Eddy Viscosity Model for Supercritical Fluids

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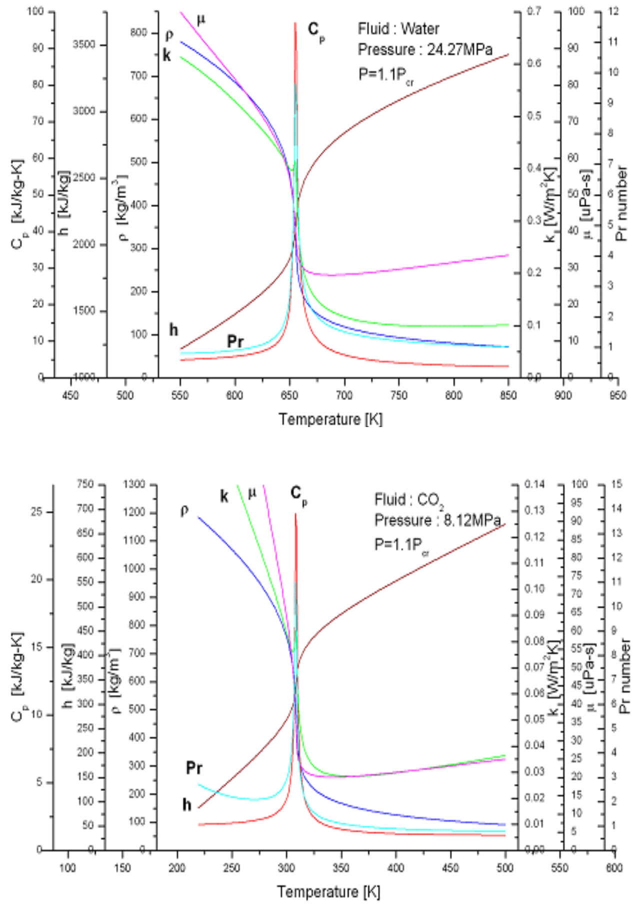
Issues in Supercritical Fluids

- Severe property change
- Property change appear in middle of flow domain
- M-shaped velocity profile



Property variations of water & CO_2 at a pressure of $1.1P_{cr}$

Issues in Supercritical Fluids

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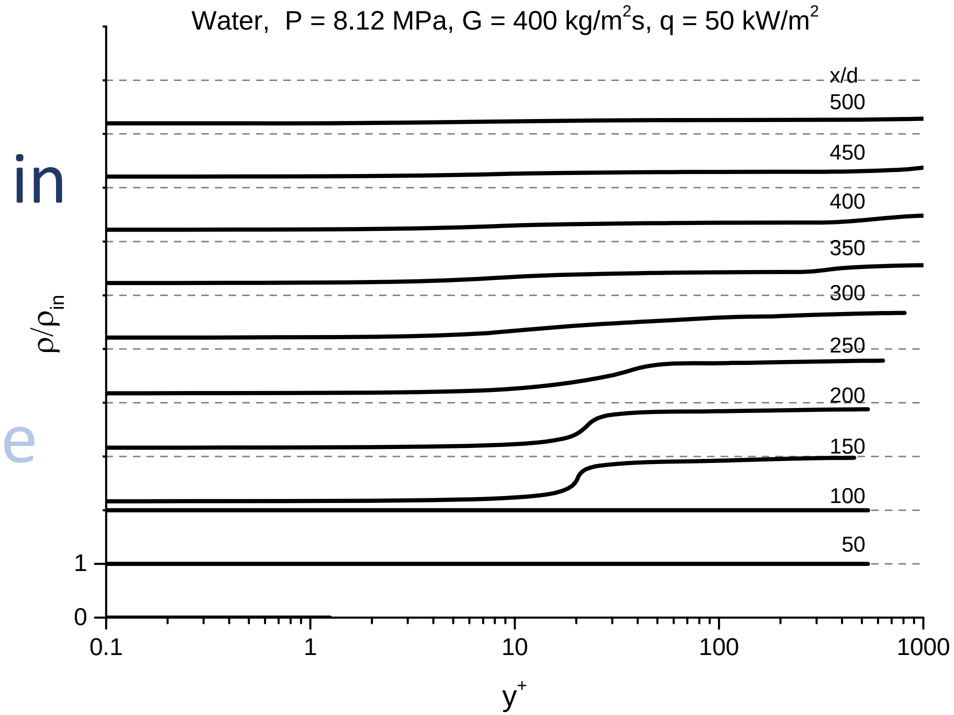

