A Modification of the Linear Theory to Predict Sloshing in the Spent Fuel Pool under High Frequency Seismic Conditions

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1. Introduction

Spent fuel pools of nuclear power plants are filled with fluid(coolant). Fluid shows different dynamic behaviors according to its depth and experiences sloshing under an external excitation[1]. It has been reported that fluid sloshing, subject to seismic loads, could be a reason of serious accidents. Therefore, sloshing dynamics is critical in the seismic evaluation of nuclear facilities. Several analytical and numerical methods, including analytical solutions, computational fluid dynamics (CFD), and finite element analysis (FEA), have been used to predict fluid behaviors.

A linear theory introduced by Faltinsen [2] has been used to predict the behaviors of fluid, as a response to external excitations including seismic waves or ship motions, filling in pools or tanks in mechanical engineering or ocean engineering fields. Previous studies have shown that the analytical solutions using the linear theory are well matched with experimental and numerical results under low frequency excitations. The theory excludes nonlinear factors in fluid dynamics [3, 4].

Free-surface height consists of impulsive and convective components in the linear solution. The amplitude of convective components decreases as the excitation frequency increase, while the amplitude of impulsive component decreases. Thus, excitation component could be mainly shown in the linear solution of the free-surface height under a high-frequency excitation, while sloshing behaviors is affected by convective component. Therefore, the linear theory should be modified to predict free-surface height in high frequency excitation conditions. In this study, the linear theory is modified to predict the free-surface in a spent fuel pool model and compared the calculated results using the modified theory to the numerical results using computational fluid dynamics.

2. Linear theory

In this study, two-dimensional rectangular pool is considered (Fig. 1). Free-surface height in the rectangular pool can be calculated with Eq. (1). Here, ξ_1 and ξ_2, which are calculated from Eq. (2) and (3), are the excitation frequency and the natural frequency of the water, respectively.

$$\xi(x,t) = \xi_1(x,t) + \xi_2(x,t) \tag{1}$$

$$\xi_1(x,t) = \frac{A}{g}\left(x\omega^2 + \sum_{n=0}^{\infty} C_n \cdot \omega \cdot \sin(k_n x)\right)\sin(\omega t) \tag{2}$$

$$\xi_2(x,t) = -\frac{A}{g}\sum_{n=0}^{\infty} \omega_n\left(C_n + \frac{H_n}{\omega^2}\right)\sin(k_n x)\sin(\omega_n t) \tag{3}$$

Where,

$$\omega_n^2 = g \cdot k_n \cdot \tanh(k_n h) \tag{4}$$

$$k_n = \frac{2n+1}{L}\pi \tag{5}$$

$$H_n = \omega^3\left(\frac{4}{L}\right)\left(\frac{(-1)^n}{k_n^2}\right) \tag{6}$$

$$C_n = \frac{H_n}{\omega_n^2 - \omega^2} \tag{7}$$

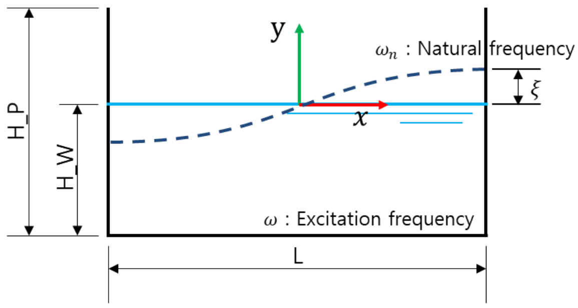


Fig. 1. A rectangular pool filled with water.

A custom Matlab code (Matlab 2018a, MathWorks, Inc., Natick, MA, USA) was developed based on the linear theory and calculated sloshing height under two excitation conditions with low and high frequencies.

For the low frequency excitation, the sloshing height was predicted in the same condition with the previous literature [3]. Sloshing height of the water at the left wall were calculated using the established linear theory and was compared to the previous experimental and analytical results. Here, the width and height of the t are 0.96 m and 1.0 m, respectively. Depth of the water is 0.624 m. Excitation frequency and amplitude are 0.7017 Hz and 0.005 m, respectively.

