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Modeling of Eddy Viscosity for Density-Varying Fluids

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1. Introduction

In the numerical simulation of a fluid at supercritical pressure flowing upward in vertical heated channels, difficulties arises due to the dramatic variation of physical properties, especially density, when fluid temperature crosses the pseudocritical temperature. The decrease in density due to temperature increase generates buoyancy force near-wall region, alters boundary layer velocity profile, and make the convectional turbulence models inappropriate, which are without exception based on typical turbulent boundary layer and homogeneous turbulence. However, the boundary layer in a supercritical fluid flow under strong buoyancy is quite different from the typical profile, and accordingly so turbulence properties are. When a velocity peak appears in the inner region (including log-law layer) the result of numerical simulation begins to deviates from the experimental data.

Many different turbulence models were tried to simulate the fluid flow under strong property (especially density) variation and failed without exception in reproducing experimental data. Few models were claimed to work for particular cases but failed to show a similar performance in other applications [1], [2]. This failure is considered to be an inherent deficiency of the presently available turbulence models including the Reynolds stress and v^2-f models.

In this paper a new eddy viscosity model is presented. The model adopts a different approach in using the length and time scale rather than the velocity and length scale. The new approach allowed an easy incorporation of the influence of density gradient.

2. Methods and Results

In this section firstly the method of modeling the eddy viscosity is presented and the short discussion of the turbulent Prandtl number follows.

2.1 Eddy Viscosity Model

The eddy viscosity, $\nu_t = C_\mu k^2/\varepsilon$, has a dimension of $m^2 s^{-1}$, and was commonly modeled to be proportional to the product of velocity, $k^{1/2}$, and length scales, $k^{3/2}/\varepsilon$. The proportionality constant, C_μ, was determined by evaluating the homogeneous turbulence data. In the present study the eddy viscosity was treated differently to be proportional to the product of square of the length scale, $\mathcal{L}$, and time scale, $\mathcal{T}$.

$$\bar{\nu}_t \propto \mathcal{L}^2/\mathcal{T} \tag{1}$$

where $\mathcal{L}$ and $\mathcal{T}$ are defines as the maximum of the Taylor micro scale, λ, and the integral length scale, ℓ.

$$\mathcal{L} = max(C_1\lambda, C_2\ell) \tag{2}$$

In the same manner the time scale is obtained as the maximum of the Kolmogorov time scale, τ_K, and the integral time scale, τ_I.

$$\mathcal{T}_c = max(C_3\tau_K, C_4\tau_I) \tag{3}$$

In reviewing the DNS data, it was identified that the density gradient clearly demonstrates its influence on the turbulence and Reynolds stress. Intuitively, it is natural to consider the density gradient interacts with turbulence, and the interaction can be expressed as the product of density gradient and characteristic velocity, $\mathcal{U}$. The product of $(\partial\rho/\partial y)/\rho$ and $\mathcal{U}$ has a dimension of inverse time scale, or frequency. A large value of $\mathcal{U}(\partial\rho/\partial y)/\rho$ corresponds to a larger eddy viscosity and more active mixing. In the buffer layer there are two outstanding velocity scale, the friction velocity and the square root of the turbulence kinetic energy. In the present study a mixed velocity, $\sqrt{u^* k^{1/2}}$, was used as the velocity scale. The density-gradient time scale, $\mathcal{T}_\rho$, can be expressed as

$$\frac{1}{\mathcal{T}_\rho} = C_5 \frac{1}{\rho}\frac{\partial\rho}{\partial y}\sqrt{u^* k^{1/2}} \tag{4}$$

Incorporating the density-gradient time scale, the time scale can be expressed as follows:

$$\frac{1}{\mathcal{T}} = max\left(\frac{1}{\mathcal{T}_c}, \frac{1}{\mathcal{T}_\rho}\right) \tag{5}$$

The constants C_i, summarized in Table 1, were determined so that the time and length scales converge to the near-wall or outer boundary layer condition.

Table 1. The values of constants appeared in the time and length scale

Constants	C_1	C_2	C_3	C_4	C_5
Value	$\kappa\alpha$	0.3	6.0	1.0	1.67α*

*α is the value of $\partial u^+/\partial y^+$ at $y^+ = 5$.