Issues in Supercritical Fluids

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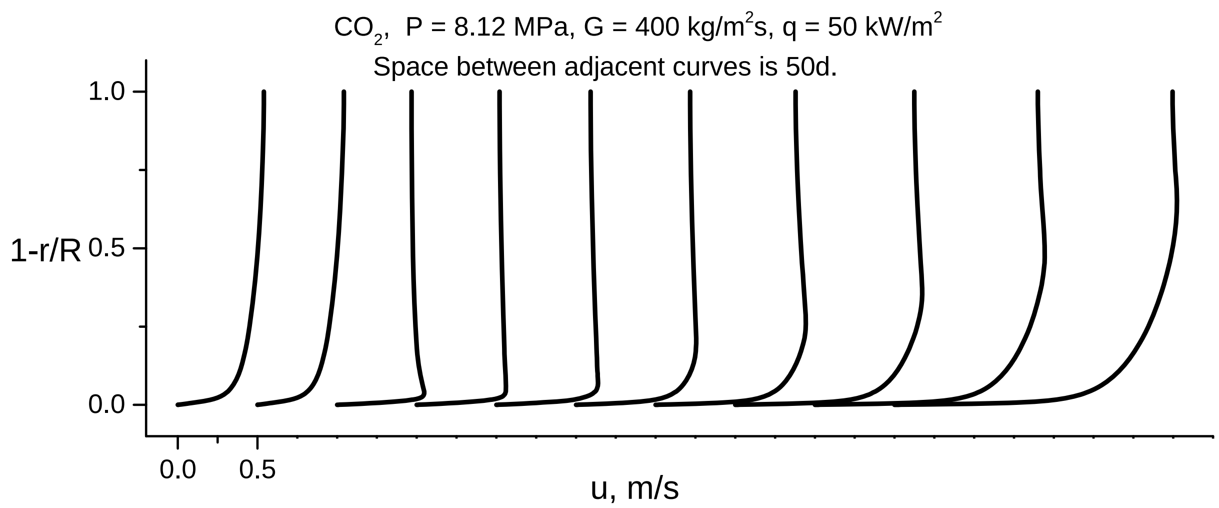


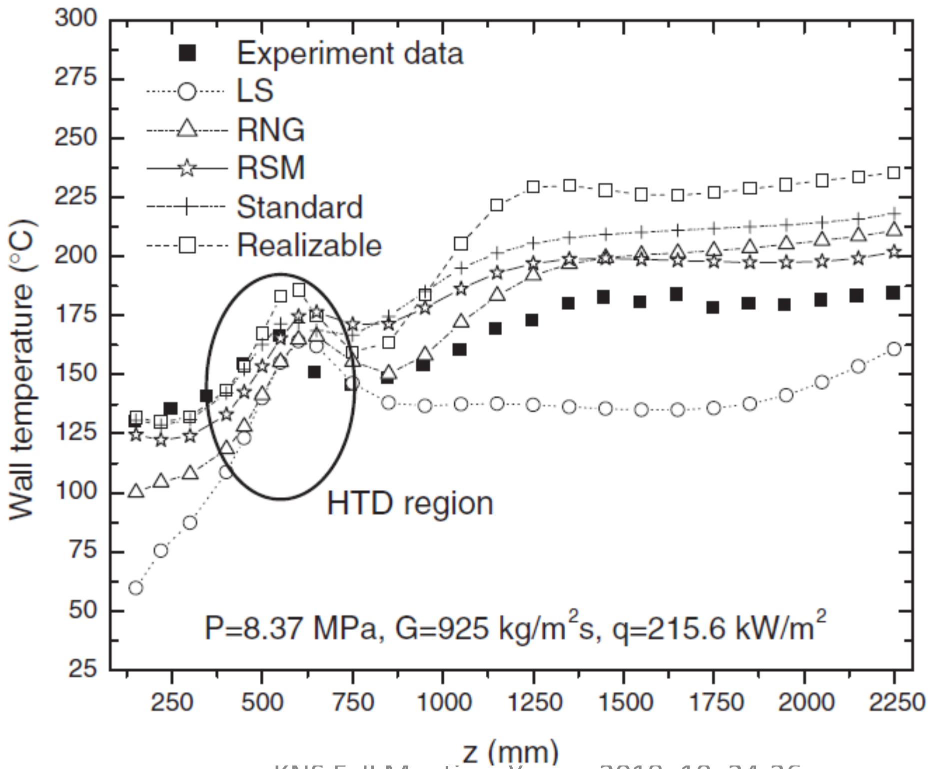
Objectives

- Are the existing eddy viscosity models only option for Reynolds stress?
  - If not, what is alternative?

- How to incorporate the influence of property (density) variation in the eddy viscosity model?

Performance of Turbulence Models

- Most of the existing turbulence models failed to reproduce deteriorated heat transfer
- No model incorporates property variation effect



Turbulence Scales

	Time Scale	Length Scale
Kolmogorov	$\tau_K = (\nu/\varepsilon)^{1/2}$	$\eta = (\nu^3/\varepsilon)^{1/4}$
Taylor		$\lambda = \left(15\nu\overline{u'^2}/\varepsilon\right)^{1/2}$
Integral	$\tau_I = k/\varepsilon$	$\ell = k^{3/2}/\varepsilon$

