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Minimum-Jerk Trajectory Generation for Control Applications

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1. Introduction

Control applications such as reactor operation or remote handling in decommissioning of nuclear facilities involve trajectory generation. For example, we want the reactor power change following smooth trajectories without abrupt changes in control reactivity or we want to move remote handling manipulators smoothly for safe handling of hazardous radioactive materials. These trajectory generation problems can be modeled as a point-to-point motion problem in control theory.

It is well known that, in optimal control theory, simple trapezoidal velocity models can achieve time-optimal motions. Fig.1 shows a trapezoidal velocity model. As shown in Fig. 1, when the velocity reaches the maximum value at the time instant t_1, ,the acceleration jumps from a non-zero constant value to zero. The rate of changes in the acceleration is called jerk. The jumps also occur at other time instants, when there are non-zero jerks. In trapezoidal velocity model, the discontinuities of the acceleration are caused by the jerks exhibiting instant but infinite values. The issues with the trapezoidal velocity profiles include overshoots, and excitation of unnecessary vibration modes in control systems; you would feel uncomfortable if you sit in a car that accelerates or decelerates suddenly.

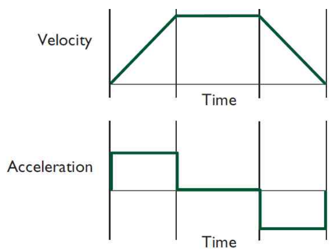

Fig. 1. Trapezoidal velocity and its acceleration profile

The s-curve was introduced to remedy the short comings' of the trapezoidal velocity model [1]. Most of the research on s-curve or trapezoidal motion profiles in motion control community has been focusing on time-optimality of the motion. This is partially due to the majority of the motion control applications are in manufacturing where productivity and efficiency are the priorities. This study, however, summarizes the general model of polynomial s-curve motion profiles focusing on the minimization of jerk so that the resulting profiles exhibit smooth and safe motion.

2. Trajectory Modeling

This section summarizes the work of Nguyen et al.[1], which introduced the generalization of s-curve model. Nguyen proposed to use the concept of "template." The template of a model is defined as the highest order derivative of position profile whose peak value is finite.

2.1 2nd order Model

In trapezoidal velocity model (Fig.2), its position is determined by 2nd order polynomials. So it is sometimes called 2nd order polynomial model. As mentioned earlier, the jerk in trapezoidal model exhibits infinite value when the acceleration makes a jump in its value, which is potentially problematic for physical control systems. The trapezoidal profile has three phases:

$$a = \begin{cases} a_{m\,ax} & t_0 \le t \le t_1 \\ 0 & t_1 < t < t_2 \\ -a_{m\,ax} & t_2 \le t \le t_3 \end{cases}$$

Since the acceleration is finite and the jerk is infinite, the template, T_2, of 2nd order model is the acceleration profile given above:

$$T_2 = \begin{cases} A_{peak} & t_0 \le t \le t_1 \\ 0 & t_1 < t < t_2 \\ -A_{peak} & t_2 \le t \le t_3 \end{cases}$$

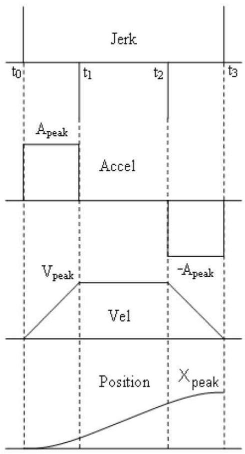

Fig. 2. Trapezoidal trajectory model[1]

2.2 3rd order Model

For polynomial models whose orders are higher than 2, the jerks exhibit finite values and their velocity profiles are smooth during motions [1]. The 3rd order model has seven phases:

$$J = \begin{cases} J_1 > 0, t_0 \le t \le t_1 \\ J_2 = 0, t_1 \le t \le t_2 \\ J_3 < 0, t_2 \le t \le t_3 \\ J_4 = 0, t_3 \le t \le t_4 \\ J_5 < 0, t_4 \le t \le t_5 \\ J_6 = 0, t_5 \le t \le t_6 \\ J_7 > 0, t_6 \le t \le t_7 \end{cases}$$

