraulic phenomena such as natural circulation, heat transfer, pressure drop and critical heat flux, can show different aspects unlike those under the stationary condition of the land-based reactors. Therefore, it is necessary to evaluate thermal-hydraulic phenomena under dynamic condition through experimental and analytical studies. Especially, in the experimental research, the scaling method is important to design and model the motion platform properly, not distorting the physical phenomena under dynamic condition compared to those of the prototype. However, there is no general form of similarity criteria for a single-phase natural circulation test loop under dynamic motion condition.

According to this necessity of scaling criteria under dynamic motion condition, the similarity criteria under dynamic motion was derived in this study. In addition, the non-dimensional governing equation for the single-phase natural circulation was suggested for practical application.

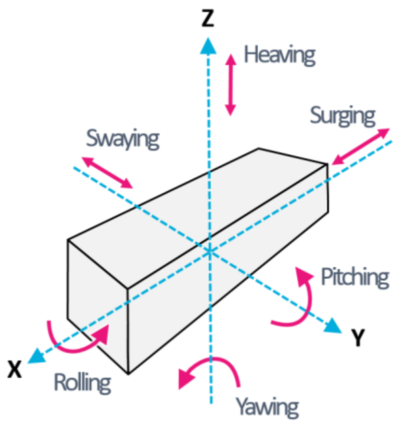

Fig. 1. Six-degree of freedom of the body under dynamic motion

2. Similarity criteria under dynamic motion condition

In this section, the governing equation was established under dynamic motion by adding the external force term due to the dynamic motion to the existing momentum equation for stationary condition. In addition, similarity criteria for momentum equation in differential form was derived through the scaling analysis reflecting the characteristics of dynamic motion. For practical applications, the non-dimensional form of governing equations for single-phase natural circulation was derived.

2.1 Governing equation

In the dynamic motion environment, additional external forces can be acted on fluid volume compared to the case of the stationary condition. The previous researches analyzing these external forces have been carried out from the viewpoint of the dynamics [1] or coordinate transformation [2]. In short, the governing equation under dynamic condition is derived as follows,

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=-\nabla p+\nabla\cdot\boldsymbol{\tau}+\rho\Big(\mathbf{g}-\frac{d^2\mathbf{R}}{dt^2}-2\boldsymbol{\Omega}\times\mathbf{u}-\frac{D\boldsymbol{\Omega}}{Dt}\times\mathbf{r}-\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\Big) \quad (1)$$

where ρ, $\mathbf{u}$, p, $\boldsymbol{\tau}$ and $\mathbf{g}$ indicates the fluid density, velocity, pressure, shear stress and gravitational acceleration, respectively. For additional acceleration, the last four terms on the right hand side (RHS) of Eq. (1) are implemented as a form of body force. The general meaning of these accelerations are described in Table I.

Table I: Additional acceleration under dynamic motion

Mathematical form	Meaning of acceleration	Related dynamic motion in 6-DOF
$d^2\mathbf{R}/dt^2$	Frame acceleration	Heaving, Swaying, Surging
$2\boldsymbol{\Omega}\times\mathbf{u}$	Coriolis acceleration	Rolling, Pitching, Yawing
$(D\boldsymbol{\Omega}/Dt)\times\mathbf{r}$	Euler acceleration	
$\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})$	Centrifugal acceleration	

These kinds of acceleration exert forces on the fluid in various directions. As shown in Fig. 2, assuming that the fluid is flowing in the direction of the center of rotation, the Coriolis force and the Euler force act on the tangential direction of rotation. The centrifugal force acts in the radial direction of rotation, and the acceleration term of the frame itself can act on any direction. However, since the Coriolis, Euler, and centrifugal terms already consider the effect of rotational motion, it is appropriate to express frame acceleration as the translational (linear) motion.

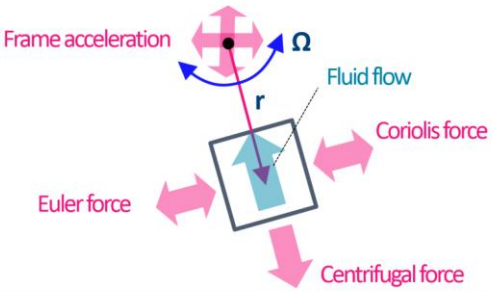

Fig. 2. Forces acting on the moving body under dynamic motion

This form of additional acceleration terms had been modelled as the same form in the momentum equation of thermo-hydraulic analysis codes including dynamic motion model such as RETRAN-02/GRAV [3], RETRAN-03/MOV [4] and INT [5], RELAP5/3D [6] and MARS [7].