New Modeling of Eddy Viscosity

- Traditional
  - $\nu_t \propto (velocity\ scale) \times (length\ scale) = \mathcal{U} \times \mathcal{L}$
- $\mathcal{U}$ cannot be a characteristic scale over the entire domain
- Present method
  - $\nu_t \propto \mathcal{L}^2/\mathcal{T}$
  - Convenient to incorporate the density influence

New Modeling of Eddy Viscosity

- Length scales
  - $\mathcal{L} = max(C_1\lambda, C_2\ell) = C_2\ell\, max\left[\frac{C_1}{C_2}\left(\frac{10}{Re_t}\right)^{1/2}, 1.0\right]$
- Time scales
  - $\mathcal{T}_c = max(C_3\tau_K, C_4\tau_I) = C_4\tau_I max\left[\frac{C_3}{C_4}\left(\frac{1}{Re_t}\right)^{1/2}, 1.0\right]$

    ✓ $Re_t = \left(\tilde{k}^2/\bar{\nu}\tilde{\varepsilon}\right)^{1/2}$

Time Scale due to Density Gradient

- Time scale due to density gradient: τ_ρ
  - $\frac{1}{\rho}\frac{\partial \rho}{\partial y}$ (m) may combine with $\sqrt{k}$ or u^* (m/s) leading to frequency dimension (1/s)
  - $1/\tau_\rho = C_5 f_\rho \frac{1}{\rho}\frac{\partial \rho}{\partial y}\left(u^*\sqrt{k}\right)^{1/2}$
    - ✓ $f_\rho = 0.5\left[1 + tanh\left(\frac{y^+_{T_{pc}} - 12}{5}\right)\right]$: reflects pressure scrambling, (to be verified).
- Resulting time scale
  - $\frac{1}{\mathcal{T}} = max\left(\frac{1}{\mathcal{T}_c}, \frac{1}{\tau_\rho}\right)$

Eddy Viscosity

- The final form of the eddy viscosity
  - $\nu_t = f_{VD} C_2^2 \ell^2 \left\{ max \left[\frac{C_1}{C_2} \left(\frac{10}{Re_t} \right)^{1/2}, 1.0 \right] \right\}^2$

    $\times \frac{1}{C_4 \tau_I} max \left\{ \frac{1}{max \left[\frac{C_3}{C_4} \left(\frac{1}{Re_t} \right)^{1/2}, 1.0 \right]}, \frac{C_4 \tau_I}{\tau_\rho} \right\}$

    $= \underbrace{\frac{C_2^2}{C^4}}_{C_\mu} \underbrace{f_{VD} \{\cdots\}^2 \{\cdots\}}_{f_\mu} \frac{k^2}{\varepsilon}$
  - f_{VD}: van-Driest damping function
    - ✓ $f_{VD} = 1 - e^{-y^+/70}$

Determination of Constants C_i

- Near-wall behaviors and log-law
  - Time and length scales; $\mathcal{T}$ and $\mathcal{L}$
  - Mean and fluctuation properties: k, ε, U, u_i' and $\widetilde{u'v'}$

Near-Wall Behavior of Turbulence

- $u' = b_1 y + b_2 y^2 + \cdots$
- $v' = c_1 y^2 + c_2 y^3 + \cdots$
- $w' = d_1 y + d_2 y^2 + \cdots$
- $\widetilde{u'v'} = \widetilde{b_1 c_1} y^3 + \cdots$
- $\tilde{k} = \left[\left(\widetilde{b_1^2} + \widetilde{d_1^2}\right)/2\right] y^2 + \cdots$
- $\tilde{\varepsilon} = \nu \left(\frac{\partial\sqrt{\tilde{k}}}{\partial y}\right)^2 = \nu \left(\widetilde{b_1^2} + \widetilde{d_1^2}\right)$

Time Scale in ε - Equation

- The time scale was replaced with the newly defined one.

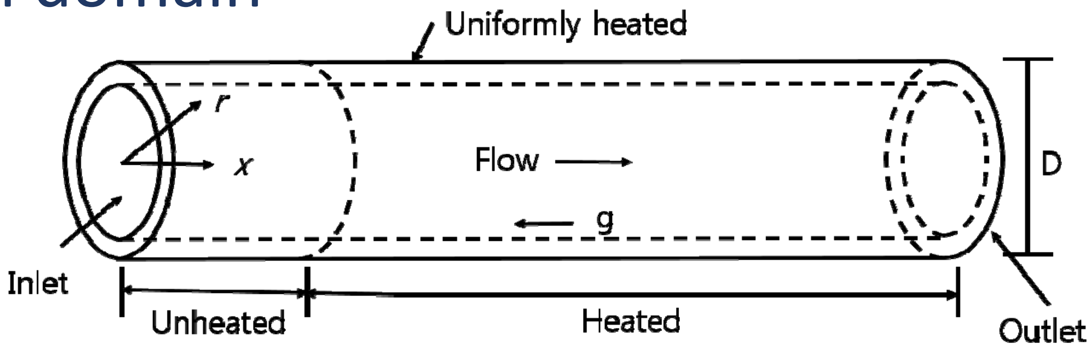
$$\nabla \cdot (\bar{\rho}\vec{v}\tilde{\varepsilon}) = \nabla \cdot \left[\left(\bar{\mu} + \frac{\bar{\mu}_t}{\sigma_\varepsilon}\right)\nabla\tilde{\varepsilon}\right] + C_{\varepsilon 1} f_{\varepsilon 1} \frac{1}{\mathcal{T}} (P_k + G_k) - \bar{\rho} C_{\varepsilon 2} f_{\varepsilon 2} \frac{\tilde{\varepsilon}}{\mathcal{T}}$$