In this case, the jerk of the model has finite amplitude and the template, by using the template T_2, is as follows:

$$T_3 = \begin{cases} T_2 & t_0 \le t \le t_3 \\ -T_2 & t_4 \le t \le t_7 \end{cases}$$

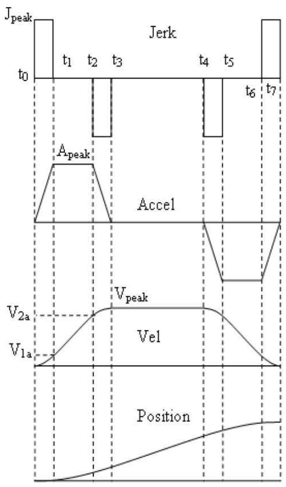

Fig. 3. 3rd order trajectory model[1]

2.3 n^{th} order Model

By induction, the template of nth order s-curve model can be expressed in a recursive manner:

$$T_n = \begin{cases} T_{n-1} & t_0 \le t \le t_{2^{n-1}-1} \\ -T_{n-1} & t_{2^{n-1}} \le t \le t_{2^n-1} \end{cases}$$

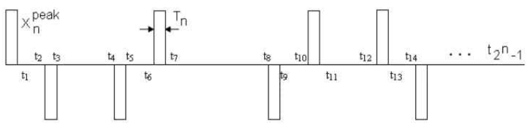

Fig. 4. Template of the nth order s-curve model[1]

3. Minimum-Jerk Trajectory

Our interest lies in finding smooth trajectories with minimum jerk. More specifically, we want to find a family of trajectories that minimizes the following cost functional:

$$J_t(p(t)) := \frac{1}{2}\int_{t_i}^{t_f} \dddot{p}^2(\tau)d\tau = \frac{1}{2}\int_{t_i}^{t_f} \mathcal{L} d\tau$$

where $\dddot{p(t)}$ denotes the jerk of "position" variable, and $\mathcal{L} := \dddot{p}^2$

The optimization of quadratic cost functions is one of the most well studied area [2]. For any function p(t) which is sufficiently differentiable in the interval $t_i \le t \le t_f$, the necessary condition of optimality suggests that p(t) is a solution of the following Euler equation [2].

$$\frac{\partial \mathcal{L}}{\partial p} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial p}\right) \cdots + (-1)^n \frac{d^n}{dt^n}\left(\frac{\partial \mathcal{L}}{\partial p^{(n)}}\right) = 0$$

Thus we have

$$\frac{d^6 p}{dt^6} = 0$$

which implies that, among the general nth order s-curve trajectories, following 5th order polynomial form will exhibit minimum-jerk [2]:

$$p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

3. Example

The theory developed in the previous section is applied to a reactor power tracking scenario. The Palo Verde power plant model is used in simulations [3]. Control efforts for trapezoidal trajectory and minimum-jerk trajectory are compared.

3.1 Trajectory for Power Increase

The example scenario assumes a power increase from 70% of rated power to 100% of rated power. The average rate of power increase is set to 3%/sec.

To determine the parameters for the 5th order polynomial, six boundary conditions are required:

$$P(t_0) = P_{20\%}$$
$$\dot{P}(t_0) = 0$$
$$\ddot{P}(t_0) = 0$$
$$P(t_0 + \Delta T) = P_{100\%}$$
$$\dot{P}(t_0 + \Delta T) = 0$$
$$\ddot{P}(t_0 + \Delta T) = 0$$

where $\Delta T = \frac{P_{100\%} - P_{20\%}}{0.05\ P_{100\%}}$

Trapezoidal model requires the determination of three parameters: acceleration (a_{max}) acceleration & deceleration time (ΔT_{AD}), constant velocity time (ΔT_{CV}). These parameters are calculated to meet the following constraints,

$$P_{100\%} - P_{20\%} = \int_{t_0}^{t_3} \dot{P}(\tau)d\tau$$

$$\Delta T = \Delta T_{CV} + 2 \cdot \Delta T_{AD}$$

Since there are only two constraints for three parameters, we can either add a constraint or fix one of the parameters (possibly by trial and error.) One method to fix ΔT_{CV} is to set ΔT_{CV} equal to the time between two time instances where the rate of change of acceleration is zero, i.e.