2.2 Similarity criteria

In order to derive the similarity criteria under dynamic motion condition, it is necessary to derive the non-dimensional form of governing equation. In this way, one can get dimensionless numbers describing physics in any dimension. For stationary environment, the non-dimensional governing equation is expressed as follows,

$$\frac{\partial \mathbf{u}^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla^* \mathbf{u}^* = -\mathbf{Eu}\nabla^* p^* + \frac{1}{\mathbf{Re}}\nabla^{*2}\mathbf{u}^* - \frac{1}{\mathbf{Fr}^2}\hat{\mathbf{z}} \quad (2)$$

$$\nabla^* = \frac{\nabla}{L_c},\ t^* = \frac{t}{t_c}, \mathbf{u}^* = \frac{\mathbf{u}}{u_c}, p^* = \frac{p}{p_c}, t_c = \frac{L_c}{u_c}, p_c = \rho u_c^2 \quad (3)$$

$$\mathbf{Eu} = p_c / \rho u_c^2,\ \ \mathbf{Re} = \rho u_c L_c / \mu,\ \ \mathbf{Fr} = u_c / \sqrt{L_c g} \quad (4)$$

where L_c, t_c, u_c and p_c are characteristic length, time, velocity and pressure, respectively. As shown in Eq. (4), derived dimensionless numbers are Euler number (*Eu*), Reynolds number (*Re*) and Froude number (*Fr*).

In the case of dynamic motion condition, the non-dimensional form of governing equation can be derived based on Eq. (1). In addition, for scaling analysis, one can set scaling parameters related to dynamic motion as shown in Fig. 3.

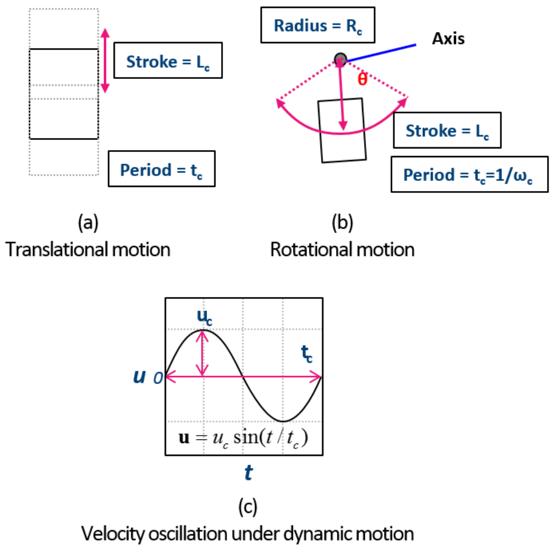

Fig. 3. Characteristic parameters for scaling analysis under dynamic motion, (a) translational motion (b) rotational motion (c) velocity oscillation

Unlike the stationary conditions, the characteristic length and time are set to stroke of motion and period of motion, respectively. Specifically, as depicted in Fig. 3, in the case of translational motion, one needs stroke and period for scaling parameters, however, for the rotational case, the radius of gyration, stroke and period are necessary.

As the scaling parameters show different forms between translational and rotational motions, the governing equations and related scaling parameters are categorized into two cases as below.

Translational motion:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\boldsymbol{\tau} + \rho(\mathbf{g} - \frac{d^2\mathbf{R}}{dt^2}) \quad (5)$$

$$\nabla^* = \frac{\nabla}{L_c},\ t^* = \frac{t}{t_c}, \mathbf{u}^* = \frac{\mathbf{u}}{u_c}, p^* = \frac{p}{p_c}, p_c = \rho u_c^2 \quad (6)$$

Rotational motion:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \nabla\cdot\boldsymbol{\tau} + \rho(\mathbf{g} - 2\boldsymbol{\Omega}\times\mathbf{u} - \frac{D\boldsymbol{\Omega}}{Dt}\times\mathbf{r} - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})) \quad (7)$$

$$\nabla^* = \frac{\nabla}{L_c},\ t^* = \frac{t}{t_c}, \mathbf{u}^* = \frac{\mathbf{u}}{u_c}, p^* = \frac{p}{p_c}, \mathrm{r}^* = \frac{r}{R_c}, \Omega^* = \frac{\Omega}{\omega_c}, \quad t_c = \frac{1}{\omega_c}, p_c = \rho u_c^2 \quad (8)$$

As the Eq. (5) and (7) are non-dimensionalized using scaling parameters of Eq. (6) and (8) respectively, dimensionless form of governing equation and related dimensionless numbers are derived as follows.