Numerical Method

- Favre-averaged governing equations
- FVM
- $y_P^+ < 0.1$
- Semi-local scale: $y^+ = (\tau_w/\rho)^{1/2} y/\nu$
- Property-dependent turbulent Prandtl number, Pr_{t-v}
- Reynolds stress: Boussinesq approximation
- Turbulent heat flux: SGDH
- Buoyancy production in *k*-Eq. : GGDH

Domain & Boundary Conditions

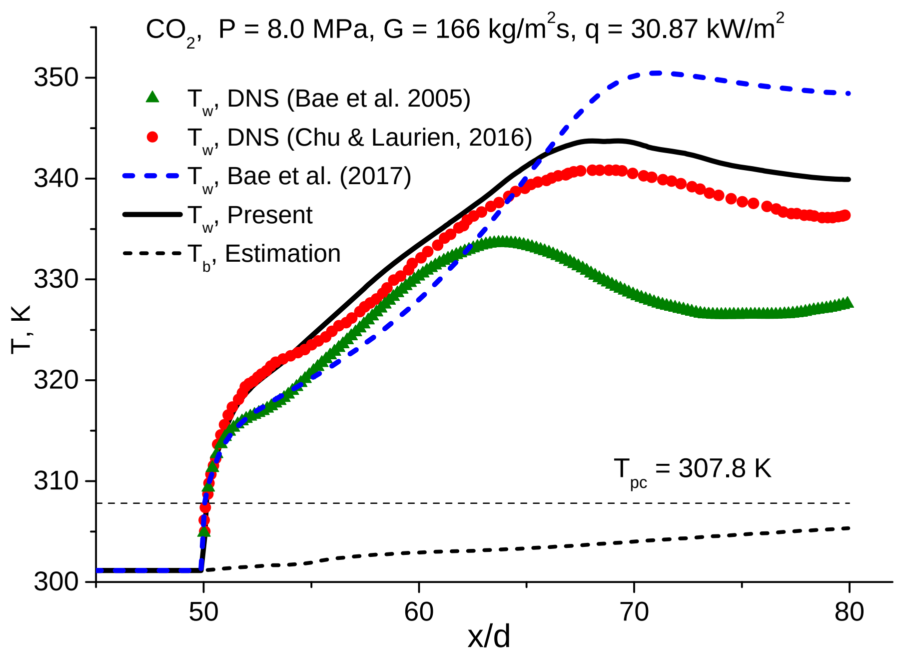
- Calculation domain

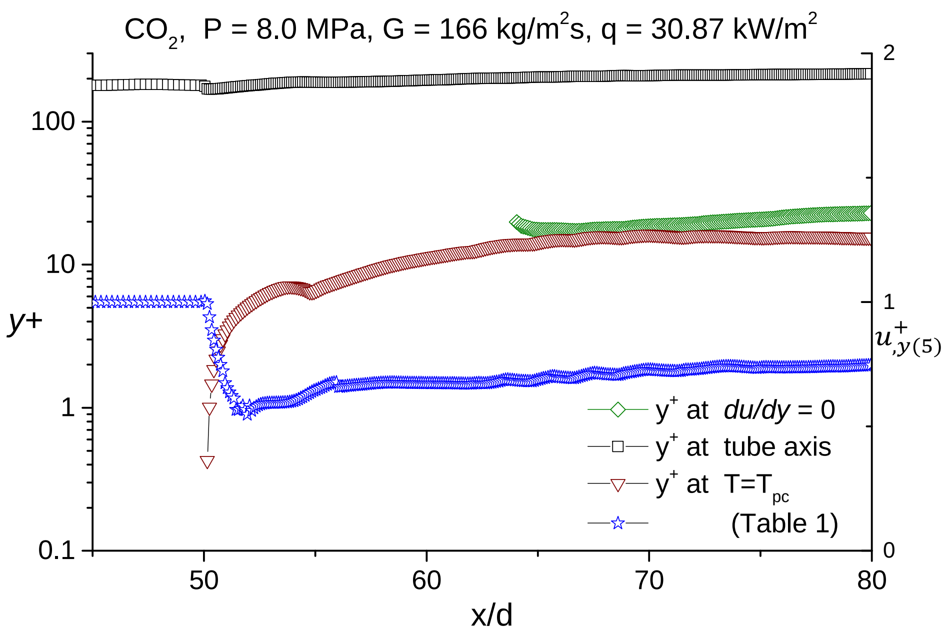


- Boundary conditions
  - Inlet: $\tilde{u}, \tilde{v}, \tilde{T}, \tilde{k}$ and $\tilde{\varepsilon}$ are given
  - Wall: $\tilde{u} = \tilde{v} = \tilde{k} = 0, \tilde{T} = T_w$ and $\tilde{\varepsilon} = 4\nu\tilde{k}_p / y_p^2$