$$\dddot{P}(t_1) = 0, \dddot{P}(t_2) = 0$$

$$\Delta T_{CV} = t_2 - t_1 = \frac{\sqrt{(2a_4)^2 - 10a_5 \cdot a_3}}{5a_5}$$

3.2 Simulation Result

Fig.4 shows three difference power reference trajectories; red, green, and blue line represent 5th order, trapezoidal and linear profile, respectively.

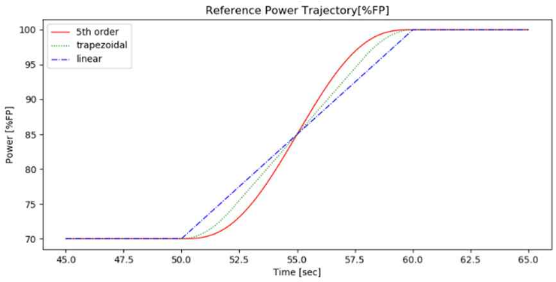

Fig. 4. Comparison of power reference trajectories

Fig.5 shows the results of power reference tracking, using the tracking control method proposed in [3]. Control reactivity inputs are compared in Fig.6. As expected from the theory, 5th order power reference trajectory results in smooth control reactivity input.

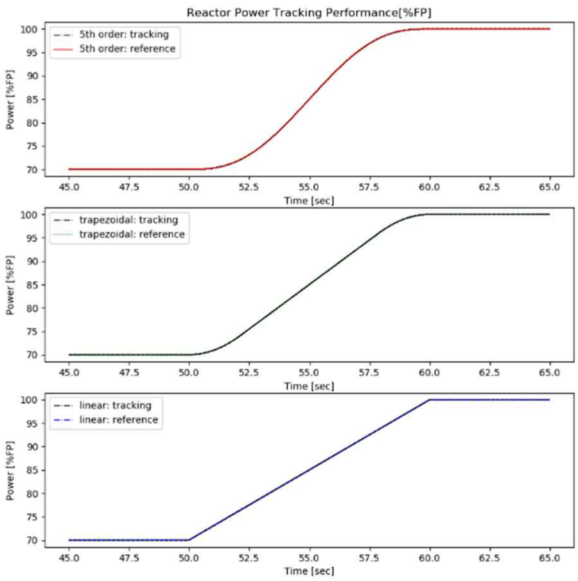

Fig. 5. Tracking of power reference trajectories

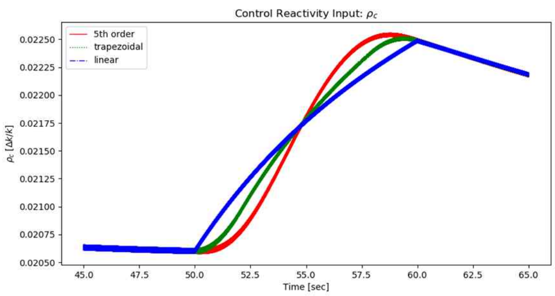

Fig. 6. Comparison of control reactivity input

4. Conclusion

This study summarizes the general model of polynomial s-curve motion profiles focusing on the minimization of jerk so that the resulting profiles exhibit smooth and safe motion. In reactor control application, the jerk minimization of power reference trajectory can result in the minimization of wear and tear in the control rod drive mechanism. Simulations show that the tracking of minimum-jerk power reference trajectory exhibits a smoother control reactivity profile.

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[3] S. Joo, J. Lee, and G. Park, “Robust and Optimal Nuclear Reactor Power Tracking using Integral Sliding Mode Control”