Translational motion:

$$\mathbf{St_L}\frac{\partial \mathbf{u}^*}{\partial t^*} + \mathbf{u}^*\cdot\nabla^*\mathbf{u}^* = -\mathbf{Eu}\nabla^* p^* + \frac{1}{\mathbf{Re}}\nabla^{*2}\mathbf{u}^* - \frac{1}{\mathbf{Fr}^2}\hat{\mathbf{z}} - \mathbf{St_L}^2\frac{d^2\mathbf{R}^*}{dt^{*2}} \tag{9}$$

Rotational motion:

$$\mathbf{St_L}\frac{\partial \mathbf{u}^*}{\partial t^*} + \mathbf{u}^*\cdot\nabla^*\mathbf{u}^* = -\mathbf{Eu}\nabla^* p^* + \frac{1}{\mathbf{Re}}\nabla^{*2}\mathbf{u}^* - \frac{1}{\mathbf{Fr}^2}\hat{\mathbf{z}} - \left[2\cdot\mathbf{St_L}(\mathbf{\Omega}^*\times\mathbf{u}^*) + \mathbf{St_R}^2\{\mathbf{\Omega}^*\times(\mathbf{\Omega}^*\times\mathbf{r}^*)\} + \mathbf{St_R}^2\left(\frac{d\mathbf{\Omega}^*}{dt^*}\times\mathbf{r}^*\right)\right] \tag{10}$$

Two kinds of Strouhal numbers:

$$\mathbf{St_L} = \frac{L_c}{u_c t_c},\quad \mathbf{St_R} = \frac{\sqrt{R_c^2\theta_c}}{u_c t_c} \tag{11}$$

As a result, the Strouhal number is obtained in non-dimensional governing equation under the dynamic motion condition. Depending on the case of translation and rotation, Strouhal number is divided into two types. First, the 'St_L' in Eq. (11), it is expressed as ratio of 'stroke of motion' to 'moving distance of fluid during a single period'. In addition, it appears in the time derivative on the left hand side (LHS) of Eq. (9) and (10), the translational acceleration on the RHS of Eq. (9) and Coriolis acceleration term on the RHS of Eq. (10). On the other hand, the 'St_R' in Eq. (11), it is expressed as the ratio of product of 'radius of gyration' and 'angle of rotation' to 'moving distance of fluid during a single period'. It appears in the centrifugal and Euler acceleration terms on the RHS of Eq. (10).

Generally, the Strouhal number has the physical meaning of the ratio of inertial forces from external dynamic motion and fluid motion. It is recommended that similarity in Strouhal number should be satisfied for designing thermal-hydraulic experiments simulating dynamic motion.

2.3 Application for single-phase natural circulation

In the previous section, the differential form of non-dimensional governing equation is derived and its similarity criteria is suggested. However, in designing an experimental test loop under dynamic motion condition, it is necessary to derive the equations as an integral form in order to apply them to thermal-hydraulic analysis code such as RELAP5/3D or MARS. Therefore, for this application, the momentum equation on the single-phase natural circulation under dynamic condition is derived based on the Eq. (9) and (10). First, one can expand the non-dimensional numbers (*St, Eu, Re* and *Fr*) from Eq. (9) and (10), and add '$\cos\theta$'for inclination effect on gravity term. Thereafter, multiply both sides by 'ρu_c^2', divide both sides by 'L_c' and integral both sides along the natural circulation loop, then, the Eq. (9) and (10) are substituted into,

$$\omega_c u_c \sum(\rho_i l_i)\frac{\partial \mathbf{u}^*}{\partial t^*} = -\left\{\frac{1}{2}\sum \rho_i u_i^2\left(\frac{fl}{d}+K\right)_i\right\} + \Delta\rho g l_{hc}\cos\theta - \sum\rho_i l_i\left[\omega_c^2 L_c\frac{d^2\mathbf{R}^*}{dt^{*2}} + \omega_c^2 R_c\{\mathbf{\Omega}^*\times(\mathbf{\Omega}^*\times\mathbf{r}^*)\} + \omega_c^2 R_c\left(\frac{d\mathbf{\Omega}^*}{dt^*}\times\mathbf{r}^*\right)\right] \tag{12}$$

where 'i' indicate the i-th node position of the loop, 'f' and 'K' means the frictional loss and head loss coefficient, respectively, 'd' is the diameter of pipe, and '$\Delta\rho$' and 'l_{hc}' is the density difference and height difference between the heater and cooler of the loop, respectively. From the Eq. (12), one can use relation of '$\rho_i u_i a_i = \rho_r u_r a_r$' from the continuity equation ('r' denotes the reference position, commonly at the inlet of the heater), and adopt following scaling parameters.