  - Exit: $\partial\tilde{u}/\partial x = \partial\tilde{v}/\partial x = \partial\tilde{T}/\partial x = \partial\tilde{k}/\partial x = \partial\tilde{\varepsilon}/\partial x =$ Constants
  - Symmetry Line: $\partial\tilde{u}/\partial r = \partial\tilde{v}/\partial r = \partial\tilde{T}/\partial r = \partial\tilde{k}/\partial r = \partial\tilde{\varepsilon}/\partial r = 0$

Result - I

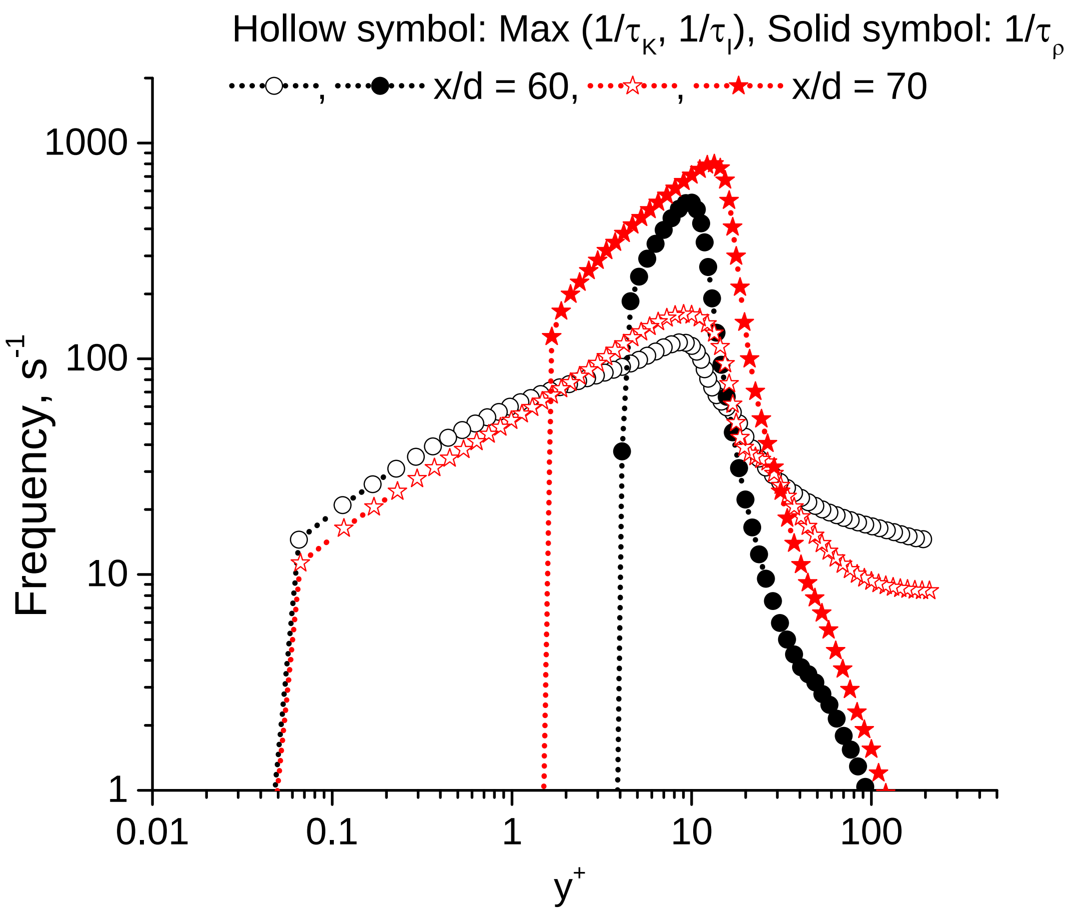
- Comparison with DNS Chu and Laurien (2016)
  - $y^+ < 15$
  - The influence of density gradient was not fully demonstrated.