Finally, the eddy viscosity is expressed as

$$\bar{\nu}_t = f_{VD}\frac{\mathcal{L}^2}{\mathcal{T}} \tag{6}$$

where f_{VD} is the commonly-used van-Driest type damping function.

2.2 Turbulent Prandtl number

In order to incorporate the influence of property variation the following turbulent Prandtl number [3] was used.

$$Pr_t = \sigma_t - \sigma_t \left[1 - \frac{1 + min\left(\frac{\tilde{u}}{\bar{\rho}}\left|\frac{\bar{\rho}_{,y}}{\tilde{u}_{,y}}\right|, 1.0\right)}{1 + \frac{\tilde{T}}{\bar{\rho}}\left|\frac{\bar{\rho}_{,y}}{\tilde{T}_{,y}}\right| + \frac{\tilde{T}}{\bar{C}_p}\left|\frac{\bar{C}_{p,y}}{\tilde{T}_{,y}}\right|} \right] f_1 f_2 \qquad (7)$$

$\phi_{,y}$ represents the partial differentiation of $\partial\phi/\partial y$.

3. Results and Discussions

An axisymmetric flow through a vertically-oriented tube at a supercritical pressure was simulated. The flow conditions are summarized in Table 2.

Table 2. Configuration of the present case

Medium	CO_2	Total/Heated length (x/d)	80/30
Direction	Upward	T_{in} (K)	301.15
G (kg/m^{2}s)	166	P (MPa)	8.0
q (kW/m^2)	30.87	T_{pc} (K)	307.8
d (mm)	2	Re_{in}	5486

The wall temperature distribution obtained from the present study is compared with the DNS data [4]. The present results closely follow the DNS data indicating the reasonableness of the present method. The maximum difference in temperature was smaller than 4°C.

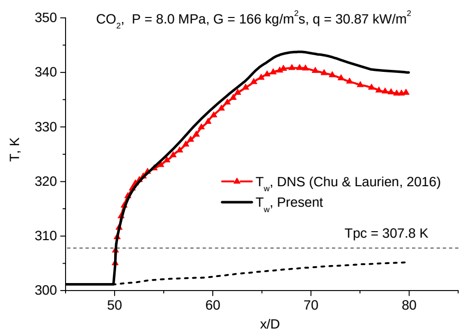

Figure 1. Wall temperature distribution

As is shown in Figure 2, the value of y^+ at $T = T_{pc}$ remains smaller than 20, which includes the viscous sublayer and most part of the buffer layer. In this region the density gradient interacts with the strong turbulence generation, and the existing turbulence models are susceptible to failure since they are not modeled for this situation. Thus, a special treatment proposed in this paper is needed. The value of κα is also shown in Figure 2, which accounts for the existence of $\partial u/\partial y = 0$ associated with the increase in length scale.

The inverse time scale (frequency) is shown in Figure 3. Higher frequency (small time scale) corresponds to stronger mixing. The large value of frequency in the buffer region is evident indicating the necessity of consideration of the time scale related to density gradient.

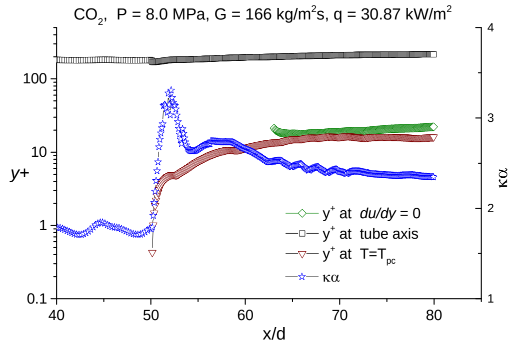

Figure 2. Variation of y^+.

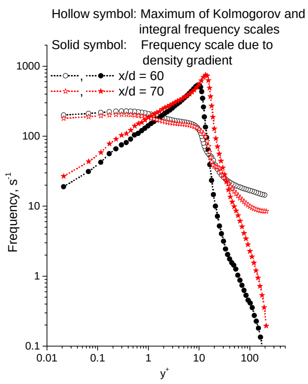

Figure 3. Variation of frequency

4. Conclusions

The eddy viscosity was modeled using length and time scale instead of commonly-used velocity and length scales. The additional time scale associated with density-gradient was developed. The governing equations with the newly developed eddy viscosity successfully reproduced the DNS data indicating the plausibility of the present method,

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