For the high frequency excitation, the width and height of the pool, and, depth of the water were set to be 0.52 m, 0.835 m, and 0.61 m, respectively. Excitation frequency and amplitude were 10 Hz and 0.005 m. The sloshing height was calculated on the left wall of the pool and compared to the numerical results using CFD software Ansys Fluent (Ansys Inc., Canonsburg, PA, USA).

The calculated sloshing height using the linear theory well matched to the experiments in the low frequency excitation condition [3], but slight differences are found around 5, 10, and 16 second at which the sloshing height is low (Fig. 2).

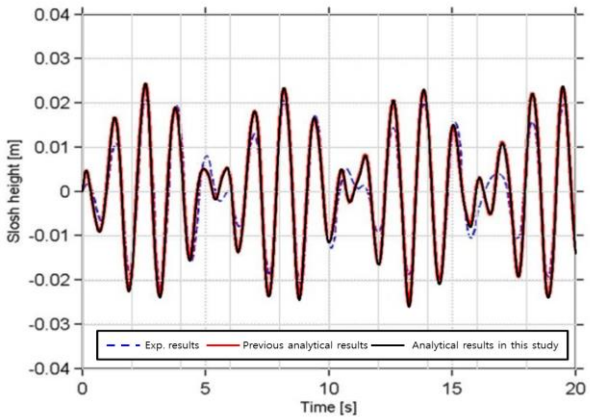


Fig. 2 Comparison of the analytical sloshing height to the experimental results in the low frequency excitation condition

However, in the high frequency excitation condition, low amplitude oscillation of the water with the frequency same as the excitation frequency was predicted in the analytical result, while the numerical result showed that first mode of the natural frequency of the water was a major component of the sloshing motion (Fig. 3).

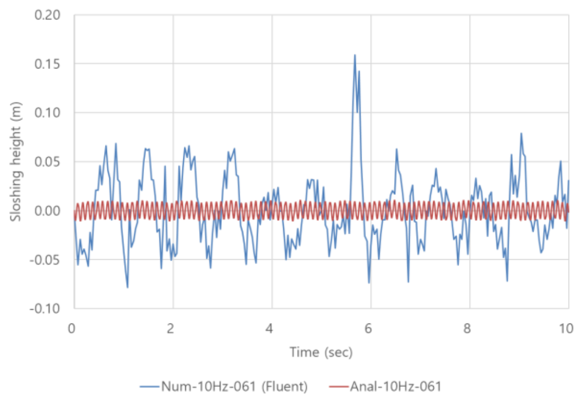


Fig. 3 Comparison of the analytical sloshing height to the numerical result in the high frequency excitation condition

3. Modified linear theory

Wei et al. have asserted that convective component of the sloshing is manly observed in high frequency excitation conditions. Goudarzi and Sabbagh-Yazdi explained that the first mode of the filled fluid is the major behavior of the convective component of the sloshing [3]. Fig. 3 also showed that the first mode of the water filled in the pool mainly occurred with the high amplitude and excitation frequency with low amplitude also occurred in the sloshing height.

Therefore, the authors have calculated a scale factor for the first mode sloshing behavior of the water. The variables calculated in Eq. (6), and (7) were calculated from low frequency to high frequency, and linear correlation were found between excitation frequency and two variables from Eq. (6) and (7). The calculated scale factor based on the correlation was then adapted to the modified linear theory method (Fig. 4).