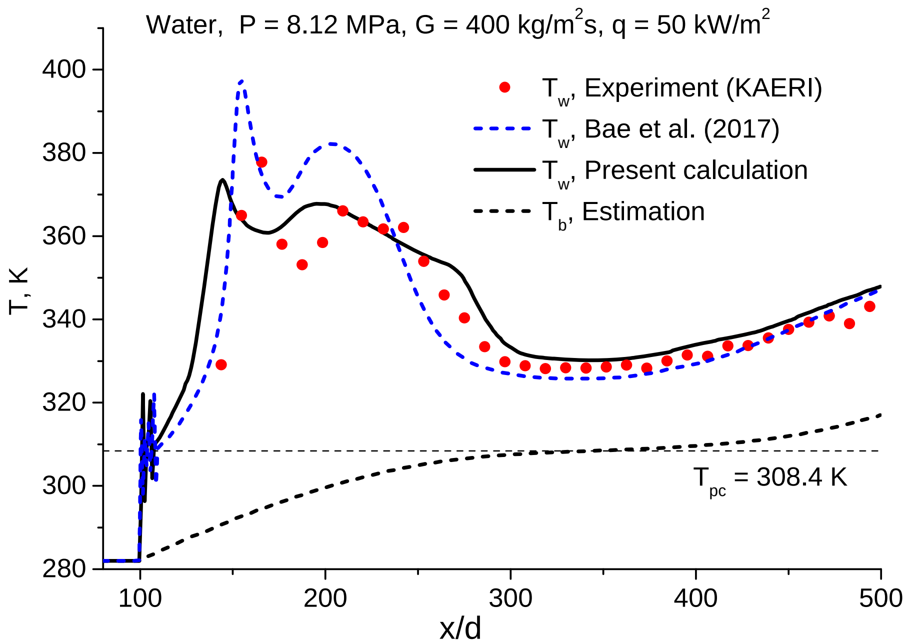
Result - I

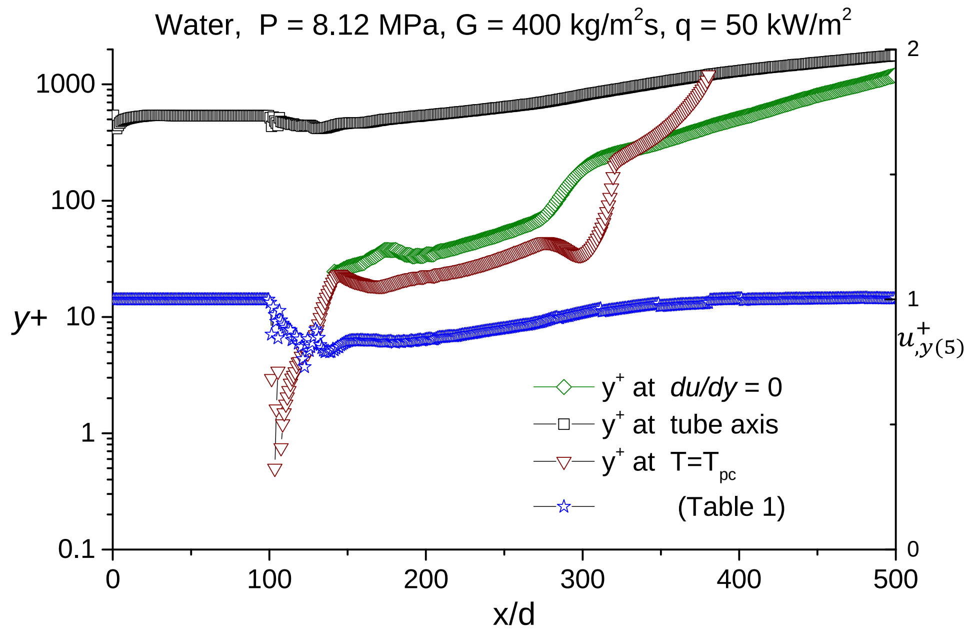
- Time scale



Result - II

- CO_2, P = 8.12 MPa, G = 400 kg/m^{2}s, q = 50 kW/m^2