$$\left(l_i^* = \frac{l_i}{L_c},\ l_{hc}^* = \frac{l_{hc}}{L_{hc}},\ \rho_i^* = \frac{\rho_i}{\rho_0},\ \rho_r^* = \frac{\rho_r}{\rho_0},\ \Delta\rho^* = \frac{\Delta\rho}{\Delta\rho_0},\ A_i^* = \frac{A_i}{A_0},\ u_r^* = \frac{u_r}{u_c},\ t^* = \frac{t}{1/\omega_c}, R^* = \frac{R}{L_c}, r^* = \frac{r}{R_c},\ \Omega^* = \frac{\Omega}{\omega_c}\right) \tag{13}$$

The parameters in Eq. (13) are described in Fig. 4 including that 'L_{hc}', 'u_0', 'ρ_0' and 'A_0' represent the initial state of the dynamic motion. Finally, the dimensionless form of momentum equation for single-phase natural circulation is derived under dynamic motion conditions as follows.

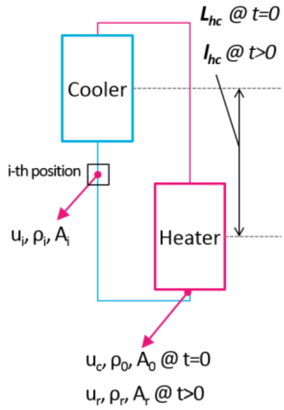

Fig. 4. Conceptual natural circulation loop under dynamic motion and related parameters

$$\mathbf{St_L}\cdot\sum\frac{l_i^*}{A_i^*}\frac{\partial u_r^*}{\partial t^*} = \mathbf{Ri}\cdot\Delta\rho^* l_{hc}^*\cos\theta - \frac{1}{2}(u_r^*)^2\sum\mathbf{Fi} + \sum\frac{\rho_i^* l_i^*}{\rho_r^*}\left[\mathbf{St_L}^2\frac{d^2\mathbf{R}^*}{dt^{*2}} + \mathbf{St_R}^2\{\mathbf{\Omega}^*\times(\mathbf{\Omega}^*\times\mathbf{r}^*)\} + \mathbf{St_R}^2\left(\frac{d\mathbf{\Omega}^*}{dt^*}\times\mathbf{r}^*\right)\right] \tag{14}$$

$$\mathbf{St_L} = \frac{L_c}{u_c t_c},\quad \mathbf{St_R} = \frac{\sqrt{R_c^2\theta_c}}{u_c t_c},\quad \mathbf{Fi} = \left(\frac{\rho_r^*}{\rho_i^*}\right)\left(\frac{1}{a_i^*}\right)^2\left(\frac{fl}{d}+K\right),\quad \mathbf{Ri} = \frac{\Delta\rho_0 g l_{hc}}{\rho_r u_c^2} \tag{15}$$

By applying the specific conditions of single-phase natural circulation to the generalized governing equation, Eq. (9) and (10), we derived the Eq. (14). Therefore, Eq. (14) suggested in this study is a generalized form of governing equation for single-phase natural circulation under dynamic conditions.

Additionally, Yan and Wen [8] has also established governing equations under ocean motion for single-phase natural circulation. However, since it did not originate from general form of momentum equation under dynamic motion such as Eq. (9) and (10) in this study, it did not provide a non-dimensionalized formation for dynamic motion, as shown in the LHS and the last term of RHS in Eq. (14).

On the other hand, the equation derived from this study can provide the basis for the derivation of various types of scaling methods for single-phase natural circulation under dynamic motion.

3. Conclusions

In the present study, the governing equation under dynamic motion was established based on the previous researches and the additional acceleration terms were categorized into translational and rotational ones according to the six-degree of freedom in motion.

In order to derive the similarity criteria under dynamic condition for a single-phase test loop, first, the non-dimensional form of governing equation was derived and corresponding similarity criteria was suggested. In result, the two kinds of Strouhal numbers were obtained for the case of translational and rotational motion. Furthermore, for the practical application in the thermal-hydraulic system analysis code, the non-dimensional governing equation under dynamic motion was derived for a single-phase natural circulation loop. This equation suggested the general form of single-phase natural circulation under dynamic motion. Moreover, it can provide the fundamental form for the derivation of various scaling methods in single-phase natural circulation under dynamic motion.

For the future works, the verification study could be carried out using thermal-hydraulic system codes containing dynamic motion model such as MARS for various dynamic environments.

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