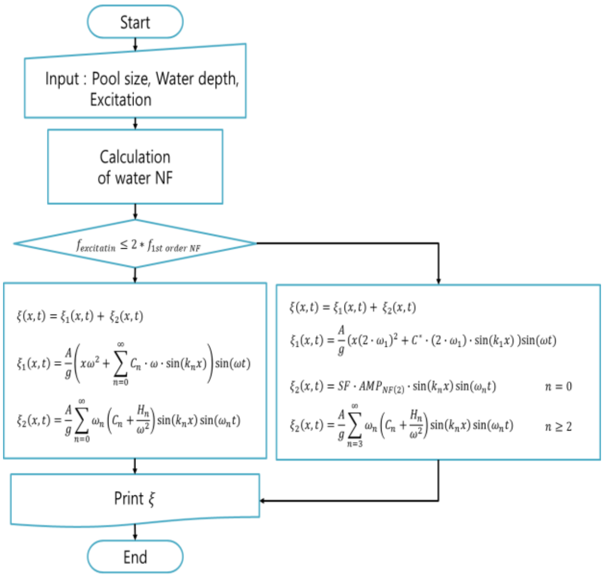
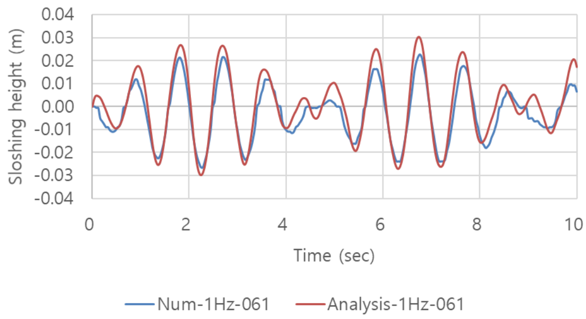


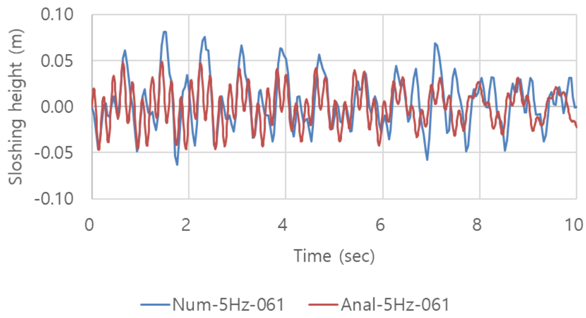
Fig. 4 Computational algorithm of the modified linear theory to calculate height of the free-surface in the high frequency excitation condition

The sloshing heights were calculated in various excitation conditions and compared to the numerical results. The same testing conditions with the high frequency excitation condition for the original linear theory was used, but the frequency varied to 1 Hz, 5 Hz, and 10 Hz.

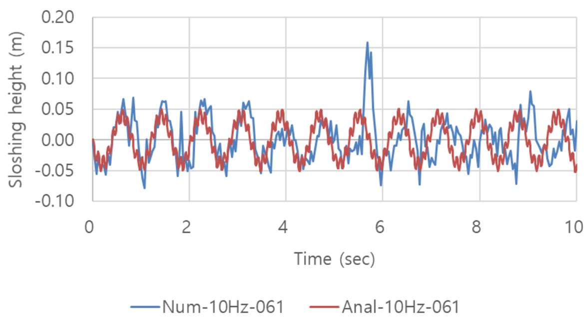
The modified linear theory well predicted sloshing height in all testing conditions. In case of the 1 Hz excitation condition, the original linear theory was used and well matched with the numerical results on the aspects of amplitude as well as frequency (Fig. 5 (a)). For the excitation frequency of 5 Hz, the modified linear theory well predicted the sloshing height (Fig. 5 (b)), while the original theory showed different sloshing height compared to the numerical results (Fig. 3). The first mode sloshing height was mainly shown in the 10 Hz excitation frequency in both numerical and analytical results (Fig. 5 (c)).



(a) Excitation frequency = 1 Hz



(b) Excitation frequency = 5 Hz



(c) Excitation frequency = 10 Hz

Fig. 5 Comparison of the analytical sloshing height using modified linear theory to the numerical result in various excitation frequency, (a) 1 Hz, (b) 5 Hz, and (c) 10 Hz

7. Discussions

The linear theory introduced by Faltinsen [2] has been widely used to estimate fluid behavior subject to external stimulation including seismic load. Even though the theory estimated the shape of the free surface in the low frequency excitation conditions, it doesn't in the high frequency condition. Thus, the authors suggested the modified linear theory for estimating free surface in the high frequency excitation conditions.

The modified linear theory could well predict the sloshing height of the water not only in the low frequency excitation conditions but also in the high frequency conditions. However, even the modified linear theory could not be described the delayed motion and abnormal peaks, shown in the numerical results, which might be occurred by the nonlinear factors of the water. Despite these limitations of the modified linear theory, this method could be highly useful for calculating sloshing height subject to external excitation as a simple calculation method.

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