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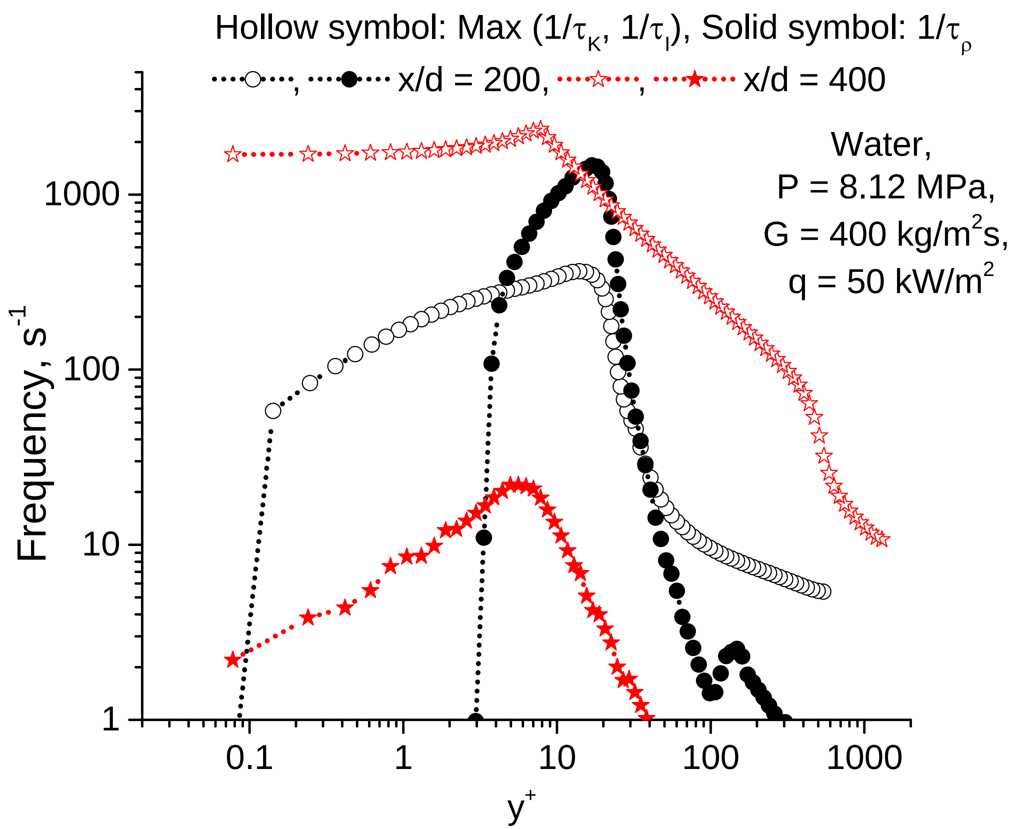
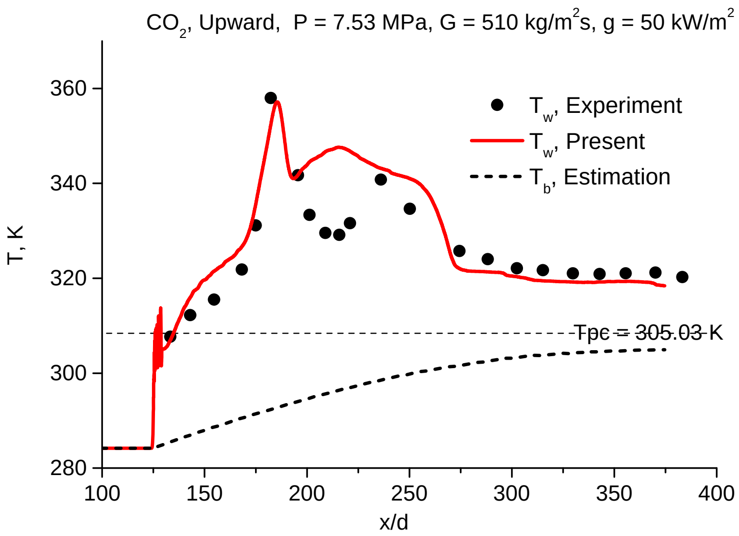
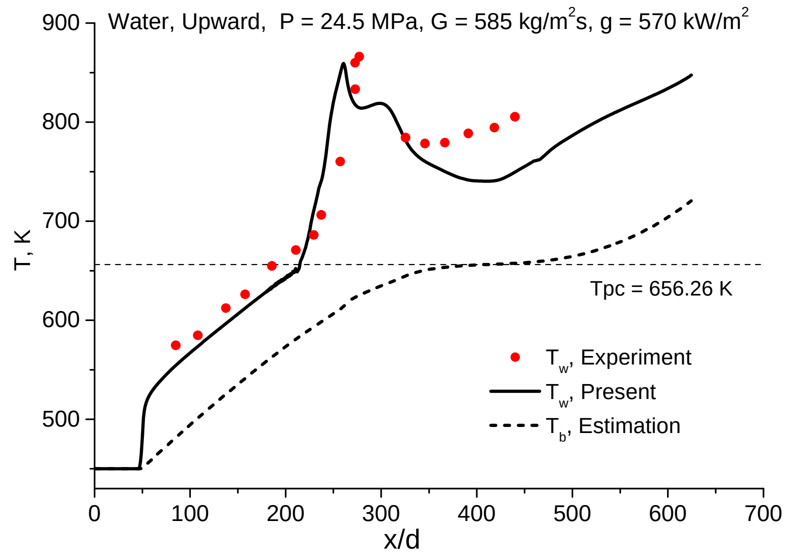
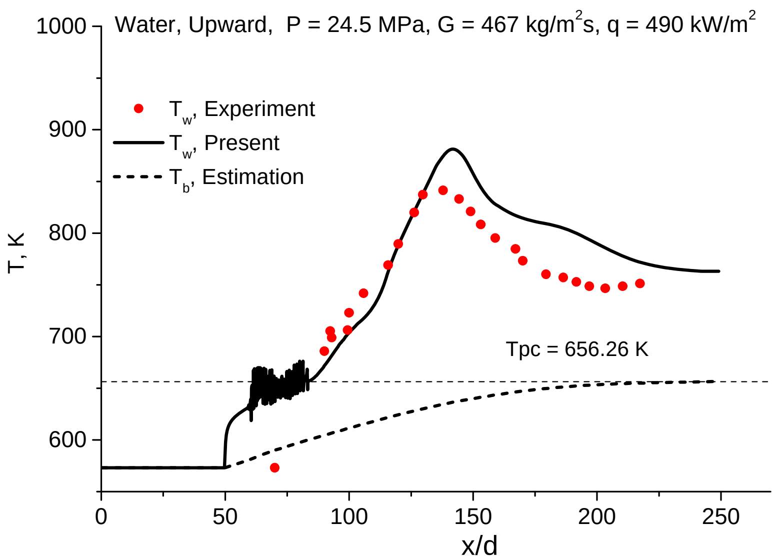


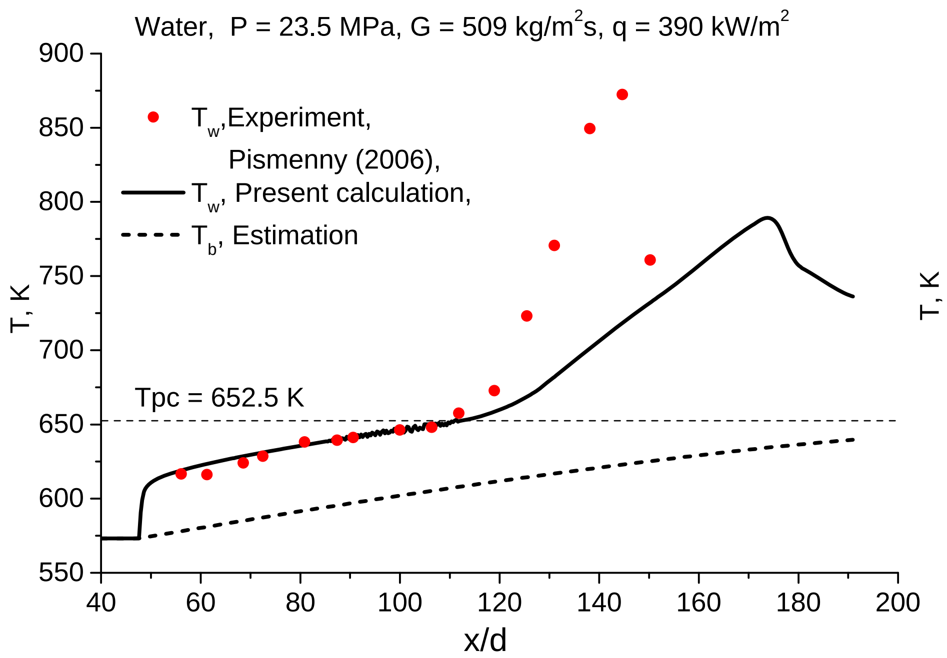
Additional Good Results

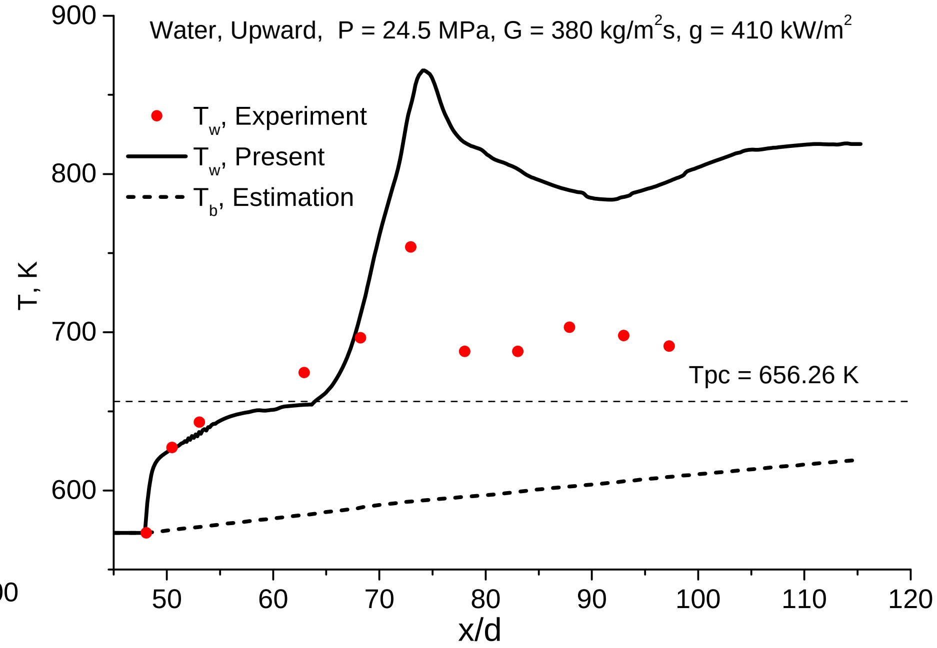






Cases of Poor Performance





Conclusions

- The eddy viscosity was defined by a combination of $\mathcal{L}$ and $\mathcal{T}$ instead of $\mathcal{U}$ and $\mathcal{L}$
- Influence of density gradient was accounted for with $\frac{1}{\rho}\frac{\partial\rho}{\partial y} \times \left(k^{1/2}u^*\right)^{1/2}$
- The model including the new ν_t and Pr_{t-v} successfully reproduced the experimental data.